Stochastic particle production in the early universe

Marcos A. G. García

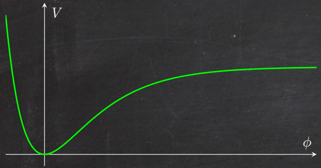
Rice University

Based on 1705.????, with

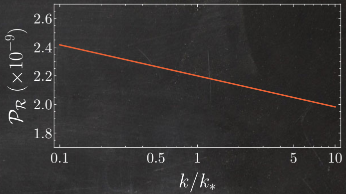
M. Amin, O. Wen (Rice U.)
H. Xie (U. Wisconsin)

Motivation

Cosmological inflation is the early period of accelerated expansion, $a \sim e^{Ht}$

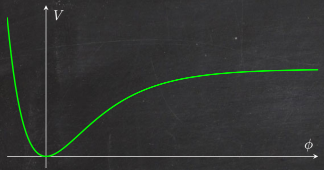


$$\xrightarrow[g \to g + \delta g]{\phi \to \phi + \delta\phi}$$

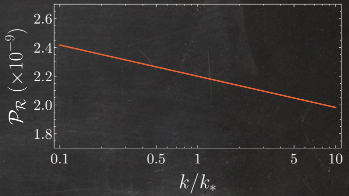


Motivation

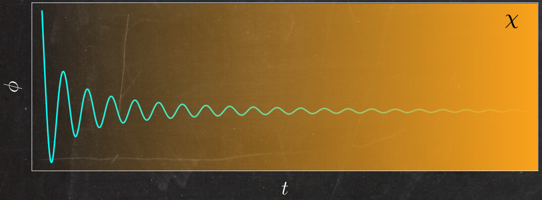
Cosmological inflation is the early period of accelerated expansion, $a \sim e^{Ht}$

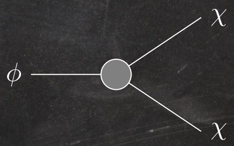


$$\xrightarrow[]{\substack{\phi \to \phi + \delta\phi \\ g \to g + \delta g}}$$



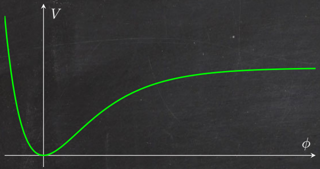
When inflation ends, the Universe reheats…



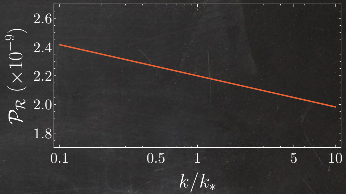


Motivation

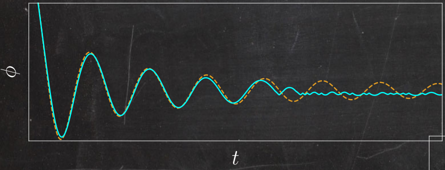
Cosmological inflation is the early period of accelerated expansion, $a \sim e^{Ht}$



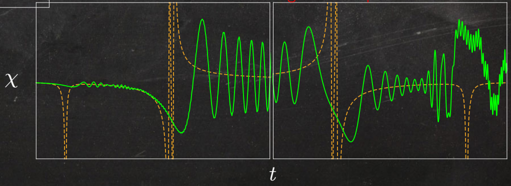
$$\xrightarrow[\;g \to g+\delta g\;]{\phi \to \phi+\delta\phi}$$



When inflation ends, the Universe reheats…in a potentially complicated way



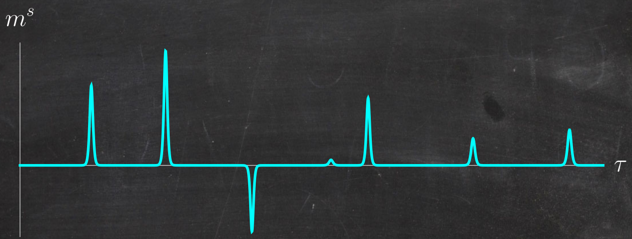
Ellis, G, Nagata, Nanopoulos, Olive 2017



Stochastic Particle Production

Consider $N_{\rm f}$ coupled (scalar) fields. Assume the evolution of fluctuations contains localized non-adiabatic events with random strengths at random intervals, and that the fields are otherwise free

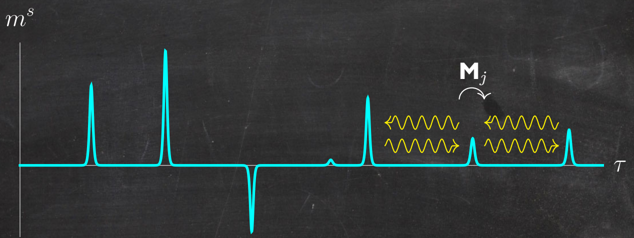
$$\left[\mathbb{1}\,\partial_\tau^2 + \boldsymbol{\omega}^2 + \mathbf{m}^s(\tau)\right]\cdot\boldsymbol{\chi}(\tau,\mathbf{k}) = 0\,, \qquad \omega_a^2 = k^2 + m_a^2$$



Stochastic Particle Production

Consider $N_{\rm f}$ coupled (scalar) fields. Assume the evolution of fluctuations contains localized non-adiabatic events with random strengths at random intervals, and that the fields are otherwise free

$$\left[\mathbb{1}\,\partial_\tau^2 + \boldsymbol{\omega}^2 + \mathbf{m}^s(\tau)\right]\cdot \boldsymbol{\chi}(\tau,\mathbf{k}) = 0\,, \qquad \omega_a^2 = k^2 + m_a^2$$



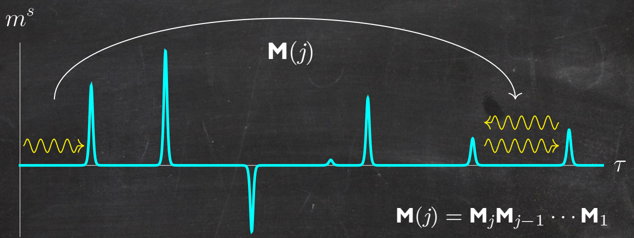
After the j-th event,

$$\chi_j^a(\tau) \equiv \frac{1}{\sqrt{2\omega_a}}\left[\beta_j^a e^{i\omega_a\tau} + \alpha_j^a e^{-i\omega_a\tau}\right]\,, \qquad \begin{pmatrix}\boldsymbol{\beta}_j\\ \boldsymbol{\alpha}_j\end{pmatrix} = \mathbf{M}_j \begin{pmatrix}\boldsymbol{\beta}_{j-1}\\ \boldsymbol{\alpha}_{j-1}\end{pmatrix}$$

