Non-thermal Particle Production in the Early Universe

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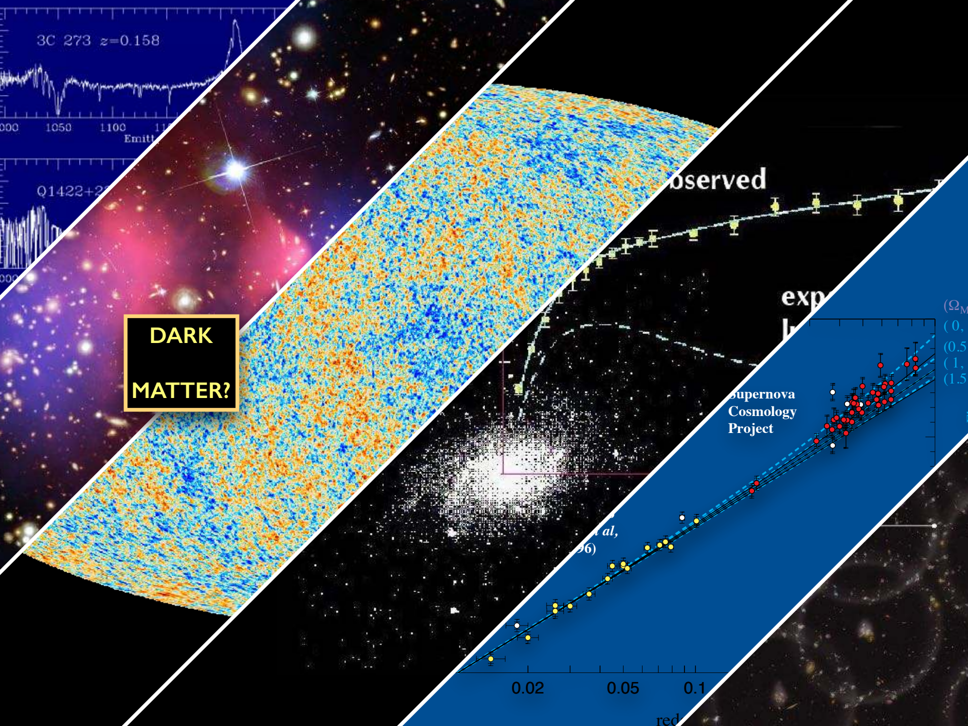
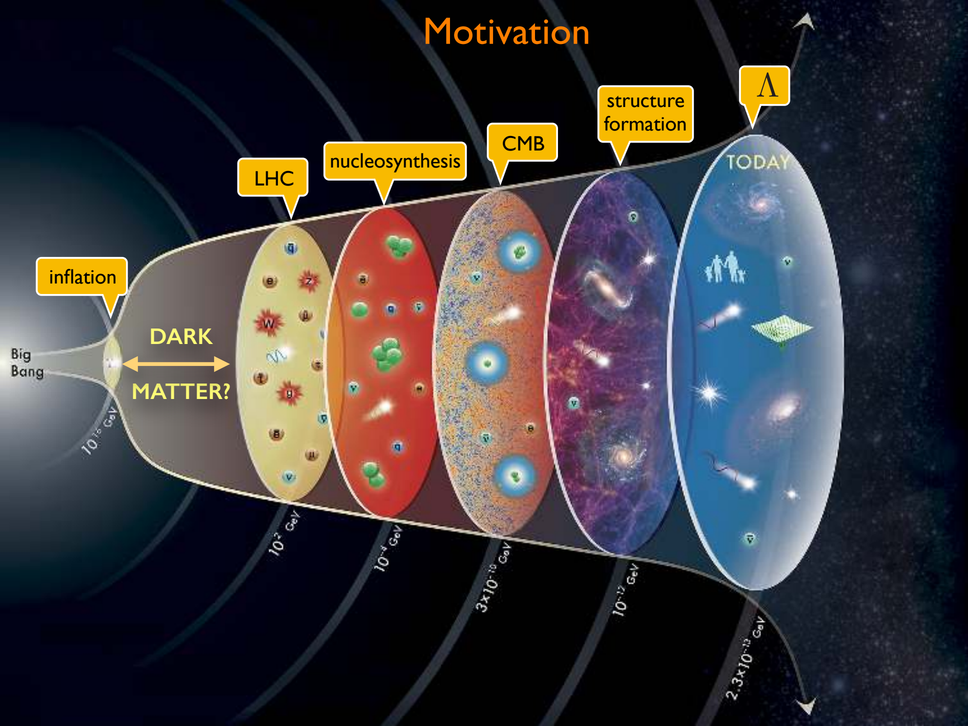
1709.01549, MG, Y. Mambrini, K. Olive, M. Peloso
1806.01865, MG, M. Amin

1706.02319, M. Amin, MG, H. Xie, O. Wen
18xx.xxxxx, MG, M. Amin, D. Green


3C 273 z=0.158
1050
1100
DARK
MATTER?
Cosmology
Project
0.02
0.05
0.1

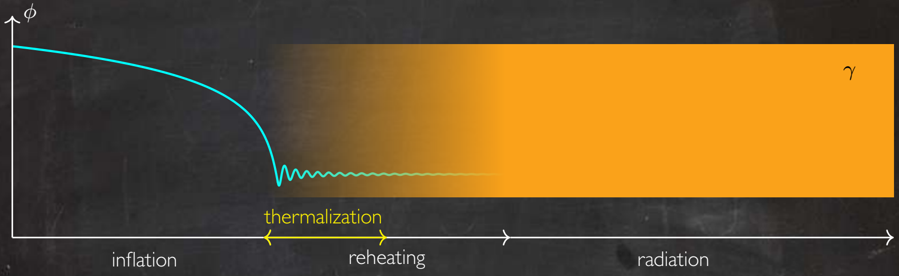
Motivation



Λ
structure formation
CMB
nucleosynthesis
LHC
TODAY
inflation
DARK
MATTER?
Big Bang

Freeze-in vs. Freeze-out

The reheating and thermalization processes after inflation have a finite duration



Freeze-in vs. Freeze-out

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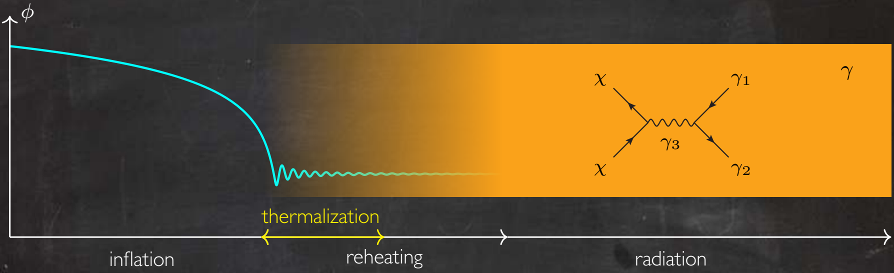
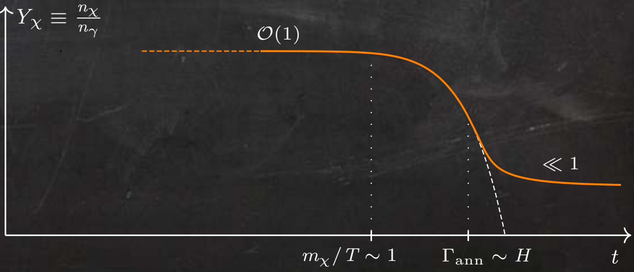


