Curvature Perturbations From Stochastic Particle Production During Inflation

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Work in progress with
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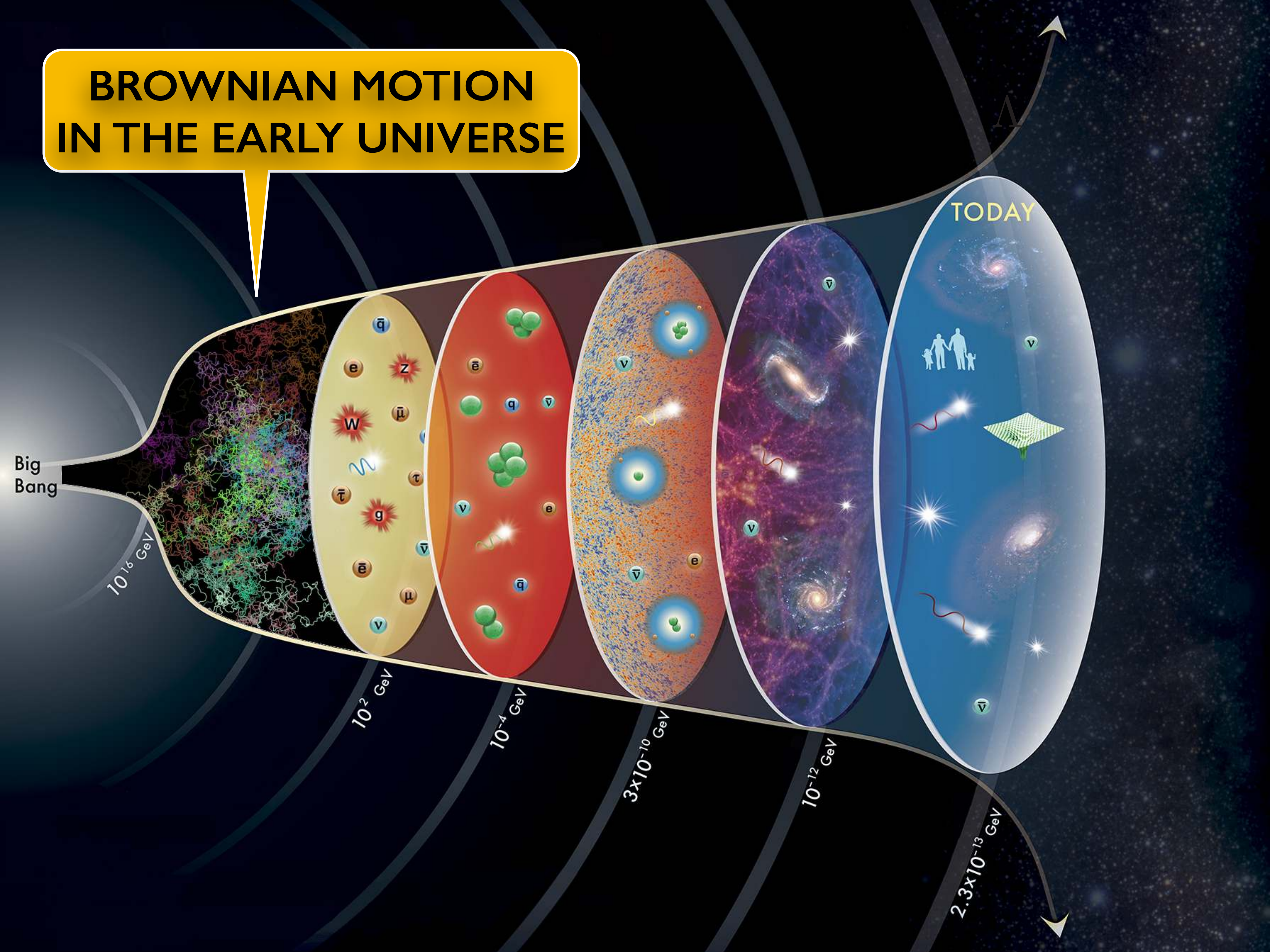
RICE


BROWNIAN MOTION IN THE EARLY UNIVERSE
Big Bang
10^{16} GeV
10^{2} GeV
10^{-4} GeV
3×10^{-10} GeV
10^{-12} GeV
2.3×10^{-13} GeV
TODAY
Λ

What if the early universe is not simple?

$$\mathcal{S} \;=\; \int d^4x\,\sqrt{-g}\,\left[\frac{1}{2}G_{ij}\partial_\mu\phi^i\partial^\mu\phi^j - V(\boldsymbol{\phi})\right]$$

$$\downarrow$$

$$\ddot{\phi}^i + \Gamma^i_{jk}\dot{\phi}^j\dot{\phi}^k + 3H\dot{\phi}^i + G^{ij}V_{,j} \;=\; 0$$

(non-linear, complicated)

$$\downarrow$$

$$\left[\partial_t^2 + \left(\mathbf{P}(t) + 3H\right)\partial_t + \frac{k^2}{a^2} + \mathbf{M}^2(t)\right]\chi \;=\; 0$$