Stochastic Particle Production

Consider $N_{\rm f}$ coupled (scalar) fields. Assume the evolution of fluctuations contains localized non-adiabatic events with random strengths at random intervals, and that the fields are otherwise free

$$\left[\mathbb{1}\,\partial_\tau^2 + \boldsymbol{\omega}^2 + \mathbf{m}^s(\tau)\right]\cdot\boldsymbol{\chi}(\tau,\mathbf{k}) = 0\,, \qquad \omega_a^2 = k^2 + m_a^2$$



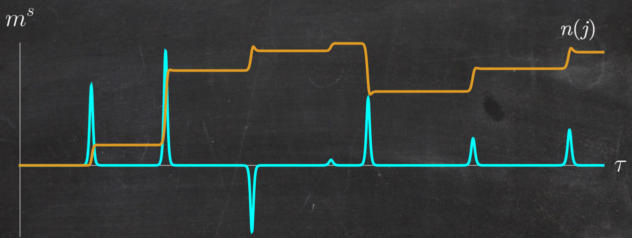
After the j-th event,

$$\chi_j^a(\tau) \equiv \frac{1}{\sqrt{2\omega_a}}\left[\beta_j^a e^{i\omega_a\tau} + \alpha_j^a e^{-i\omega_a\tau}\right]\,, \qquad \begin{pmatrix}\boldsymbol{\beta}_j\\ \boldsymbol{\alpha}_j\end{pmatrix} = \mathbf{M}(j)\begin{pmatrix}\boldsymbol{\beta}_0\\ \boldsymbol{\alpha}_0\end{pmatrix}$$

Stochastic Particle Production

Consider N_{f} coupled (scalar) fields. Assume the evolution of fluctuations contains localized non-adiabatic events with random strengths at random intervals, and that the fields are otherwise free

$$\left[\mathbb{1}\,\partial_\tau^2 + \boldsymbol{\omega}^2 + \mathbf{m}^s(\tau)\right] \cdot \boldsymbol{\chi}(\tau, \mathbf{k}) = 0\,, \qquad \omega_a^2 = k^2 + m_a^2$$



A random walk (with drift) for the occupation number

$$n_a(j) = \frac{1}{2\omega_a}\left(|\dot{\chi}_j^a|^2 + \omega_a^2|\chi_j^a|^2\right) - \frac{1}{2} = |\beta_j^a|^2$$

Stochastic Particle Production

Consider $N_{\rm f}$ coupled (scalar) fields. Assume the evolution of fluctuations contains localized non-adiabatic events with random strengths at random intervals, and that the fields are otherwise free

$$\left[\mathbb{1}\,\partial_\tau^2 + \boldsymbol{\omega}^2 + \mathbf{m}^s(\tau)\right]\cdot\boldsymbol{\chi}(\tau,\mathbf{k}) = 0\,, \qquad \omega_a^2 = k^2 + m_a^2$$

Randomness and non-adiabaticity are encoded in $\mathbf{M}$ $\Rightarrow$ define $P(\mathbf{M};\tau)$

$$\partial_\tau P(\mathbf{M};\tau) = -\partial_{\mathbf{M}}\left[\frac{\langle\delta\mathbf{M}\rangle_{\mathbf{M}_2}}{\delta\tau}P(\mathbf{M};\tau)\right] + \frac{1}{2!}\partial^2_{\mathbf{M}}\left[\frac{\langle\delta\mathbf{M}^2\rangle_{\mathbf{M}_2}}{\delta\tau}P(\mathbf{M};\tau)\right]$$

A general transfer matrix can be parametrized as

$$\mathbf{M} = \begin{pmatrix} \mathbf{u} & 0 \\ 0 & \mathbf{u}^* \end{pmatrix} \begin{pmatrix} \sqrt{1+\mathbf{n}} & \sqrt{\mathbf{n}} \\ \sqrt{\mathbf{n}} & \sqrt{1+\mathbf{n}} \end{pmatrix} \begin{pmatrix} \mathbf{v} & 0 \\ 0 & \mathbf{v}^* \end{pmatrix}$$

where $\mathbf{u}, \mathbf{v} \in U(N_{\rm f})$, and $\mathbf{n} = \mathrm{diag}(n_1, n_2, \cdots) \quad \Rightarrow \quad N_{\rm f}(2N_{\rm f}+1)$ variables in FP equation!

A general transfer matrix can be parametrized as

$$\mathbf{M} = \begin{pmatrix} \mathbf{u} & 0 \\ 0 & \mathbf{u}^* \end{pmatrix} \begin{pmatrix} \sqrt{1+\mathbf{n}} & \sqrt{\mathbf{n}} \\ \sqrt{\mathbf{n}} & \sqrt{1+\mathbf{n}} \end{pmatrix} \begin{pmatrix} \mathbf{v} & 0 \\ 0 & \mathbf{v}^* \end{pmatrix}$$

where $\mathbf{u}, \mathbf{v} \in U(N_{\rm f})$, and $\mathbf{n} = {\rm diag}(n_1, n_2, \cdots) \quad \Rightarrow \quad N_{\rm f}(2N_{\rm f}+1)$ variables in FP equation!

Is it possible to derive universal or quasi-universal results?

A general transfer matrix can be parametrized as

$$\mathbf{M} = \begin{pmatrix} \mathbf{u} & 0 \\ 0 & \mathbf{u}^* \end{pmatrix} \begin{pmatrix} \sqrt{1+\mathbf{n}} & \sqrt{\mathbf{n}} \\ \sqrt{\mathbf{n}} & \sqrt{1+\mathbf{n}} \end{pmatrix} \begin{pmatrix} \mathbf{v} & 0 \\ 0 & \mathbf{v}^* \end{pmatrix}$$

where $\mathbf{u}, \mathbf{v} \in U(N_f)$, and $\mathbf{n} = \mathrm{diag}(n_1, n_2, \cdots) \quad \Rightarrow \quad N_f(2N_f+1)$ variables in FP equation!