Freeze-out:

- Thermal equilibrium
- Thermal production
- $Y_\chi \longleftrightarrow \langle \sigma v \rangle_{\rm ann}$
- IR dominated



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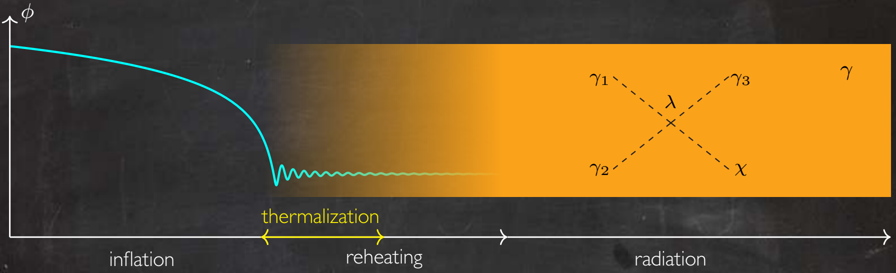
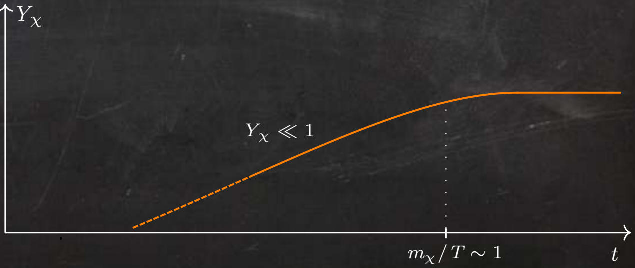


Freeze-in:

- Not in equilibrium
- Thermal production
- $Y_\chi \longleftrightarrow \langle \sigma v \rangle_{\rm prod}$
- IR dominated



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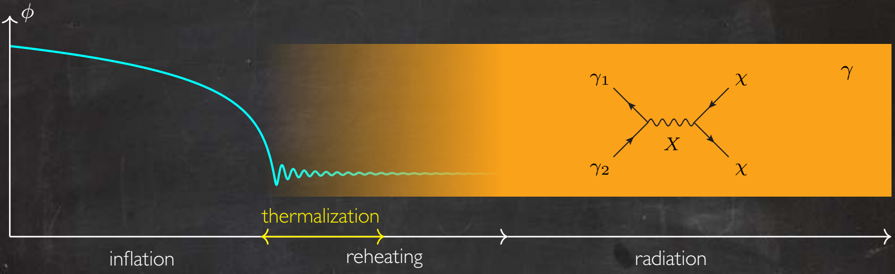
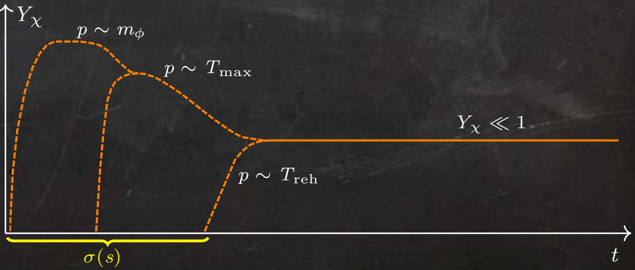


Freeze-in:

- Not in equilibrium
- Non-thermal and thermal production
- $Y_\chi \longleftrightarrow \langle\sigma v\rangle_{\rm prod}$
- UV dominated



Reheating and Thermalization

(i) Instantaneous inflaton decay *and* thermalization:

$$T_{\rm reh} = \left(\frac{40}{\pi^2 g_{\rm reh}}\right)^{1/4} (\Gamma_\phi M_P)^{1/2}$$

Reheating and Thermalization

(i) Instantaneous inflaton decay *and* thermalization:

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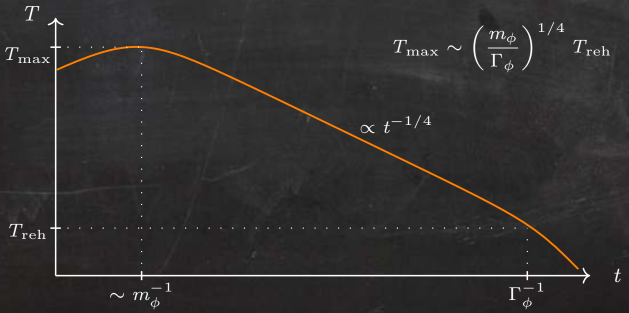
(ii) Instantaneous thermalization:

$$\begin{aligned}\dot\rho_\phi + 3H\rho_\phi + \Gamma_\phi\rho_\phi &= 0\\ \dot\rho_\gamma + 4H\rho_\gamma - \Gamma_\phi\rho_\phi &= 0\\ \rho_\phi + \rho_\gamma &= 3M_P^2 H^2\end{aligned}$$



$$T_{\rm max} \sim \left(\frac{m_\phi}{\Gamma_\phi}\right)^{1/4} T_{\rm reh}$$

(iii) The path to thermal equilibrium:

Immediately after reheating starts, n_γ is *not* determined by a temperature, but by counting

How many inflatons are left over?

$$n_\phi = \frac{\rho_\phi}{m_\phi} = \frac{\rho_{\rm end}}{m_\phi}\left(\frac{a}{a_{\rm end}}\right)^{-3} e^{-\Gamma_\phi t}$$

How many inflatons have decayed?

$$n_\gamma \simeq \frac{\rho_{\rm end}}{m_\phi}\left(\frac{a}{a_{\rm end}}\right)^{-3}\left(1-e^{-\Gamma_\phi t}\right)$$

In thermal equilibrium we would have

$$\tilde{g} n_\gamma^{\rm T} \equiv \left(\sum_B g_B + \frac{3}{4}\sum_F g_F\right)\frac{\zeta(3)}{\pi^2} T^3$$

At $T_{\rm max}$, $\tilde{g} n_\gamma^{\rm T}/n_\gamma > 1$ for $\Gamma_\phi/m_\phi \lesssim 10^{-10}$ (Planck suppressed)

The spectrum of γ is determined by the decay of the ϕ-condensate, $\quad f_\phi = (2\pi)^3 n_\phi \delta^3(\mathbf{p})$

$$\frac{\partial f_\gamma}{\partial t} - Hk\frac{\partial f_\gamma}{\partial k} = \frac{2\pi^2}{k^2} n_\phi \Gamma_\phi \delta(k - m_\phi/2)$$

The solution predicts very energetic particles, $\langle k \rangle \sim m_\phi$,

$$\boxed{f_\gamma(k) \;\simeq\; 24\pi^2 \frac{n_\gamma}{m_\phi^3}\left(\frac{m_\phi}{2k}\right)^{3/2} \theta(m_\phi/2 - k)}$$

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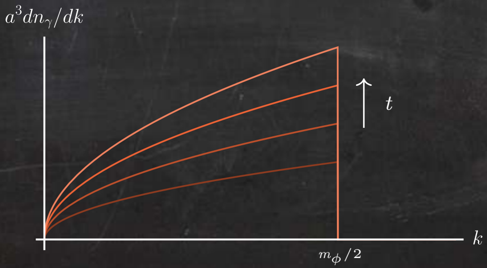