(linear, complicated)

$$(\mathbf{P})^i_k \;=\; 2\Gamma^i_{jk}\dot{\phi}^j$$

$$(\mathbf{M}^2)^i_k \;=\; (G^{ij}V_{,j})_{,k} + \frac{\dot{\phi}^i}{H}V_{,k} + 3G_{jk}\dot{\phi}^i\dot{\phi}^j + \cdots$$

Consider a spectator field in an expanding universe

$$\left(\frac{d^2}{dt^2} - \frac{\nabla^2}{a^2} + 3H\frac{d}{dt} + M^2 + m^2(t)\right)\chi(t,\mathbf{x}) \;=\; 0$$

$$a(t) \;=\; e^{H(t-t_0)}$$

(de Sitter)

$$M^2 \;=\; 2H^2$$ (conformal)

$$M^2 \;=\; 0$$ (massless)

$$m^2(t) \;=\; \sum_j m_j\,\delta(t-t_j)$$

(localized non-adiabatic events)

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$$a(t) \;=\; e^{H(t-t_0)}$$

(de Sitter)

$$\frac{k}{aH} \;=\; |k\tau| \;\gtrless\; 1$$

(sub-, super-horizon)

$M^2 \;=\; 2H^2$ (conformal)

$M^2 \;=\; 0$ (massless)

$$X_k \;\equiv\; a\chi_k$$
$$\;=\; \alpha_{k,j} f_k(\tau) + \beta_{k,j} f_k^*(\tau)$$

$$f_k(\tau) \;=\; \frac{e^{-ik\tau}}{\sqrt{2k}} \times \begin{cases} 1 \\ \left(1 - \frac{i}{k\tau}\right) \end{cases}$$

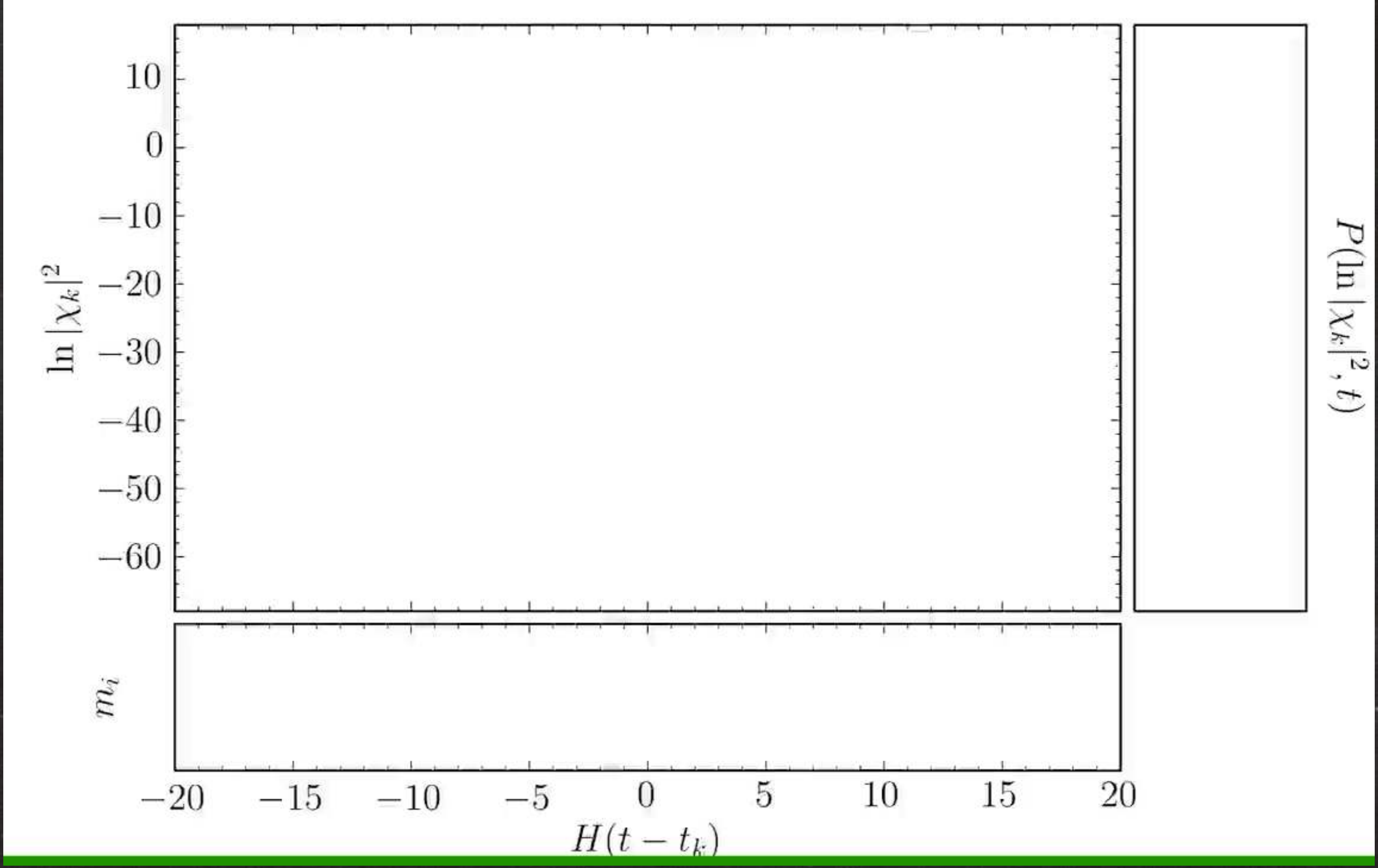
$$m^2(t) \;=\; \sum_j m_j\,\delta(t - t_j)$$