Is it possible to derive universal or quasi-universal results?

The Maximum Entropy Ansatz

Assume the building block P maximizes the Shannon entropy

$$S[P] = -\int P(\mathbf{M};\delta\tau)\, \ln P(\mathbf{M};\delta\tau)\, d\mathbf{M}$$

subject to the constraints:

- The local mean particle production rate is known and fixed, $\mu_j \equiv \frac{1}{N_f}\frac{\langle n_j\rangle_{\delta\tau}}{\delta\tau}$
- Coarse-grained continuity, $\mathbf{M}_{\tau+\delta\tau} \xrightarrow{\delta\tau\to 0} \mathbf{M}_\tau$

A general transfer matrix can be parametrized as

$$\mathbf{M} = \begin{pmatrix} \mathbf{u} & 0 \\ 0 & \mathbf{u}^* \end{pmatrix} \begin{pmatrix} \sqrt{1+\mathbf{n}} & \sqrt{\mathbf{n}} \\ \sqrt{\mathbf{n}} & \sqrt{1+\mathbf{n}} \end{pmatrix} \begin{pmatrix} \mathbf{v} & 0 \\ 0 & \mathbf{v}^* \end{pmatrix}$$

where $\mathbf{u},\mathbf{v} \in U(N_{\rm f})$, and $\mathbf{n} = {\rm diag}(n_1, n_2, \cdots) \quad \Rightarrow \quad N_{\rm f}(2N_{\rm f}+1)$ variables in FP equation!

Is it possible to derive universal or quasi-universal results?

The Maximum Entropy Ansatz

Then (Mello, Pereyra, Kumar 1988; Amin, Baumann 2016),

- P is flat (Haar) wrt $\mathbf{u}$, $dP(\{\mathbf{u},\mathbf{n},\mathbf{v}\}) = P(\{\mathbf{n},\mathbf{v}\})\, d\mu(\mathbf{u})$
- A closed set of equations for the moments of $n = \sum_a n_a$ is obtained,

$$\partial_\tau \langle \ln(1+n)\rangle \xrightarrow{\tau\to\infty} \frac{2N_{\rm f}}{N_{\rm f}+1}\,\mu$$

i.e. exponential growth

Exact Results

Consider the approximation

$$m^{\rm s}_{ab}(\tau) = 2\sqrt{\omega_a\omega_b}\sum_{j=1}^{N_{\rm s}}\Lambda_{ab}(\tau_j)\delta(\tau-\tau_j)\,,$$

where τ_j are uniformly distributed, and

$$\langle\Lambda_{ab}\rangle = 0, \qquad \langle\Lambda_{ab}\Lambda_{cd}\rangle = \sigma^2_{ab}(\delta_{ac}\delta_{bd}+\delta_{ad}\delta_{bc})$$

The transfer matrix takes the form

$$\mathbf{M}_j = \mathbb{1} + i\underbrace{\begin{pmatrix}\mathbf{a}_j^* & 0\\ 0 & \mathbf{a}_j\end{pmatrix}\begin{pmatrix}\mathbf{\Lambda}_j & \mathbf{\Lambda}_j\\ -\mathbf{\Lambda}_j & -\mathbf{\Lambda}_j\end{pmatrix}\begin{pmatrix}\mathbf{a}_j & 0\\ 0 & \mathbf{a}_j^*\end{pmatrix}}_{\mathbf{m}_j}, \qquad \mathbf{a}_j \equiv {\rm diag}(e^{i\omega_1\tau_j}, e^{i\omega_2\tau_j}, \cdots)$$

Exact Results

Consider the approximation

$$m^{\rm s}_{ab}(\tau) = 2\sqrt{\omega_a\omega_b}\sum_{j=1}^{N_{\rm s}} \Lambda_{ab}(\tau_j)\delta(\tau-\tau_j)\,,$$

where τ_j are uniformly distributed, and

$$\langle\Lambda_{ab}\rangle = 0, \qquad \langle\Lambda_{ab}\Lambda_{cd}\rangle = \sigma^2_{ab}(\delta_{ac}\delta_{bd}+\delta_{ad}\delta_{bc})$$

The transfer matrix takes the form

$$\mathbf{M}_j = \mathbb{1} + i\underbrace{\begin{pmatrix}\mathbf{a}_j^* & 0\\ 0 & \mathbf{a}_j\end{pmatrix}\begin{pmatrix}\mathbf{\Lambda}_j & \mathbf{\Lambda}_j\\ -\mathbf{\Lambda}_j & -\mathbf{\Lambda}_j\end{pmatrix}\begin{pmatrix}\mathbf{a}_j & 0\\ 0 & \mathbf{a}_j^*\end{pmatrix}}_{\mathbf{m}_j}, \qquad \mathbf{a}_j \equiv {\rm diag}(e^{i\omega_1\tau_j}, e^{i\omega_2\tau_j},\cdots)$$

Will focus on the total occupation number. Define $\mathbf{R} = \mathbf{M}\mathbf{M}^\dagger$:

$$n(j) \;=\; \frac{1}{4}{\rm Tr}\left[\mathbf{M}(j)\mathbf{M}^\dagger(j) - \mathbb{1}\right] \;\equiv\; \frac{1}{4}{\rm Tr}\left[\mathbf{R}(j)-\mathbb{1}\right]$$

Exact Results

Consider the approximation

$$m^{\rm s}_{ab}(\tau) = 2\sqrt{\omega_a\omega_b}\sum_{j=1}^{N_{\rm s}}\Lambda_{ab}(\tau_j)\delta(\tau-\tau_j)\,,$$

where τ_j are uniformly distributed, and

$$\langle\Lambda_{ab}\rangle = 0, \qquad \langle\Lambda_{ab}\Lambda_{cd}\rangle = \sigma^2_{ab}(\delta_{ac}\delta_{bd}+\delta_{ad}\delta_{bc})$$