These particles need to *slow-down* and *multiply*

Thermalization through gauge interactions is *tricky*

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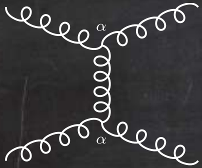
If a Boltzmann eq. can be used to track the evolution of $f_\gamma(p)$, the elastic collision term would be

$$\mathcal{C}^{2\leftrightarrow 2}[f_\gamma(p)] = \frac{1}{2}\int \frac{d^3\mathbf{k}\, d^3\mathbf{p}'\, d^3\mathbf{k}'}{(2\pi)^9} \frac{|\mathcal{M}(p,k;p',k')|^2}{(2p_0)(2k_0)(2p_0')(2k_0')}(2\pi)^4\delta^{(4)}(p+k-p'-k')$$
$$\times \left\{ f_p f_k[1\pm f_{p'}][1\pm f_{k'}] - f_{p'}f_{k'}[1\pm f_p][1\pm f_k] \right\},$$



$$\frac{|\mathcal{M}(p,k;p',k')|^2}{(2p_0)(2k_0)(2p_0')(2k_0')} \sim \frac{\alpha^2}{|\mathbf{k}-\mathbf{k}'|_\perp^4}$$

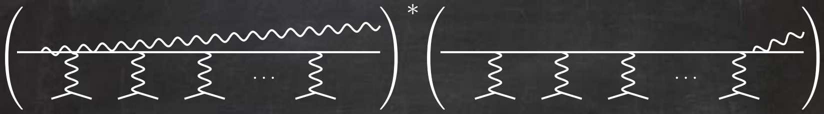
Screening scale determined by effect of successive scatterings

$$m_s^2 \sim \alpha \int d^3\mathbf{k}\;\; f_\gamma(k)/k \qquad \Rightarrow \qquad \frac{\Gamma_{\rm elas}}{H} \sim \alpha\, m_\phi t$$

(A. Kurkela, G. Moore, 1107.5050; J. Ellis, MG et. al., 1512.05701)

Thermalization through gauge interactions is *tricky*

Inelastic processes can have formation times which exceed the mean free time between scatterings



$\neq 0$

(L. Landau, I. Pomeranchuk, Dokl. Akad. Nauk Ser. Fiz. 92 (1953) 535; A. Migdal, Phys. Rev. 103 (1956) 1811)

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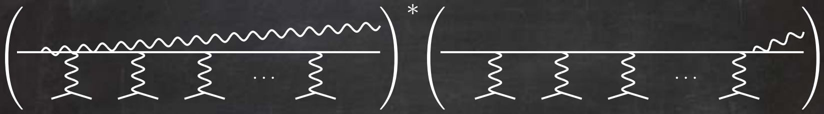

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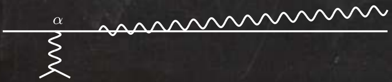
Hard branching including LPM effect



$$t_\gamma \sim \sqrt{\frac{\tau E}{q_\perp^2}}$$

(P. Arnold, G. Moore, L. Yaffe, hep-ph/0209353)

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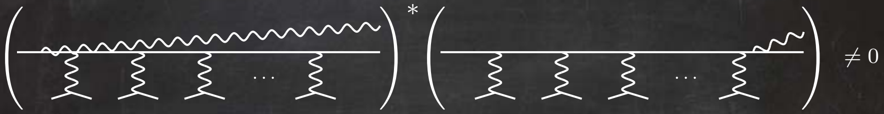

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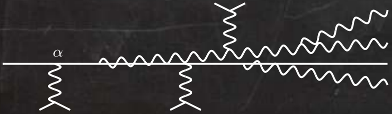

$$\frac{\Gamma_{\rm inelas}}{H} \sim \alpha \left(\frac{M_P}{m}\right) (\Gamma_\phi t)^{1/2}$$

(J. Ellis, MG et. al., 1512.05701)

(A. Kurkela, G. Moore, 1107.5050)

$$\frac{\partial f_\gamma}{\partial t} - Hk\frac{\partial f_\gamma}{\partial k} = \left| \quad \right|^2 + \left| \quad \right|^2 + \cdots$$

$$\equiv -\mathcal{C}^{2\leftrightarrow 2}[f_\gamma] - \mathcal{C}^{\text{“}1\leftrightarrow 2\text{”}}[f_\gamma] + \cdots$$

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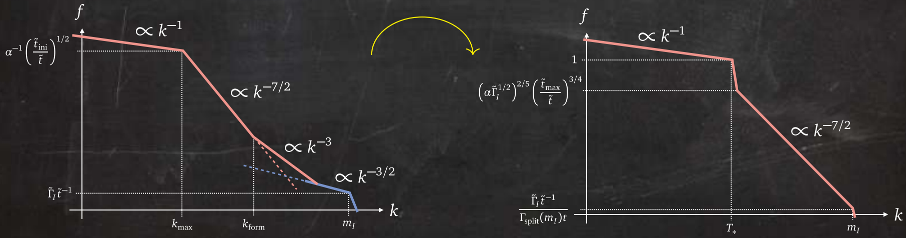
(K. Harigaya, K. Mukaida, 1312.3097; K. Mukaida, M. Yamada, 1506.07661)



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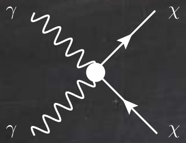
$$\boxed{\Gamma_\phi t_{\rm th} \simeq \alpha^{-16/5}\left(\frac{\Gamma_\phi m_\phi^2}{M_P^3}\right)^{2/5} \ll 1}$$

$$T_{\rm th} \simeq \alpha^{4/5} m_\phi \left(\frac{24}{\pi^2 g_{\rm reh}}\right)^{1/4} \left(\frac{\Gamma_\phi M_P^2}{m_\phi^3}\right)^{2/5}$$

Non-equilibrium DM production



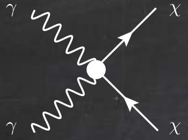
The interaction between DM and the relativistic background is *very* weak

$$\frac{\partial f_\chi}{\partial t} - p_1 \frac{\partial f_\chi}{\partial p_1} = -\frac{1}{2p_1} \int \frac{g_\chi\, d^3\mathbf{p}_2}{(2\pi)^3 2p_2} \frac{g_\gamma\, d^3\mathbf{k}_1}{(2\pi)^3 2k_1} \frac{g_\gamma\, d^3\mathbf{k}_2}{(2\pi)^3 2k_2} (2\pi)^4 \delta^{(4)}(k_1 + k_2 - p_1 - p_2)$$

$$\times \Big[|\mathcal{M}|^2_{\chi\chi\to\gamma\gamma} \quad f_\chi(p_1)\, f_\chi(p_2) \ [1 + f_\gamma(k_1)]\, [1 + f_\gamma(k_2)]$$

$$- |\mathcal{M}|^2_{\gamma\gamma\to\chi\chi} \quad f_\gamma(k_1)\, f_\gamma(k_2)\, [1 - f_\chi(p_1)\,]\, [1 - f_\chi(p_2)\,] \Big]$$

Non-equilibrium DM production



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$$\times\left[\,|\mathcal{M}|^2_{\chi\chi\to\gamma\gamma}\quad \cancel{f_\chi(p_1)\, f_\chi(p_2)}\ [1+f_\gamma(k_1)]\,[1+f_\gamma(k_2)]\right.$$

$$\left. -\,|\mathcal{M}|^2_{\gamma\gamma\to\chi\chi}\quad f_\gamma(k_1)\, f_\gamma(k_2)\, [1-\cancel{f_\chi(p_1)}]\,[1-\cancel{f_\chi(p_2)}]\,\right]$$