(localized non-adiabatic events)

$m_j\,,\; t_j$ random

$$\langle m_j \rangle \;=\; 0$$
$$\langle m_i m_j \rangle \;=\; \sigma^2 \delta_{ij}$$

Brute force: solve numerically



Equation of motion $\longrightarrow$ junction conditions

$$\begin{pmatrix}\beta_j\\ \alpha_j\end{pmatrix} = \mathbf{M}_j \begin{pmatrix}\beta_{j-1}\\ \alpha_{j-1}\end{pmatrix} = \mathbf{M}(j) \begin{pmatrix}\beta_0\\ \alpha_0\end{pmatrix}\,, \qquad |\alpha_j|^2 - |\beta_j|^2 = 1$$

where

$$\mathbf{M}_j = \mathbf{1} + i m_j a_j \begin{pmatrix}|f_k(\tau_j)|^2 & f_k(\tau_j)^2\\ -f_k(\tau_j)^{*2} & -|f_k(\tau_j)|^2\end{pmatrix}$$

$$\mathbf{M}(j) = \mathbf{M}_j\mathbf{M}_{j-1}\cdots\mathbf{M}_1 = \begin{pmatrix}e^{i\phi} & 0\\ 0 & e^{-i\phi}\end{pmatrix}\begin{pmatrix}\sqrt{1+n} & \sqrt{n}\\ \sqrt{n} & \sqrt{1+n}\end{pmatrix}\begin{pmatrix}e^{i\psi} & 0\\ 0 & e^{-i\psi}\end{pmatrix}$$

$$\phi(j) = \frac{1}{2}\arg\left(\beta_j\alpha_j^*\right)\,,\quad \psi(j) = -\frac{1}{2}\arg\left(\beta_j\alpha_j\right)\,,\quad n(j) = |\beta_j|^2$$

$$X_k(\tau) \;=\; (1+n)^{1/2}e^{-i(\phi+\psi)}f_k(\tau) + n^{1/2}e^{i(\phi-\psi)}f_k^*(\tau)$$

Evolution of the transfer matrix is Markovian

$$\mathbf{M}(j) \;=\; \mathbf{M}_j \mathbf{M}(j-1)$$

$$\downarrow$$

$$P_{t+\delta t}(\mathbf{M}) = \int d\mathbf{M}_j \, P_t(\mathbf{M}_j^{-1}\mathbf{M}) P_{\delta t}(\mathbf{M}_j)$$

(P is a transition probability)