The transfer matrix takes the form

$$\mathbf{M}_j = \mathbb{1} + i\underbrace{\begin{pmatrix}\mathbf{a}_j^* & 0\\ 0 & \mathbf{a}_j\end{pmatrix}\begin{pmatrix}\boldsymbol{\Lambda}_j & \boldsymbol{\Lambda}_j\\ -\boldsymbol{\Lambda}_j & -\boldsymbol{\Lambda}_j\end{pmatrix}\begin{pmatrix}\mathbf{a}_j & 0\\ 0 & \mathbf{a}_j^*\end{pmatrix}}_{\mathbf{m}_j}, \qquad \mathbf{a}_j \equiv {\rm diag}(e^{i\omega_1\tau_j}, e^{i\omega_2\tau_j},\cdots)$$

Will focus on the total occupation number. Define $\mathbf{R} = \mathbf{M}\mathbf{M}^\dagger$:

$$\mathbf{R} = \begin{pmatrix}\mathbf{u} & 0\\ 0 & \mathbf{u}^*\end{pmatrix}\begin{pmatrix}\sqrt{1+\mathbf{n}} & \sqrt{\mathbf{n}}\\ \sqrt{\mathbf{n}} & \sqrt{1+\mathbf{n}}\end{pmatrix}^2\begin{pmatrix}\mathbf{u}^\dagger & 0\\ 0 & \mathbf{u}^\top\end{pmatrix}$$

Single field

Only two parameters, n and $u = e^{i\phi}$. Computation is straightforward,

$$\langle \delta n \rangle = (2n+1)\,\sigma^2$$

$$\langle \delta\phi \rangle = 0$$

$$\langle \delta n\,\delta n \rangle = 2n(n+1)\sigma^2$$

$$\langle \delta n\,\delta\phi \rangle = 0$$

$$\langle \delta\phi\,\delta\phi \rangle = \frac{\sigma^2}{8n(n+1)}(12n^2+12n+1)$$

Single field

Only two parameters, n and $u = e^{i\phi}$. Computation is straightforward,

$$\langle \delta n \rangle = (2n+1)\,\sigma^2$$
$$\langle \delta \phi \rangle = 0$$

$$\langle \delta n\, \delta n \rangle = 2n(n+1)\sigma^2$$
$$\langle \delta n\, \delta \phi \rangle = 0$$
$$\langle \delta \phi\, \delta \phi \rangle = \frac{\sigma^2}{8n(n+1)}(12n^2 + 12n + 1)$$

No ϕ dependence! $\Rightarrow$ maximum entropy

Single field

Only two parameters, n and $u = e^{i\phi}$. Computation is straightforward,

$$\langle \delta n \rangle = (2n+1)\,\sigma^2$$

$$\langle \delta\phi \rangle = 0$$

$$\langle \delta n\,\delta n \rangle = 2n(n+1)\sigma^2$$

$$\langle \delta n\,\delta\phi \rangle = 0$$

$$\langle \delta\phi\,\delta\phi \rangle = \frac{\sigma^2}{8n(n+1)}(12n^2+12n+1)$$

No ϕ dependence! $\Rightarrow$ maximum entropy

The FP equation is

$$\frac{1}{\sigma^2}\frac{\partial}{\partial\tau}P(n;\tau) = \frac{\partial}{\partial n}\left[n(n+1)\frac{\partial}{\partial n}P(n;\tau)\right]$$

Single field

Only two parameters, n and $u = e^{i\phi}$. Computation is straightforward,

$$\langle \delta n \rangle = (2n+1)\,\sigma^2$$
$$\langle \delta \phi \rangle = 0$$

$$\langle \delta n\, \delta n \rangle = 2n(n+1)\sigma^2$$
$$\langle \delta n\, \delta \phi \rangle = 0$$
$$\langle \delta \phi\, \delta \phi \rangle = \frac{\sigma^2}{8n(n+1)}(12n^2+12n+1)$$

No ϕ dependence! $\Rightarrow$ maximum entropy

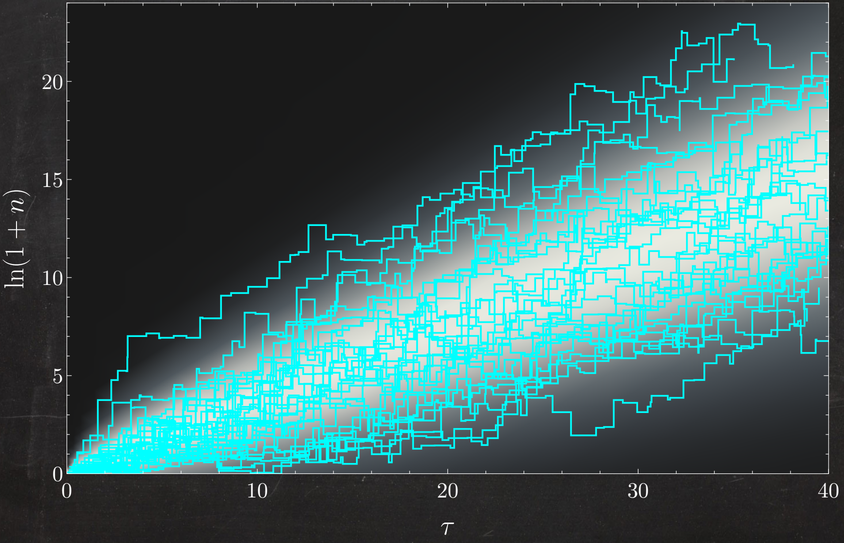
The FP equation is

$$\frac{1}{\sigma^2}\frac{\partial}{\partial \tau}P(n;\tau) = \frac{\partial}{\partial n}\left[n(n+1)\frac{\partial}{\partial n}P(n;\tau)\right]$$

with solution $(n \gg 1)$

$$P(n;\tau)\,dn = \frac{1}{\sqrt{4\pi\sigma^2\tau}}\exp\left[-\frac{(\ln n - \sigma^2\tau)^2}{4\sigma^2\tau}\right]\,d\ln n$$

$$\Rightarrow \quad n = e^{\sigma^2\tau} - 1$$


20
15
10
5
0
$\ln(1+n)$
0
10
20
30
40
τ

Two fields

Six parameters now, n_1, n_2 and

$$\mathbf{u}(\phi,\theta,\psi,\varphi) = e^{-\frac{i}{2}\phi}\begin{bmatrix} \cos\frac{\theta}{2}\, e^{-\frac{i}{2}(\varphi+\psi)} & -\sin\frac{\theta}{2}\, e^{-\frac{i}{2}(\varphi-\psi)} \\ \sin\frac{\theta}{2}\, e^{\frac{i}{2}(\varphi-\psi)} & \cos\frac{\theta}{2}\, e^{\frac{i}{2}(\varphi+\psi)} \end{bmatrix}$$