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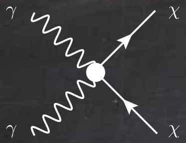

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$$\times \Bigg[|\mathcal{M}|^2_{\chi\chi\to\gamma\gamma} \quad \cancel{f_\chi(p_1)\, f_\chi(p_2)}\; [1+f_\gamma(k_1)]\,[1+f_\gamma(k_2)]$$

$$- |\mathcal{M}|^2_{\gamma\gamma\to\chi\chi} \quad f_\gamma(k_1)\, f_\gamma(k_2)\,[1-\cancel{f_\chi(p_1)}\,]\,[1-\cancel{f_\chi(p_2)}\,]\Bigg]$$

Integrating,

$$\boxed{\frac{dn_\chi}{dt} + 3Hn_\chi = 4g_\chi^2 g_\gamma^2 \int \frac{d^3\mathbf{k}_1}{(2\pi)^3 2k_1}\frac{d^3\mathbf{k}_2}{(2\pi)^3 2k_2}\,(k_1\cdot k_2)\,\sigma(s)_{\gamma\gamma\to\chi\chi}\, f_\gamma(k_1) f_\gamma(k_2)}$$

Non-thermal vs. thermal production

Before thermalization:

$$\dot{n}_\chi + 3Hn_\chi \;=\; \underbrace{18 g_\chi^2 g_\gamma^2 \frac{n_\gamma^2}{m_\phi^3} \int_0^{m_\phi^2} ds\,\sqrt{s}\,\sigma(s)\left[\ln\left(\frac{m_\phi + \sqrt{m_\phi^2 - s}}{\sqrt{s}}\right) - \frac{\sqrt{m_\phi^2 - s}}{m_\phi}\right]}_{\langle\sigma v\rangle_{\rm NT}\, n_\gamma^2}$$

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After thermalization:

$$\dot{n}_\chi + 3Hn_\chi \;=\; \frac{g_\chi^2 g_\gamma^2}{8\pi^4} \int dk_1\, dk_2\, d\cos\theta_{12}\, \frac{(k_1 k_2)^2 (1 - \cos\theta_{12})}{(e^{k_1/T} \pm 1)(e^{k_2/T} \pm 1)}\, \sigma(s)$$

$$\Big\downarrow \;\text{M.B.}$$

$$\simeq \underbrace{\frac{g_\chi^2 g_\gamma^2\, T}{2(2\pi)^4} \int_0^\infty ds\, s^{3/2} \sigma(s) K_1(\sqrt{s}/T)}_{\langle \sigma v \rangle_{\rm T} (n_\gamma^{\rm T})^2}$$

Let $\sigma(s) \sim \dfrac{s^{n/2}}{M^{n+2}}$ so that $\dfrac{\langle \sigma v \rangle}{g_\chi^2 g_\gamma^2} \propto \dfrac{1}{M^{n+2}} \begin{cases} T^n & \text{(T)} \\ m_\phi^n & \text{(NT)} \end{cases}$

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Thermal:

$$Y_\chi^{\rm T}(T_{\rm reh}) \propto \frac{M_P T_{\rm reh}^7}{g_{\rm reh}^{1/2} M^{n+2}} \times \begin{cases} \dfrac{1}{n-6}\left(T_{\rm max}^{n-6} - T_{\rm reh}^{n-6}\right), & n > -1,\ n \neq 6 \\ \ln\left(\dfrac{T_{\rm max}}{T_{\rm reh}}\right), & n = 6 \end{cases}$$

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Non-thermal:

$$Y_\chi^{\mathrm{NT}}(T_{\mathrm{reh}}) \propto g_{\mathrm{reh}}^{3/2} \frac{T_{\mathrm{reh}}^3 M_P m_\phi^{n-2}}{M^{n+2}} (\Gamma_\phi t_{\mathrm{th}})$$

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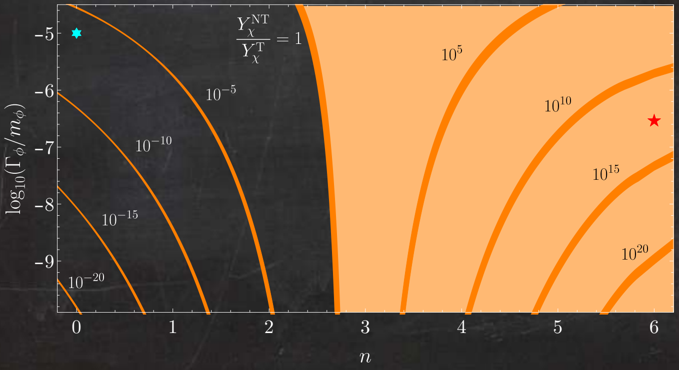
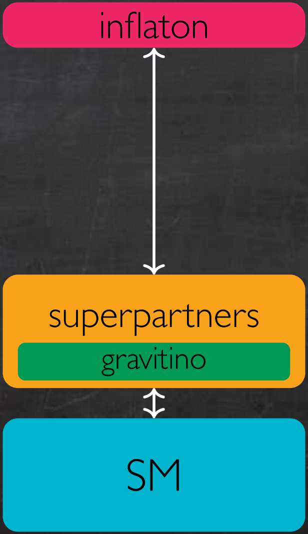


Light Gravitino

$\phi \to g + g$ and weak scale supersymmetry



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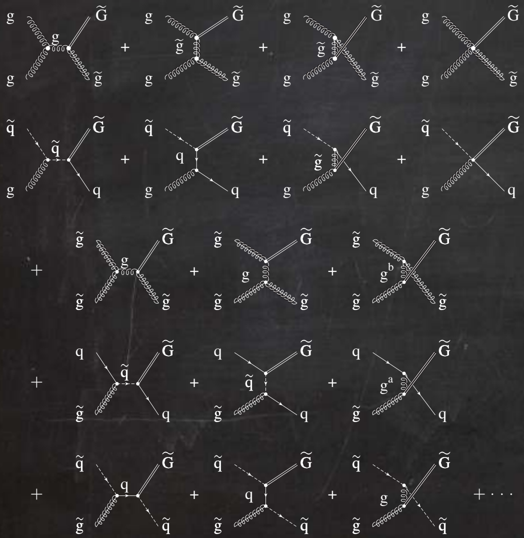

$$\langle \sigma v \rangle_{\rm NT} = \sum_{i=1}^{3} \frac{16\pi\alpha_i}{M_P^2} |f^{abc}|^2 \left(1 + \frac{m_{\tilde{g}_i}^2}{3m_{3/2}^2}\right)$$

$$\langle \sigma v \rangle_{\rm T} = \sum_{i=1}^{3} \frac{3\pi^2 c_i \alpha_i}{4\zeta(3) M_P^2} \left(1 + \frac{m_{\tilde{g}_i}^2}{3m_{3/2}^2}\right) \ln\left(\frac{k_i}{g_i}\right)$$

(M. Bolz et. al., hep-ph/0012052)

(V. Rychkov, A. Strumia, hep-ph/0701104)