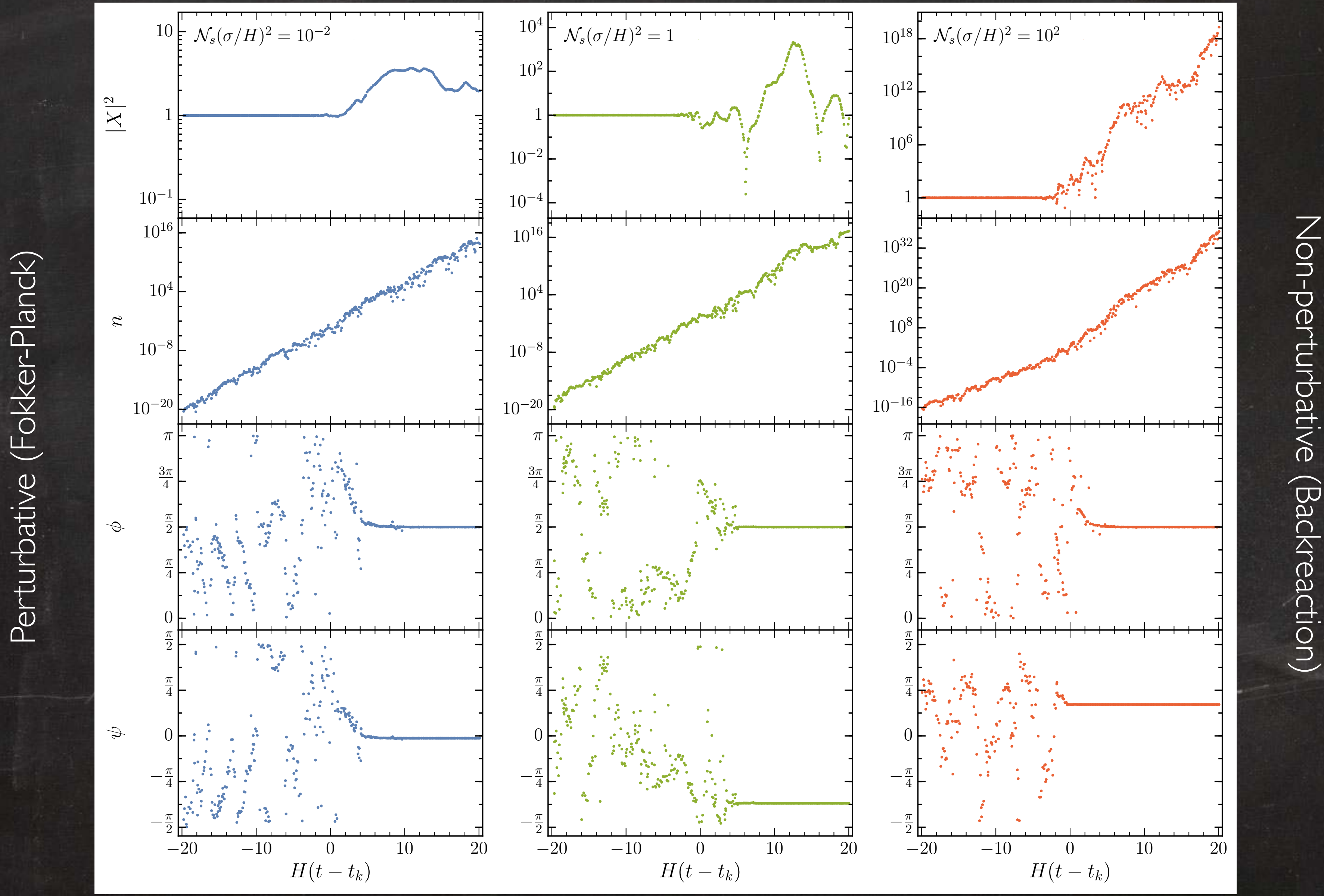
$$\downarrow$$

$$\partial_t P(\mathbf{M};t) = -\partial_{\mathbf{M}}\left[\frac{\langle \delta \mathbf{M}\rangle_{\mathbf{M}_j}}{\delta t} P(\mathbf{M};t)\right] + \frac{1}{2!}\partial^2_{\mathbf{M}}\left[\frac{\langle \delta \mathbf{M}^2\rangle_{\mathbf{M}_j}}{\delta t} P(\mathbf{M};t)\right] + \cdots$$

$$\downarrow$$

$$\boxed{\frac{\partial P}{\partial t} = -\sum_b \frac{\partial}{\partial \lambda_b}\left[\frac{\langle \delta\lambda_b\rangle_{\delta t}}{\delta t} P\right] + \frac{1}{2}\sum_{b,c} \frac{\partial^2}{\partial\lambda_b \partial\lambda_c}\left[\frac{\langle \delta\lambda_b \delta\lambda_c\rangle_{\delta t}}{\delta t} P\right], \qquad \{\lambda_a\} = \{n, \phi, \psi\}}$$

A closed form analytical approximation can be obtained for sub-horizon modes. Only marginals and low moments are available analytically for super-horizon modes