Two fields

Six parameters now, n_1, n_2 and

$$\mathbf{u}(\phi,\theta,\psi,\varphi) = e^{-\frac{i}{2}\phi}\begin{bmatrix} \cos\frac{\theta}{2}\, e^{-\frac{i}{2}(\varphi+\psi)} & -\sin\frac{\theta}{2}\, e^{-\frac{i}{2}(\varphi-\psi)} \\ \sin\frac{\theta}{2}\, e^{\frac{i}{2}(\varphi-\psi)} & \cos\frac{\theta}{2}\, e^{\frac{i}{2}(\varphi+\psi)} \end{bmatrix}$$

$\Rightarrow$ need 27 correlators

Two fields

Six parameters now, n_1, n_2 and

$$\mathbf{u}(\phi,\theta,\psi,\varphi) = e^{-\frac{i}{2}\phi}\begin{bmatrix} \cos\frac{\theta}{2}\, e^{-\frac{i}{2}(\varphi+\psi)} & -\sin\frac{\theta}{2}\, e^{-\frac{i}{2}(\varphi-\psi)} \\ \sin\frac{\theta}{2}\, e^{\frac{i}{2}(\varphi-\psi)} & \cos\frac{\theta}{2}\, e^{\frac{i}{2}(\varphi+\psi)} \end{bmatrix}$$

$\Rightarrow$ need 27 correlators

Let

$$\langle(\Lambda_{11})^2\rangle_{\delta\tau} = \sigma_1^2\,, \qquad \langle(\Lambda_{22})^2\rangle_{\delta\tau} = \sigma_2^2\,, \qquad \langle(\Lambda_{12})^2\rangle_{\delta\tau} = \langle(\Lambda_{12})^2\rangle_{\delta\tau} = \sigma_\perp^2\,.$$

Two fields

Six parameters now, n_1, n_2 and

$$\mathbf{u}(\phi,\theta,\psi,\varphi) = e^{-\frac{i}{2}\phi}\begin{bmatrix} \cos\frac{\theta}{2}\, e^{-\frac{i}{2}(\varphi+\psi)} & -\sin\frac{\theta}{2}\, e^{-\frac{i}{2}(\varphi-\psi)} \\ \sin\frac{\theta}{2}\, e^{\frac{i}{2}(\varphi-\psi)} & \cos\frac{\theta}{2}\, e^{\frac{i}{2}(\varphi+\psi)} \end{bmatrix}$$

$\Rightarrow$ need 27 correlators

$$\langle\delta f_1^{(1)}\delta f_1^{(1)}\rangle = \tilde{f}_1^2\,\gamma_1(\theta),$$

$$\langle\delta f_1^{(1)}\delta f_2^{(1)}\rangle = 2\tilde{f}_1\tilde{f}_2\cos(2\psi)\,\gamma_3(\theta),$$

$$\langle\delta f_1^{(1)}\delta\theta^{(1)}\rangle = -\frac{2\tilde{f}_1}{\Delta f}\left[\tilde{f}_1 + \tilde{f}_2\cos(2\psi)\right]\gamma_4(\theta),$$

$$\langle\delta f_1^{(1)}\delta\psi^{(1)}\rangle = -\tilde{f}_1\sin(2\psi)\left(\frac{f_2}{\tilde{f}_2}\gamma_3(\theta) - 2\frac{\tilde{f}_2}{\Delta f}\gamma_4(\theta)\cot\theta\right),$$

$$\langle\delta f_1^{(1)}\delta\varphi^{(1)}\rangle = -2\frac{\tilde{f}_1\tilde{f}_2}{\Delta f}\sin(2\psi)\gamma_4(\theta)\csc\theta,$$

$$\langle\delta f_1^{(1)}\delta\phi^{(1)}\rangle = \tilde{f}_1\frac{f_2}{\tilde{f}_2}\sin(2\psi)\gamma_3(\theta),$$

$$\langle\delta f_2^{(1)}\delta f_2^{(1)}\rangle = \tilde{f}_2^2\,\gamma_2(\theta),$$

$$\langle\delta f_2^{(1)}\delta\theta^{(1)}\rangle = -\frac{2\tilde{f}_2}{\Delta f}\left[\tilde{f}_2 + \tilde{f}_1\cos(2\psi)\right]\gamma_5(\theta),$$

$\langle\delta f_2^{(1)}\delta\psi^{(1)}\rangle = -\tilde{f}_2\sin(2\psi)\left(\frac{f_1}{\tilde{f}_1}\gamma_3(\theta) - 2\frac{\tilde{f}_1}{\Delta f}\gamma_5(\theta)\cot\right.$ [redacted]

$$\langle\delta f_2^{(1)}\delta\varphi^{(1)}\rangle = -2\frac{\tilde{f}_1\tilde{f}_2}{\Delta f}\sin(2\psi)\gamma_5(\theta)\csc\theta,$$

$$\langle\delta f_2^{(1)}\delta\phi^{(1)}\rangle = -\tilde{f}_2\frac{f_1}{\tilde{f}_1}\sin(2\psi)\gamma_3(\theta),$$

$$\langle\delta\theta^{(1)}\delta\theta^{(1)}\rangle = 2\sigma_\perp^2 + \frac{1}{\Delta f^2}\left(\tilde{f}_1^2 + \tilde{f}_2^2 + 2\tilde{f}_1\tilde{f}_2\cos(2\psi)\right)\gamma_6(\theta),$$

$$\langle\delta\theta^{(1)}\delta\psi^{(1)}\rangle = \frac{1}{\Delta f}\left[\frac{f_1\tilde{f}_2}{\tilde{f}_1}\gamma_4(\theta) + \frac{\tilde{f}_1 f_2}{\tilde{f}_2}\gamma_5(\theta)\right]\sin(2\psi) - \cos\theta\,\langle\delta\theta^{(1)}\delta\varphi^{(1)}\rangle,$$

$$\langle\delta\theta^{(1)}\delta\varphi^{(1)}\rangle = \frac{2\tilde{f}_1\tilde{f}_2\sin(2\psi)}{\Delta f^2}\gamma_6(\theta)\csc\theta,$$

$$\langle\delta f_1^{(2)}\rangle = \frac{1}{\Delta f}\left[\tilde{f}_1^2 l_1(\theta) - (f_1 f_2 - 1) l_3(\theta)\right],$$