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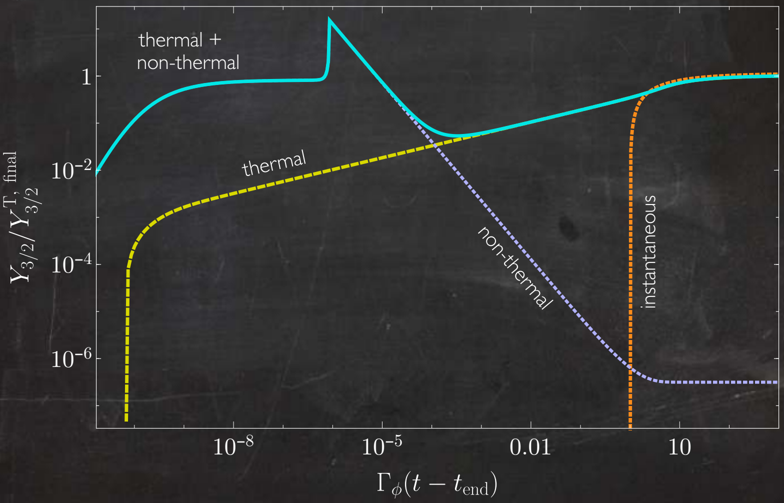
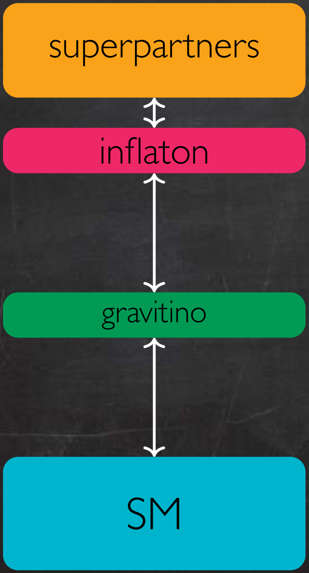


Heavy Gravitino

$\phi \to g + g$ and high scale supersymmetry (E. Dudas, Y. Mambrini, K. Olive, 1704.03008)



$$M_{\rm susy} = \frac{F}{\Lambda_{\rm mess}}\,, \quad \Lambda_{\rm mess} \geq M_{\rm susy}$$

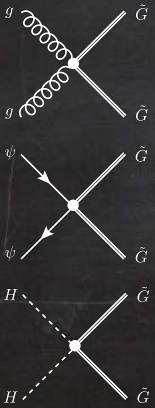
$$m_{3/2} = \frac{F}{\sqrt{3}M_P} \gtrsim 0.1\,{\rm EeV}$$

Heavy Gravitino

$\phi \to g + g$ and high scale supersymmetry

Leading-order universal Goldstino-matter interactions $(F = \sqrt{3} m_{3/2} M_P)$:

$$\mathcal{L}_{2G} = \frac{i}{2F^2} \left(G \sigma^\mu \partial^\nu \bar{G} - \partial^\nu G \sigma^\mu \bar{G} \right) T_{\mu\nu}$$



$(F_\mu^{\lambda\, a} F^a_{\nu\lambda})$

$(\bar{\psi} \bar{\sigma}_\mu \partial_\nu \psi + \cdots)$

$(\partial_\mu H \partial_\nu H^\dagger + \mathrm{h.c.})$

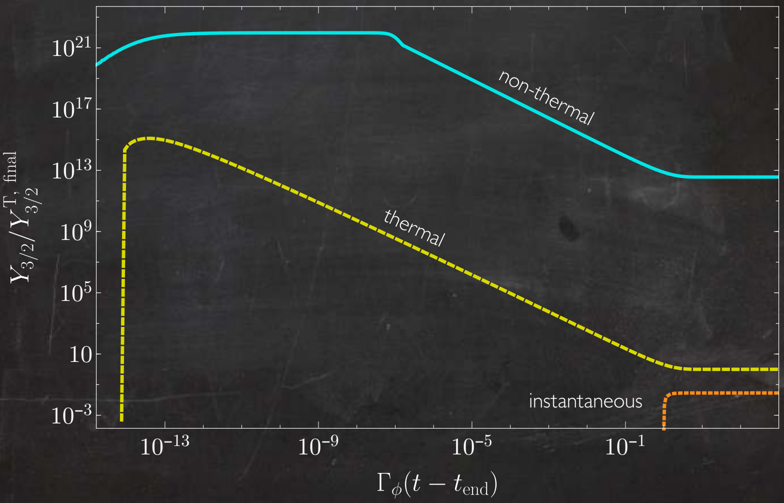
$$\langle \sigma v \rangle_{\mathrm{NT}} = \frac{154 m_\phi^6}{5(64)^2 F^4}$$

$$\langle \sigma v \rangle_{\mathrm{T}} = \frac{6400 \pi^{11} T^6}{(945)^2 \zeta(3)^2 F^4}$$

(K. Benakli et. al., 1701.06574)

Heavy Gravitino

$\phi \to g + g$ and high scale supersymmetry



Heavy Gravitino

$\phi \to g + g$ and high scale supersymmetry

Assuming instantaneous reheating and thermalization...

$$\Omega^{\rm inst}_{3/2} h^2 \simeq 0.11 \left(\frac{0.1\,{\rm EeV}}{m_{3/2}}\right)^3 \left(\frac{T_{\rm reh}}{2.2\times 10^{10}}\right)^7$$

vs. accounting for their finite duration...

$$\Omega_{3/2} h^2 \simeq 0.11 \left(\frac{0.1\,{\rm EeV}}{m_{3/2}}\right)^3 \left(\frac{T_{\rm reh}}{2.2\times 10^{8}}\right)^{19/5} \left(\frac{m_\phi}{3\times 10^{13}\,{\rm GeV}}\right)^{24/5} \left(\frac{0.030}{\alpha_3}\right)^{16/5}$$

(similar analysis applies to DM production through heavy spin-2 mediators, N. Bernal et. al. 1803.01866)

Freezing-in dark matter through a heavy invisible Z'

Gautam Bhattacharyya[a],* Maíra Dutra[b],† Yann Mambrini[b],‡ Mathias Pierre[b]§
[a] *Saha Institute of Nuclear Physics, HBNI, 1/AF Bidhan Nagar, Kolkata 700064, India*
[b] *Laboratoire de Physique Théorique (UMR8627), CNRS, Univ. Paris-Sud, Université Paris-Saclay, 91405 Orsay, France*

(1806.00016 [hep-ph])

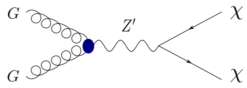


FIG. 1: *Production of dark matter through gluon fusion in the early Universe*

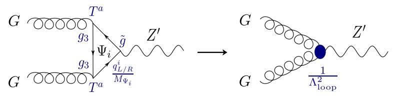
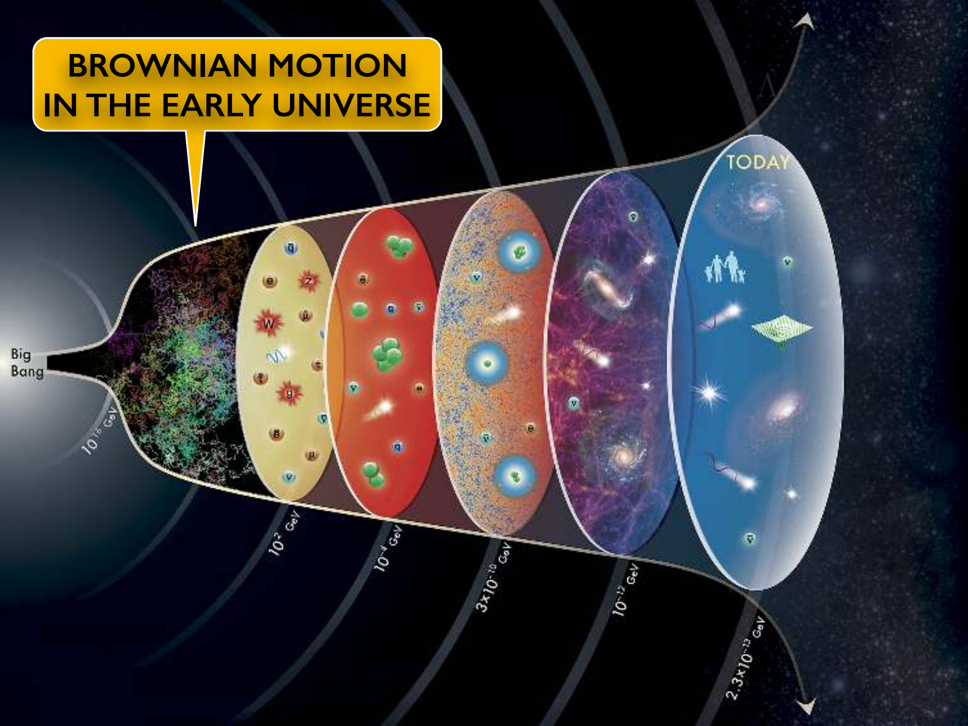