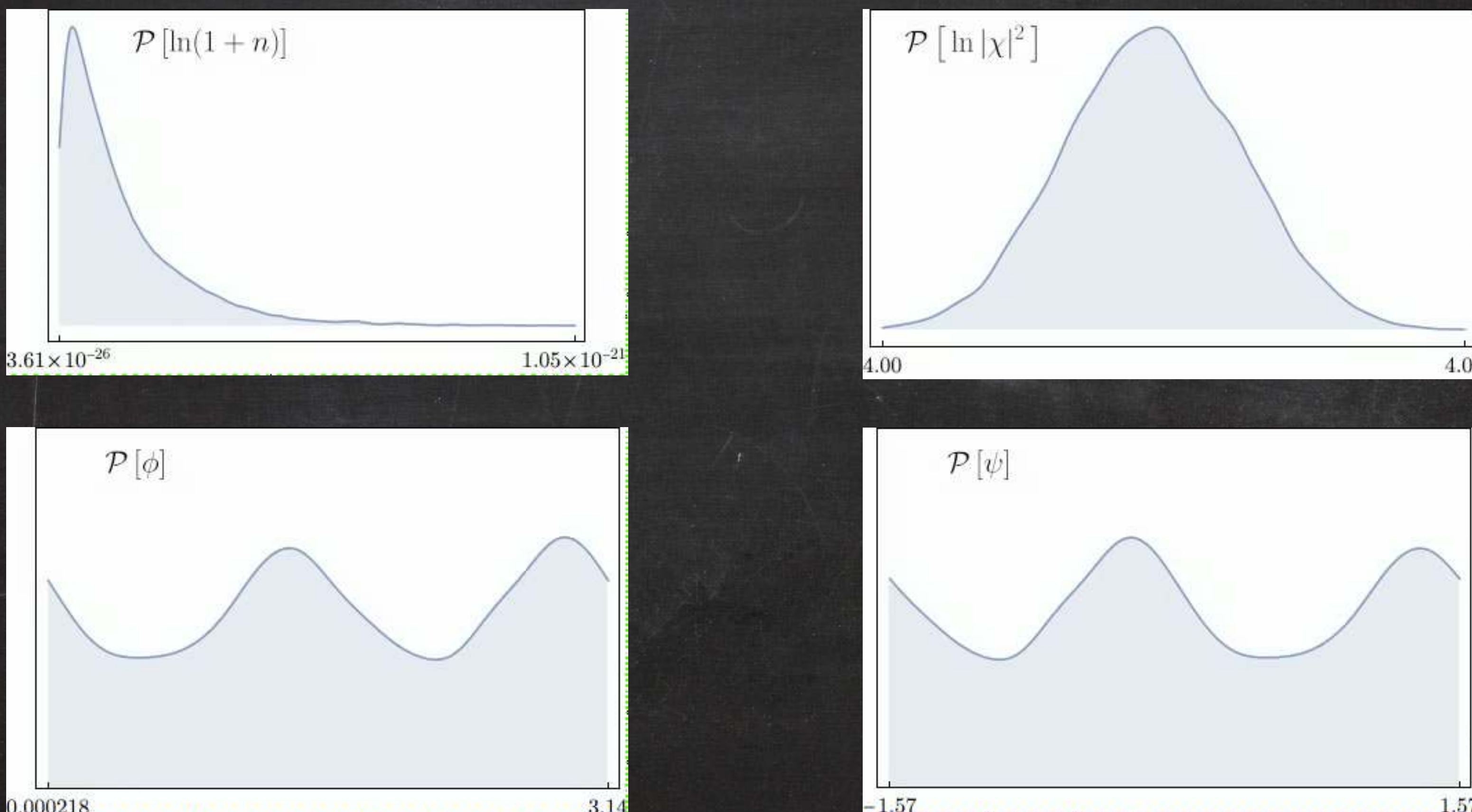


Perturbative (Fokker-Planck)
Non-perturbative (Backreaction)
$\mathcal{N}_s(\sigma/H)^2 = 10^{-2}$
$\mathcal{N}_s(\sigma/H)^2 = 1$
$\mathcal{N}_s(\sigma/H)^2 = 10^2$
$|X|^2$
n
ϕ
ψ
$H(t-t_k)$

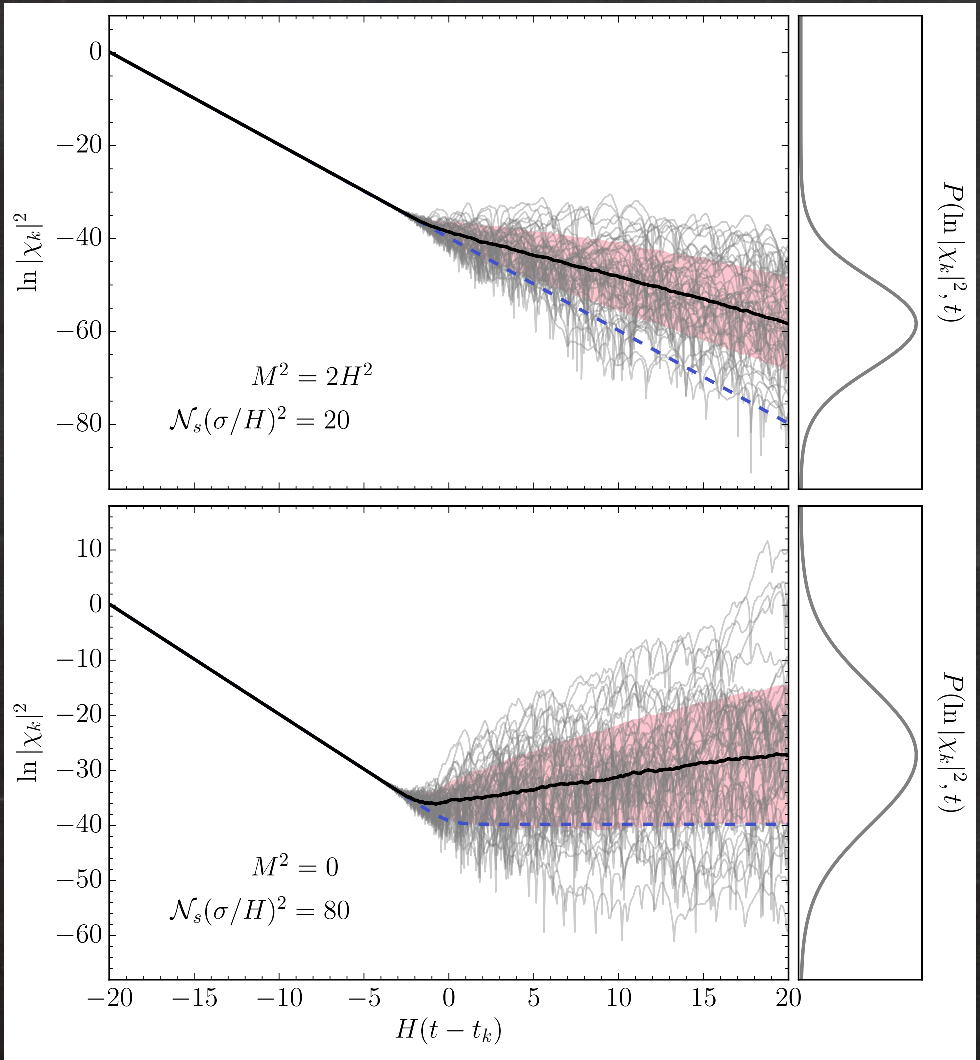
1. The strength of the non-adiabaticity is uniquely quantified by