$\langle\delta\theta^{(1)}\delta\phi^{(1)}\rangle = \frac{1}{4\Delta f}\left[\left(f_1\frac{\tilde{f}_2}{\tilde{f}_1} - f_2\frac{\tilde{f}_1}{\tilde{f}_2}\right)(\sigma_1^2 - \sigma_2^2)\sin\theta + 4\left(f_1\frac{\tilde{f}_2}{\tilde{f}_1} + f_2\frac{\tilde{f}_1}{\tilde{f}_2}\right.\right.$ [redacted] $\left.\right]\sin(2\psi)$

$\langle\delta\psi^{(1)}\delta\psi^{(1)}\rangle = \frac{1}{4}\left[\left(\frac{f_1}{\tilde{f}_1}\right)^2\gamma_1(\theta) + \left(\frac{f_2}{\tilde{f}_2}\right)^2\gamma_2(\theta)\right] -$ [redacted] $\left.\sigma_2^2\right) + 2\sin^2\theta\sigma_\perp^2 - \cos\theta\left(2\langle\delta\psi^{(1)}\delta\varphi^{(1)}\rangle + \cos\theta\,\langle\delta\varphi^{(1)}\delta\varphi^{(1)}\rangle\right),$

$\langle\delta\psi^{(1)}\delta\varphi^{(1)}\rangle = -\frac{1}{\Delta f}\left(\frac{f_1\tilde{f}_2}{\tilde{f}_1}\gamma_4(\theta)\right.$ [redacted] $\left(\sigma_1^2 + \sigma_2^2\right) - 3\sigma_\perp^2\Big]\cos\theta - \cos\theta\,\langle\delta\varphi^{(1)}\delta\varphi^{(1)}\rangle,$

$\langle\delta\psi^{(1)}\delta\phi^{(1)}\rangle$ [redacted] $\theta - \langle\delta\varphi^{(1)}\delta\phi^{(1)}\rangle\cos\theta,$

[redacted] $\left.\sigma_2^2\right)\left[5(f_1 - f_2) + \left(\tilde{f}_1\frac{f_2}{\tilde{f}_2} - \tilde{f}_2\frac{f_1}{\tilde{f}_1}\right)\cos(2\psi)\right],$

[redacted] $\left(\frac{f_1^2}{\tilde{f}_1^2} - \frac{f_2^2}{\tilde{f}_2^2}\right)\cos\theta - \frac{1}{2}\left[\frac{f_1^2}{\tilde{f}_1^2} + \frac{f_2^2}{\tilde{f}_2^2} - 2\frac{f_1}{\tilde{f}_1}\frac{f_2}{\tilde{f}_2}\cos(2\psi)\right]\gamma_3(\theta),$

[redacted] $\Big],$

[redacted] $\left.\sigma_2^2\right)\sin\theta - \frac{1}{2}\sin(2\theta)\left(\sigma_1^2 + \sigma_2^2 - 3\sigma_\perp^2\right) + \frac{1}{\Delta f}\left(\langle\delta f_2^{(1)}\delta\theta^{(1)}\rangle - \langle\delta f_1^{(1)}\delta\theta^{(1)}\rangle\right) + \frac{1}{4}\sin(2\theta)\langle\delta\varphi^{(1)}\delta\varphi^{(1)}\rangle,$

[redacted] $\left(\frac{\tilde{f}_1}{\tilde{f}_2} + \frac{\tilde{f}_2}{\tilde{f}_1}\right)\left(\sigma_1^2 + \sigma_2^2 - 2\sigma_\perp^2\right)\sin^2\theta\sin(2\psi) - \frac{1}{2}\left(\frac{f_1}{\tilde{f}_1^2}\langle\delta f_1^{(1)}\delta\phi^{(1)}\rangle - \frac{f_2}{\tilde{f}_2^2}\langle\delta f_2^{(1)}\delta\phi^{(1)}\rangle\right) - \frac{1}{2}\left(\frac{f_1}{\tilde{f}_1^2}\langle\delta f_1^{(1)}\delta\psi^{(1)}\rangle + \frac{f_2}{\tilde{f}_2^2}\langle\delta f_2^{(1)}\delta\psi^{(1)}\rangle\right)$

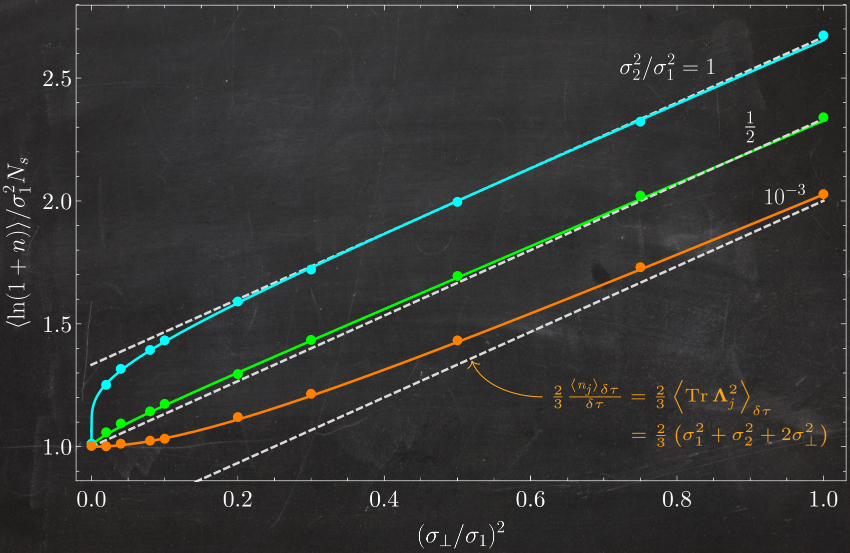
$$-\frac{1}{2}\left(\frac{f_1}{\tilde{f}_1^2}\langle\delta f_1^{(1)}\delta\varphi^{(1)}\rangle + \frac{f_2}{\tilde{f}_2^2}\langle\delta f_2^{(1)}\delta\varphi^{(1)}\rangle\right)\cos\theta + \frac{1}{2}\sin\theta\,\langle\delta\theta^{(1)}\delta\varphi^{(1)}\rangle - \cos\theta\langle\delta\varphi^{(2)}\rangle$$