FIG. 4: *Triangle diagram generated containing heavy chiral fermions Ψ_i (left panel), and the resulting effective vertex at low energy (right panel).*

$$\langle\sigma v\rangle\, n_\gamma^2 = \quad R(T) \approx \begin{cases} 2\times 10^2\, \dfrac{\alpha^2}{\Lambda^4}\dfrac{m_\chi^2}{M_{Z'}^4} T^{10} & \text{(fermionic DM)} \quad \rightarrow n=4 \\ 10^4\, \dfrac{\beta^2}{\Lambda^4 M_{Z'}^4} T^{12} & \text{(abelian DM)} \quad \rightarrow n=6 \\ 2\times 10^9\, \dfrac{\gamma^2}{\Lambda^4 M_{Z'}^4} T^{16} & \text{(non-abelian DM)} \quad \rightarrow n=10 \end{cases} \tag{14}$$

Conclusion

- UV-dominated freeze-in during reheating is realized for $\sigma(s) \sim s^{n/2}$, $n > 2$
- Thermalization time-scale determines the DM abundance at late times
- Effect important for DM production in very high scale susy models, or for heavy spin-2 mediators
- Preheating? $N_{\rm eff}$?


BROWNIAN MOTION
IN THE EARLY UNIVERSE
Big
Bang
10^15 GeV
10^2 GeV
10^-4 GeV
3x10^-10 GeV
10^-12 GeV
2.3x10^-13 GeV
TODAY
Λ

Complexity in the Early Universe

What if the early universe is not simple?

Complexity in the Early Universe

What if the early universe is not simple?

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2} G_{ij}\, \partial_\mu \phi^i \partial^\mu \phi^j - V(\boldsymbol{\phi}) \right]$$

What if the early universe is not simple?

$$\mathcal{S} = \int d^4x\,\sqrt{-g}\left[\frac{1}{2}G_{ij}\,\partial_\mu\phi^i\partial^\mu\phi^j - V(\phi)\right]$$

$$\downarrow$$

$$\left[(\partial_\tau^2 + k^2) + \mathbf{P}(\tau)\partial_\tau + \mathbf{M}(\tau)\right]\cdot\boldsymbol{\chi} \;=\; 0$$

where $d\tau = dt/a$, χ^i canonically normalized, gauge invariant perturbations, and

$$(\mathbf{P})^i_j = 2\Gamma^i_{jk}\phi^{k\,\prime}$$

$$(\mathbf{M})^i_j = -\frac{a''}{a}\,\delta^i_j - 2H\Gamma^i_{jk}\phi^{k\,\prime} + a^2\left[\frac{\dot{\phi}^i}{H}V_{,k} + (G^{ij}V_{,j})_{,k} + \cdots\right]$$

Complexity in the Early Universe

What if the early universe is not simple?

$$\mathcal{S} = \int d^4x\,\sqrt{-g}\left[\frac{1}{2}G_{ij}\,\partial_\mu\phi^i\partial^\mu\phi^j - V(\boldsymbol{\phi})\right]$$

$$\downarrow$$

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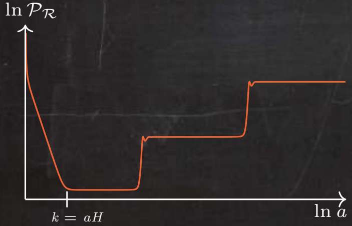
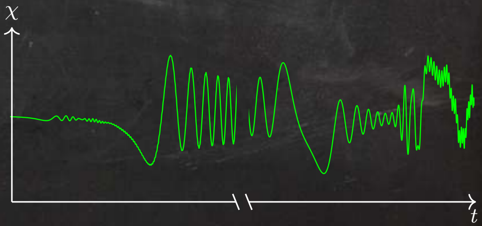
inflation



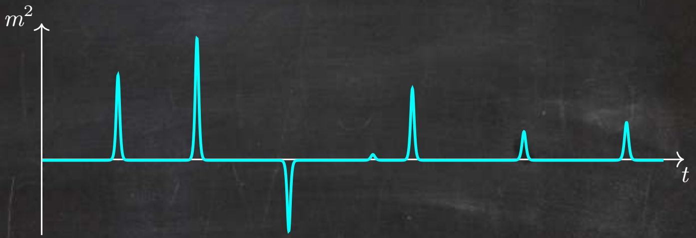
(p)reheating



Stochastic Particle Production

Spectator field in non-expanding background (M. Amin, D. Baumann 1512.02637)

$$\left(\frac{d^2}{dt^2} + k^2 + M^2 + m^2(t)\right)\chi(t) = 0$$



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$$\left(\frac{d^2}{dt^2} + k^2 + M^2 + m^2(t)\right)\chi(t) = 0$$

After the j-th event,

$$\chi_j(t) \equiv \frac{1}{\sqrt{2\omega}}\left[\beta_j e^{i\omega t} + \alpha_j e^{-i\omega t}\right], \qquad \begin{pmatrix}\boldsymbol{\beta}_j \\ \boldsymbol{\alpha}_j\end{pmatrix} = \mathbf{M}_j \begin{pmatrix}\boldsymbol{\beta}_{j-1} \\ \boldsymbol{\alpha}_{j-1}\end{pmatrix}$$

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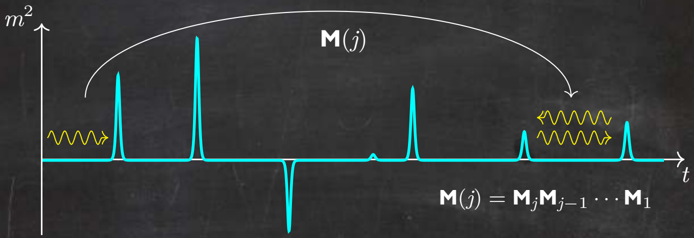

$$\mathbf{M}(j) = \mathbf{M}_j\mathbf{M}_{j-1}\cdots\mathbf{M}_1$$

After the j-th event,

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Stochastic Particle Production