$$\frac{N_s}{H\Delta t}\,\frac{\mathrm{Var}[m_j]}{H^2} \equiv \mathcal{N}_s\,\frac{\sigma^2}{H^2}$$

2. Distributions



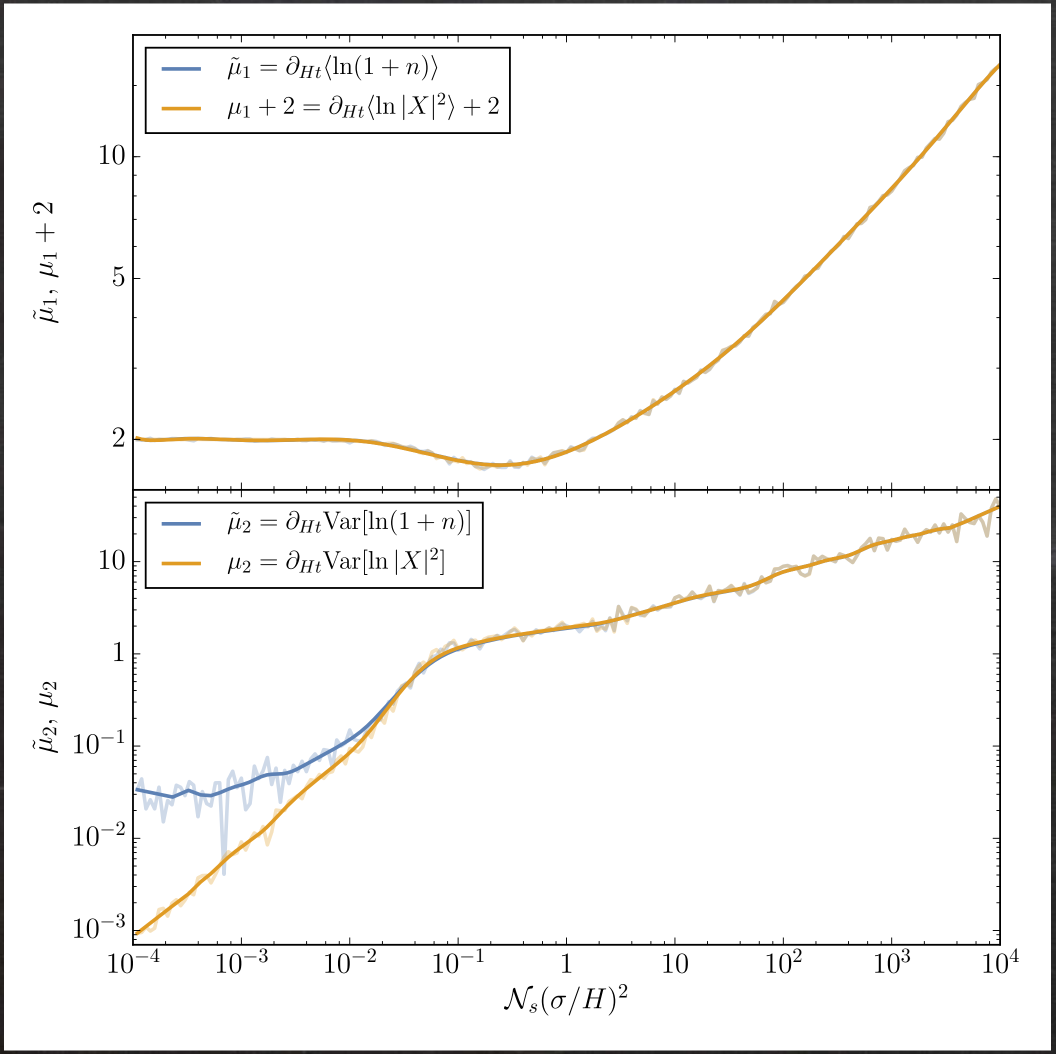
3. Inside the horizon the field is always near its vacuum state. Outside the horizon, the mean and variance grow linearly with cosmic time



$$\partial_{Ht}\langle \ln|\chi_k|^2\rangle = \mu_1 - 2$$
$$\partial_{Ht}\mathrm{Var}\left[\ln|\chi_k|^2\right] = \mu_2$$