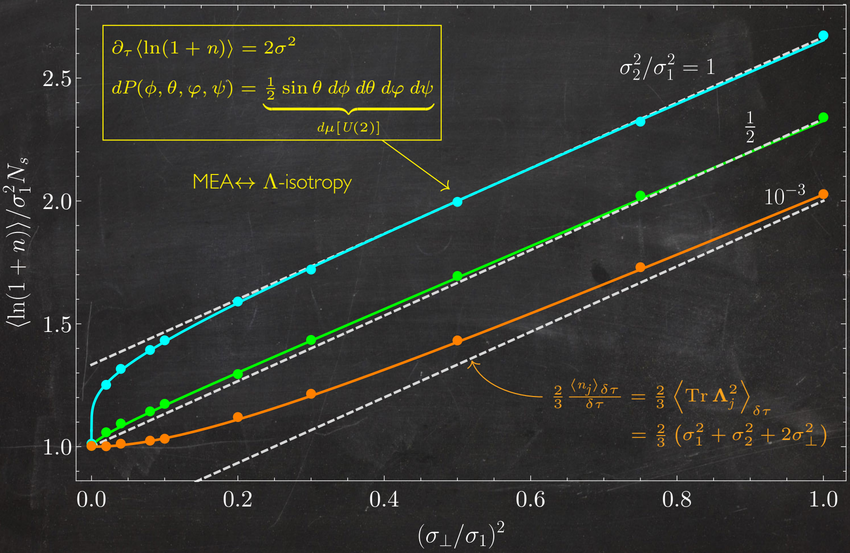
$$-\frac{1}{2}\left(\frac{\tilde{f}_1}{\tilde{f}_2} - \frac{\tilde{f}_2}{\tilde{f}_1}\right)\left[\frac{1}{4}\left(\langle\delta\varphi^{(1)}\delta\varphi^{(1)}\rangle\sin^2\theta - \langle\delta\theta^{(1)}\delta\theta^{(1)}\rangle\right)\sin 2\psi + \frac{1}{2}\langle\delta\varphi^{(1)}\delta\theta^{(1)}\rangle\sin\theta\cos 2\psi\right],$$

$$\langle\delta\varphi^{(2)}\rangle = \frac{1}{\Delta f}\left(\langle\delta f_2^{(1)}\delta\varphi^{(1)}\rangle - \langle\delta f_1^{(1)}\delta\varphi^{(1)}\rangle\right) - \cot\theta\,\langle\delta\theta^{(1)}\delta\varphi^{(1)}\rangle,$$

$$\langle\delta\phi^{(2)}\rangle = 0$$

CENSORED


$\langle \ln(1+n) \rangle / \sigma_1^2 N_s$
2.5
2.0
1.5
1.0
0.0
0.2
0.4
0.6
0.8
1.0
$(\sigma_\perp/\sigma_1)^2$
$\sigma_2^2/\sigma_1^2 = 1$
$\frac{1}{2}$
10^{-3}
$\frac{2}{3}\frac{\langle n_j \rangle_{\delta\tau}}{\delta\tau} = \frac{2}{3}\left\langle \mathrm{Tr}\, \mathbf{\Lambda}_j^2 \right\rangle_{\delta\tau}$
$= \frac{2}{3}\left(\sigma_1^2 + \sigma_2^2 + 2\sigma_\perp^2\right)$



$\partial_\tau \langle \ln(1+n) \rangle = 2\sigma^2$
$dP(\phi,\theta,\varphi,\psi) = \underbrace{\frac{1}{2}\sin\theta\, d\phi\, d\theta\, d\varphi\, d\psi}_{d\mu[U(2)]}$
MEA$\leftrightarrow$ Λ-isotropy
$\sigma_2^2/\sigma_1^2 = 1$
$\frac{1}{2}$
10^{-3}
$\frac{2}{3}\frac{\langle n_j \rangle_{\delta\tau}}{\delta\tau} = \frac{2}{3}\left\langle \mathrm{Tr}\, \mathbf{\Lambda}_j^2 \right\rangle_{\delta\tau}$
$= \frac{2}{3}\left(\sigma_1^2 + \sigma_2^2 + 2\sigma_\perp^2\right)$
$\langle \ln(1+n) \rangle / \sigma_1^2 N_s$
1.0
1.5
2.0
2.5
0.0
0.2
0.4
0.6
0.8
1.0
$(\sigma_\perp/\sigma_1)^2$

N_f fields

$N_\mathrm{f}(N_\mathrm{f}+1)$ parameters now, $n_1, n_2, \ldots, n_{N_\mathrm{f}}$ and (Tilma, Sudarshan 2002)

$$\mathbf{u} = \left(\prod_{2 \leqslant k \leqslant N} \mathbf{A}(k) \right) \cdot [SU(N-1)] \cdot e^{i\boldsymbol{\lambda}_{N^2-1}\alpha_{N^2-1}}\,, \qquad \mathbf{A}(k) = e^{i\boldsymbol{\lambda}_3 \alpha_{(2k-3)}}\, e^{i\boldsymbol{\lambda}_{(k-1)^2+1}\alpha_{2(k-1)}}$$

N_f fields

$N_\mathrm{f}(N_\mathrm{f}+1)$ parameters now, $n_1, n_2, \ldots, n_{N_\mathrm{f}}$ and (Tilma, Sudarshan 2002)

$$\mathbf{u} = \left(\prod_{2\leqslant k\leqslant N} \mathbf{A}(k)\right) \cdot [SU(N-1)] \cdot e^{i\boldsymbol{\lambda}_{N^2-1}\alpha_{N^2-1}}\,, \qquad \mathbf{A}(k) = e^{i\boldsymbol{\lambda}_3\alpha_{(2k-3)}}\, e^{i\boldsymbol{\lambda}_{(k-1)^2+1}\alpha_{2(k-1)}}$$

$\Rightarrow$ need $\mathcal{O}({N_\mathrm{f}}^4)$ correlators!