Spectator field in non-expanding background (M. Amin, D. Baumann 1512.02637)

$$\left(\frac{d^2}{dt^2}+k^2+M^2+m^2(t)\right)\chi(t)=0$$

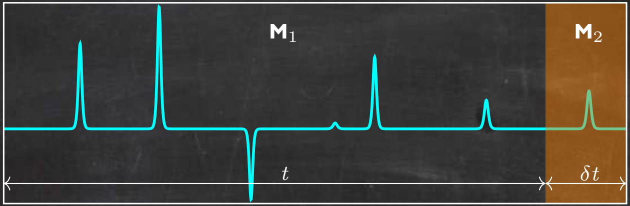
A random walk (with drift) for the occupation number

$$n(j)=\frac{1}{2\omega}\left(|\dot{\chi}_j|^2+\omega^2|\chi_j|^2\right)-\frac{1}{2}=|\beta_j|^2$$

Stochastic Particle Production

Spectator field in non-expanding background (M. Amin, D. Baumann 1512.02637)

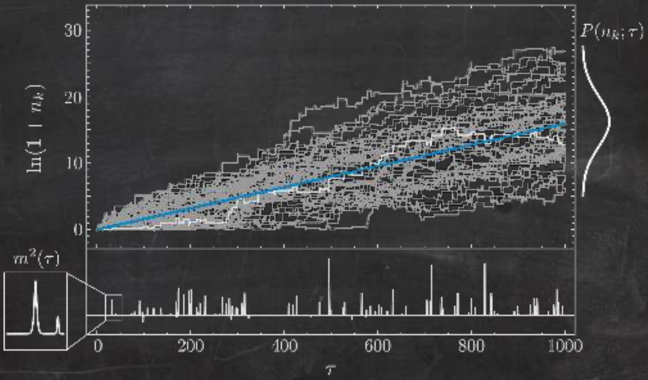
$$\left(\frac{d^2}{dt^2}+k^2+M^2+m^2(t)\right)\chi(t)=0$$



$$\partial_t P(\mathbf{M};t) = -\partial_{\mathbf{M}}\left[\frac{\langle\delta\mathbf{M}\rangle_{\mathbf{M}_2}}{\delta t}P(\mathbf{M};t)\right]+\frac{1}{2!}\partial^2_{\mathbf{M}}\left[\frac{\langle\delta\mathbf{M}^2\rangle_{\mathbf{M}_2}}{\delta t}P(\mathbf{M};t)\right]+\cdots$$

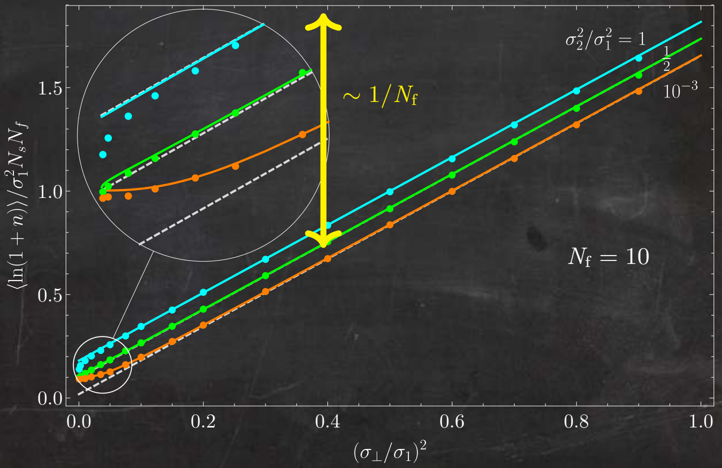
Single field:

$$\frac{1}{\sigma^2}\frac{\partial}{\partial t}P(n;t) = \frac{\partial}{\partial n}\left[n^2\frac{\partial}{\partial n}P(n;t)\right]$$



- log-normal
- late-time equilibrium $\longleftrightarrow$ maximal entropy

Multiple fields:



Similar

Conformal Spectator in dS

Include now the effect of expansion

$$\left(\frac{d^2}{dt^2} + 3H\frac{d}{dt} + \frac{k^2}{a^2} + M^2 + m^2(t)\right)\chi_{\mathbf{k}}(t) = 0$$

$a = a_0\, e^{H(t-t_0)}$
(de Sitter)

$M^2 = 2H^2$
(conformal mass)

$m^2(t) = \sum_j m_j \delta(t - t_j)$
(non-adiabatic)

Conformal Spectator in dS

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(non-adiabatic)

Or, with $X_{\mathbf{k}} = a\chi$, $\tau = a_0/a$

$$\frac{d^2 X_{\mathbf{k}}}{d\tau^2} + \left(\frac{k^2}{(a_0 H)^2} + \sum_j \frac{m_j}{\tau H}\delta(\tau - \tau_j)\right) X_{\mathbf{k}} = 0$$

In between scatterings

$$X_{\mathbf{k}}(\tau) = \alpha_{\mathbf{k},j} f_{\mathbf{k}}(\tau) + \beta_{\mathbf{k},j} f^*_{\mathbf{k}}(\tau)\,, \qquad f_{\mathbf{k}}(\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}}$$

Junction conditions

$$\begin{pmatrix}\beta_j\\ \alpha_j\end{pmatrix} = \mathbf{M}_j \begin{pmatrix}\beta_{j-1}\\ \alpha_{j-1}\end{pmatrix} = \mathbf{M}(j)\begin{pmatrix}\beta_0\\ \alpha_0\end{pmatrix}$$

with

$$\mathbf{M}_j = \mathbb{1} + i m_j a_j \begin{pmatrix}|f(\tau_j)|^2 & f(\tau_j)^2\\ -f(\tau_j)^{*2} & -|f(\tau_j)|^2\end{pmatrix}$$

and

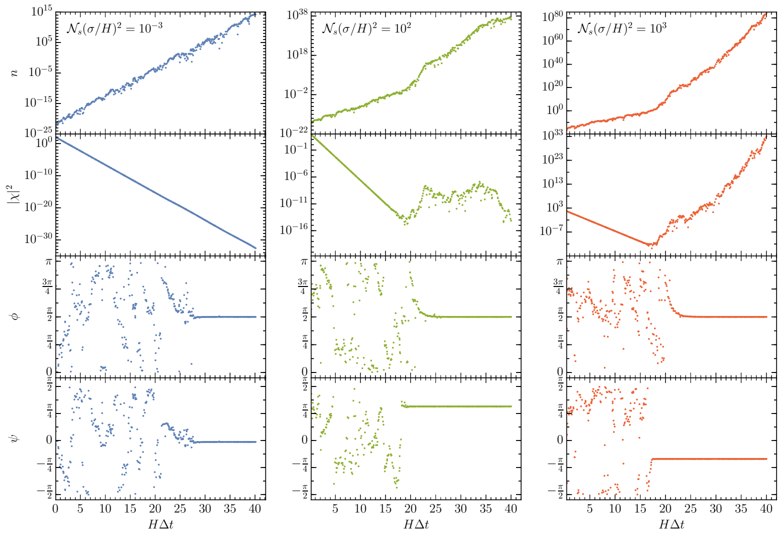
$$\mathbf{M}(j) = \mathbf{M}_j\mathbf{M}_{j-1}\cdots\mathbf{M}_1 = \begin{pmatrix}e^{i\phi} & 0\\ 0 & e^{-i\phi}\end{pmatrix}\begin{pmatrix}\sqrt{1+n} & \sqrt{n}\\ \sqrt{n} & \sqrt{1+n}\end{pmatrix}\begin{pmatrix}e^{i\psi} & 0\\ 0 & e^{-i\psi}\end{pmatrix}$$

$$\phi(j) = \frac{1}{2}\arg\left(\beta_j\alpha_j^*\right)\,,\quad \psi(j) = -\frac{1}{2}\arg\left(\beta_j\alpha_j\right)\,,\quad n(j) = |\beta_j|^2$$