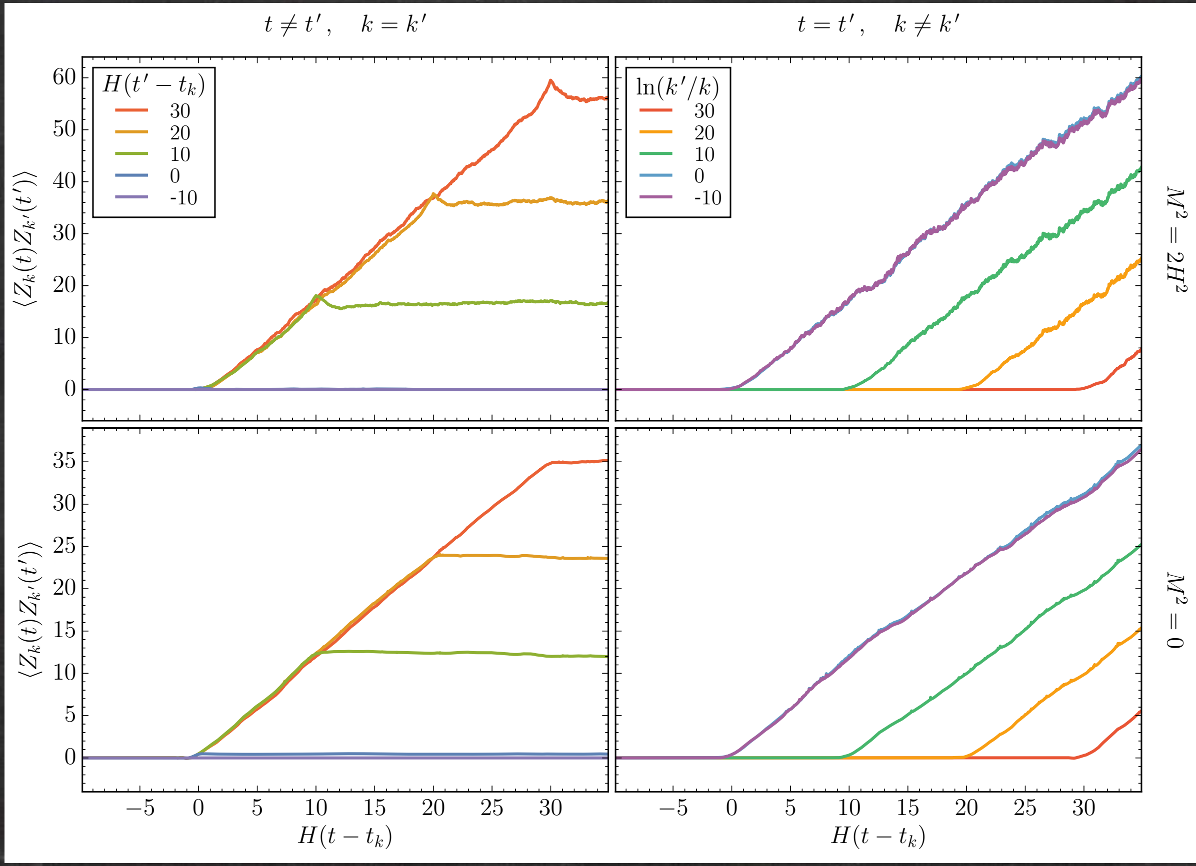
3. Inside the horizon the field is always near its vacuum state. Outside the horizon, the mean and variance grow linearly with cosmic time

(conformal)



$$\partial_{Ht}\langle \ln|\chi_k|^2\rangle = \mu_1 - 2$$
$$\partial_{Ht}\mathrm{Var}\left[\ln|\chi_k|^2\right] = \mu_2$$

3. The two-point function for $Z_k(t) \equiv \ln|\chi_k|^2 - \langle \ln|\chi_k|^2 \rangle$



$$\langle Z_k(t) Z_{k'}(t') \rangle \simeq \mu_2 H \min[t - t_k, t - t_{k'}, t' - t_k, t' - t_{k'}]$$