N_f fields

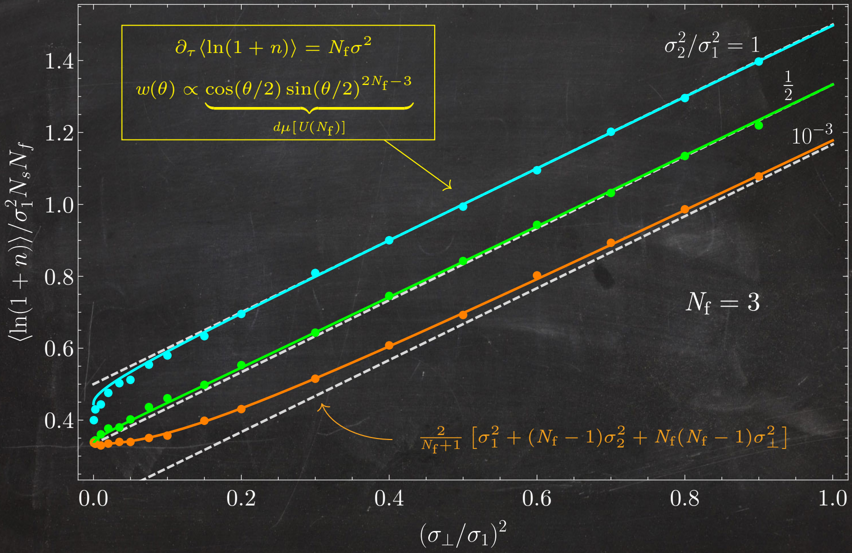
$N_\mathrm{f}(N_\mathrm{f}+1)$ parameters now, $n_1, n_2, \ldots, n_{N_\mathrm{f}}$ and (Tilma, Sudarshan 2002)

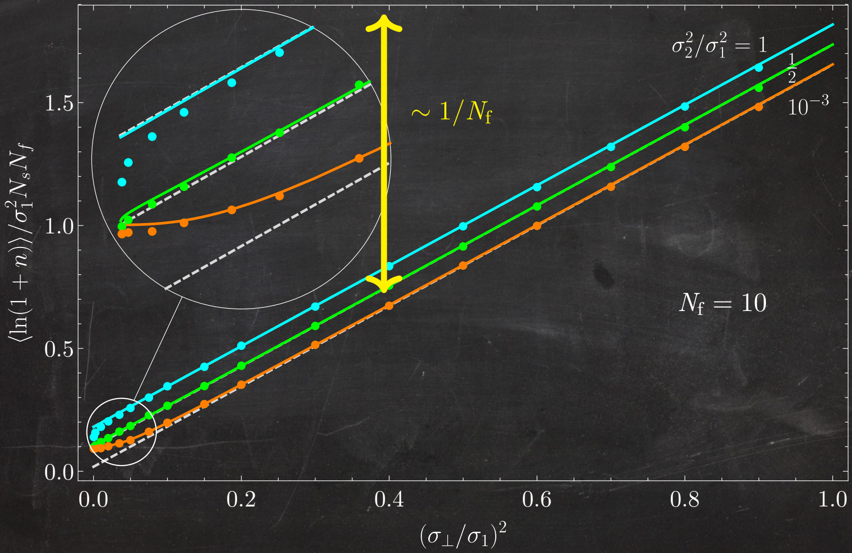
$$\mathbf{u} = \left(\prod_{2\leqslant k\leqslant N} \mathbf{A}(k)\right) \cdot [SU(N-1)] \cdot e^{i\boldsymbol{\lambda}_{N^2-1}\alpha_{N^2-1}}\,, \qquad \mathbf{A}(k) = e^{i\boldsymbol{\lambda}_3\alpha_{(2k-3)}}\, e^{i\boldsymbol{\lambda}_{(k-1)^2+1}\alpha_{2(k-1)}}$$

$\Rightarrow$ need $\mathcal{O}({N_\mathrm{f}}^4)$ correlators!

Among other assumptions, consider

$$\boldsymbol{\sigma}^2 = \begin{pmatrix} \sigma_1^2/2 & \sigma_\perp^2 & \cdots & \sigma_\perp^2 \\ \sigma_\perp^2 & \sigma_2^2/2 & \cdots & \sigma_\perp^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_\perp^2 & \sigma_\perp^2 & \cdots & \sigma_2^2/2 \end{pmatrix}$$


$\partial_\tau \langle \ln(1+n) \rangle = N_f \sigma^2$
$w(\theta) \propto \underbrace{\cos(\theta/2)\sin(\theta/2)^{2N_f-3}}_{d\mu[U(N_f)]}$
$\sigma_2^2/\sigma_1^2 = 1$
$\frac{1}{2}$
10^{-3}
$N_f = 3$
$\frac{2}{N_f+1}\left[\sigma_1^2 + (N_f-1)\sigma_2^2 + N_f(N_f-1)\sigma_\perp^2\right]$
$\langle \ln(1+n) \rangle / \sigma_1^2 N_s N_f$
$(\sigma_\perp/\sigma_1)^2$
0.4
0.6
0.8
1.0
1.2
1.4
0.0
0.2
0.4
0.6
0.8
1.0


$\sigma_2^2/\sigma_1^2 = 1$
$\frac{1}{2}$
10^{-3}
$\sim 1/N_\mathrm{f}$
$N_\mathrm{f} = 10$
$\langle \ln(1+n) \rangle / \sigma_1^2 N_s N_f$
$(\sigma_\perp/\sigma_1)^2$
0.0
0.5
1.0
1.5
0.0
0.2
0.4
0.6
0.8
1.0

Conclusion

- Avoid relying on detailed model building, and take a coarse grained approach to the particle production in the early universe
- MEA captures the universal features arising from a Central Limit Threorem (concentration of measure)…
- …as long as there's no hierarchy of couplings
- Break from weak scattering limit $\Rightarrow$ Random Matrix Theory?
- Next: include expansion and metric perturbations

Conclusion

- Avoid relying on detailed model building, and take a coarse grained approach to the particle production in the early universe
- MEA captures the universal features arising from a Central Limit Threorem (concentration of measure)…
- …as long as there's no hierarchy of couplings
- Break from weak scattering limit $\Rightarrow$ Random Matrix Theory?
- Next: include expansion and metric perturbations

Thank you