Strength of interaction (perturbativity) is quantified by

$$\frac{N_s}{H\Delta t}\frac{\mathrm{Var}[m_j]}{H^2} \equiv \mathcal{N}_s\frac{\sigma^2}{H^2}$$

Perturbative (Fokker-Planck)



Non-perturbative (Backreaction)

With $\lambda = 2n+1, \quad \tilde{\lambda} = \sqrt{\lambda^2 - 1}, \quad \xi = \frac{N_s}{2}\left(\frac{\sigma}{2k_{\rm phys}}\right)^2$

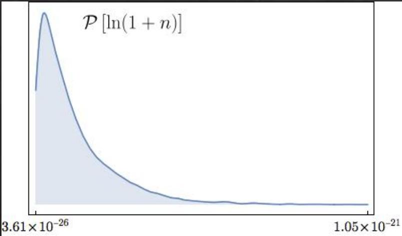
Sub-horizon:

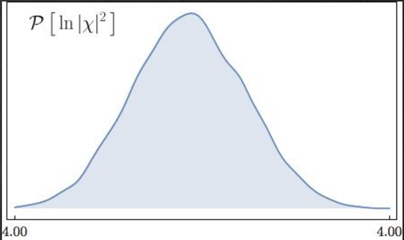
$$\frac{\partial P}{\partial \xi} = -2\frac{\partial}{\partial \lambda}(\lambda\, P) + \frac{\partial^2}{\partial \lambda^2}\left(\tilde{\lambda}^2 P\right)$$

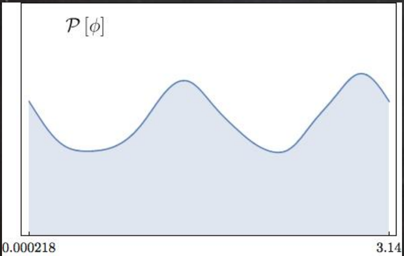
Super-horizon:

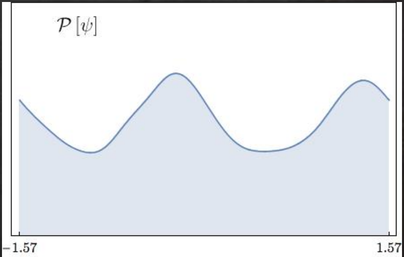
$$\frac{1}{4}\frac{\partial P}{\partial \xi} = -\frac{\partial}{\partial \lambda}\left[\lambda \cos^2\phi\, P\right] + \frac{\partial}{\partial \phi}\left[\cos^3\phi \sin\phi\, P\right] + \frac{1}{2}\frac{\partial^2}{\partial \lambda^2}\left[\lambda^2 \sin^2 2\phi\, P\right]$$
$$+ 2\frac{\partial^2}{\partial \lambda \partial \phi}\left[\lambda \cos^3\phi \sin\phi\, P\right] + \frac{1}{2}\frac{\partial^2}{\partial \phi^2}\left[\cos^4\phi\, P\right]$$

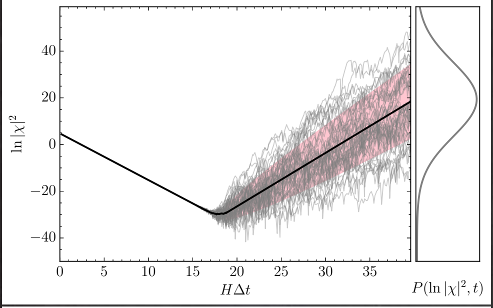
$$N_s = 1000, \quad N_{\rm real} = 1000, \quad \mathcal{N}_s(\sigma/H)^2 = 10^{-4}$$

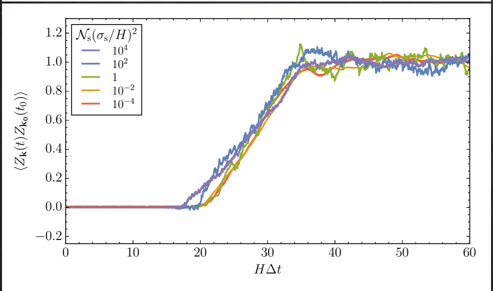












Mean
$$\langle \ln|\chi|^2 \rangle \propto H(t - t_*)$$

Variance
$$\mathrm{Var}\,[\ln|\chi|^2] \propto H(t - t_*)$$

$$\langle Z_{\mathbf{k}}(t) Z_{\mathbf{k_0}}(t_0)\rangle \propto \min(t - t_\star, t_0 - t_\star) \times \theta(t - t_\star)$$

where
$$Z_{\mathbf{k}}(t) = \ln|\chi|^2 - \langle \ln|\chi|^2 \rangle$$

GEOMETRIC RANDOM WALK

Power Spectrum

background dynamics $\longrightarrow$ particle production $\longleftrightarrow$ curvature fluctuations

$$\langle \chi_{\mathbf{k}_1} \chi_{\mathbf{k}_2} \cdots \rangle \qquad \langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \cdots \rangle$$

$$\ddot{\pi}_{\mathbf{k}} + [3H + \mathcal{O}_d]\,\pi_{\mathbf{k}} + \frac{k^2}{a^2}\pi_{\mathbf{k}} \;=\; \mathcal{O}_s\left(\langle \chi\chi \cdots \rangle_{\mathbf{k}}\right)$$

dissipation driving

(D. Green, B. Horn, L. Senatore, E. Silverstein 0902.1006
D. L. Nacir, R. Porto, L. Senatore, M. Zaldarriaga 1109.4192
M. Dias, J. Frazer, D. Marsh 1604.05970
R. Flauger, M. Mirbabayi, L. Senatore, E. Silverstein 1606.00513)

Power Spectrum

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dissipation $\qquad$ driving

$$\Delta_\zeta^2 \;\sim\; \Delta_\zeta^2\big|_{\chi=0} \;+\; \mathcal{C} \times \left(\Delta_\zeta^2\big|_{\chi=0}\right)^2 \times \mathcal{N}_s \frac{\sigma^2}{H^2}$$

Power Spectrum

background dynamics $\longrightarrow$ particle production $\longleftrightarrow$ curvature fluctuations

$\langle \chi_{\mathbf{k}_1} \chi_{\mathbf{k}_2} \cdots \rangle$ $\qquad$ $\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \cdots \rangle$

$$\ddot{\pi}_{\mathbf{k}} + [3H + \mathcal{O}_d]\,\pi_{\mathbf{k}} + \frac{k^2}{a^2}\pi_{\mathbf{k}} \;=\; \mathcal{O}_s\left(\langle \chi\chi\cdots\rangle_{\mathbf{k}}\right)$$

dissipation $\qquad$ driving

$$\langle \zeta^n \rangle \;\sim\; \langle \zeta^n \rangle_{\chi=0} \;+\; \mathcal{C}' \times \langle \zeta^2 \rangle^n_{\chi=0} \times \exp\left[\frac{n^2}{2} F\left(\mathcal{N}_s \frac{\sigma^2}{H^2}\right)\right]$$

from log-normality

(work in progress!)

The Future

- Non-conformal (massless) spectator
- Tensor fluctuations?
- (P)reheating
- Strong scattering regime: Random Matrix Theory?
- Non-linearity / Backreaction