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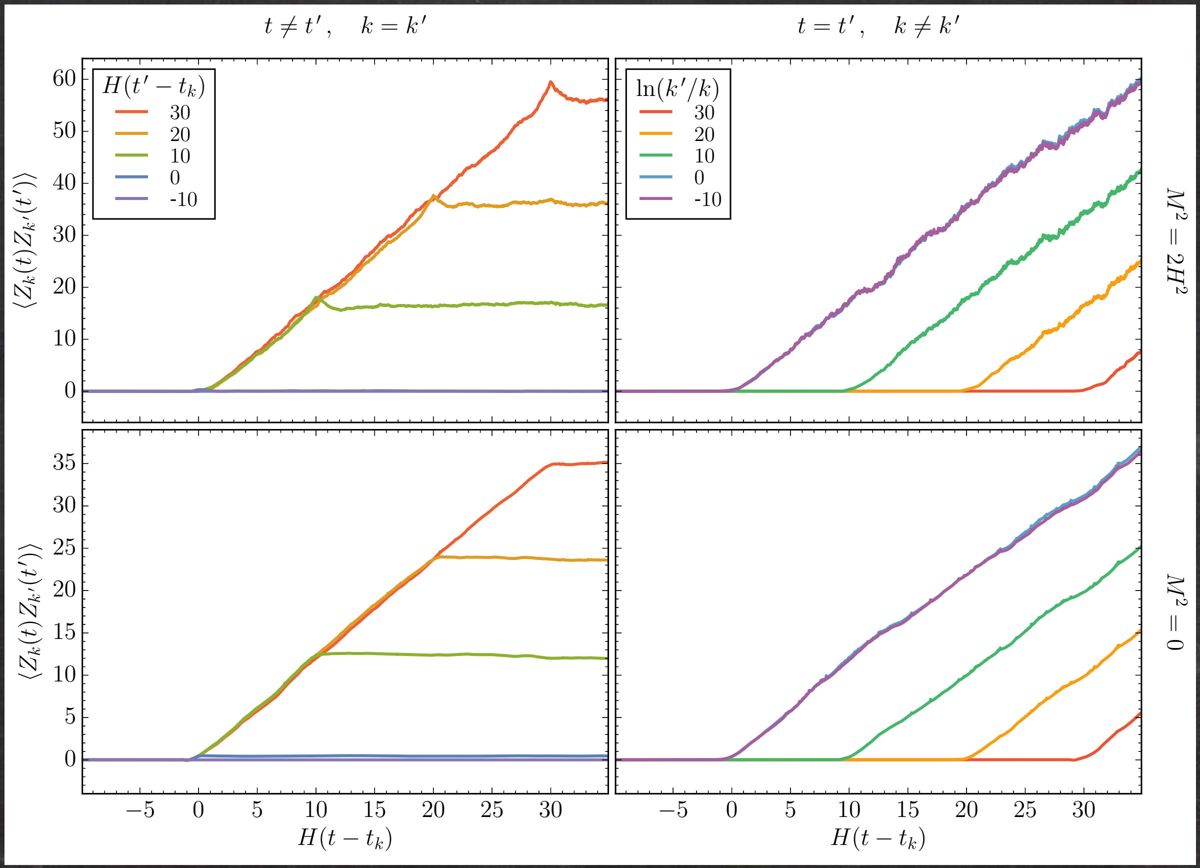


implies that $|\chi_k|^2$ performs a geometric (Brownian) random walk outside the horizon

Caveats:

- Fokker-Planck gets you everything **1**-point *inside the horizon*
- Outside the horizon, FP only gives the mean rate μ_1 in the weak scattering limit, and the distribution of one of the phases.
- No way to derive analytically the **1**-point nor the N-point distribution of the field.
- Other expansion histories unexplored

Ways forward:

- Random Matrix Theory?
- SDE approach?
- Higher spin?

The quasi-de Sitter Goldstone π couples to the spectator χ

$$\begin{aligned}\mathcal{S} &= \frac{1}{2}\int \sqrt{-g}\, d^4x \Big[c(t+\pi)\partial_\mu\pi\partial^\mu\pi + \partial_\mu\chi\partial^\mu\chi - \left(M^2 + m^2(t+\pi)\right)\chi^2\Big] \\ &= \frac{1}{2}\int \sqrt{-g}\, d^4x \left[c(t)\partial_\mu\pi\partial^\mu\pi + \partial_\mu\chi\partial^\mu\chi - \left(M^2 + m^2(t)\right)\chi^2 - \frac{dm^2(t)}{dt}\chi^2\pi + \cdots\right]\end{aligned}$$

With $\zeta \simeq H\pi$, an enhancement to the curvature power spectrum results

$$\boxed{\begin{aligned}\delta\Delta_\zeta^2(k) &= 4\pi^2(\Delta_\zeta^2)^2\frac{k^3}{H^6}\int d\tau'\, d\tau''\, G_k(\tau,\tau')G_k(\tau,\tau'')a(\tau')a(\tau'')\frac{dm^2(\tau')}{d\tau'}\frac{dm^2(\tau'')}{d\tau''} \\ &\qquad \times \int \frac{d^3\mathbf{p}}{(2\pi)^3}\,\chi_p^*(\tau')\chi_p(\tau'')\chi_{|\mathbf{p}-\mathbf{k}|}^*(\tau')\chi_{|\mathbf{p}-\mathbf{k}|}(\tau'')\end{aligned}}$$

where $\langle\zeta(\mathbf{k})\zeta(\mathbf{k}')\rangle \equiv \frac{2\pi^2}{k^3}\Delta_\zeta^2(k)\,\delta^{(3)}(\mathbf{k}+\mathbf{k}')$

Given the Dirac-delta form of the effective mass, and the lognormality of the field,

$$\langle|\chi_{k_1}(t_1)|^2\cdots|\chi_{k_n}(t_n)|^2\rangle \;=\; \exp\left[\sum_{i=1}^{n}\langle\ln|\chi_{k_i}(t_i)|^2\rangle + \frac{1}{2}\sum_{i,j=1}^{n}\langle Z_{k_i}(t_i)Z_{k_j}(t_j)\rangle\right]$$

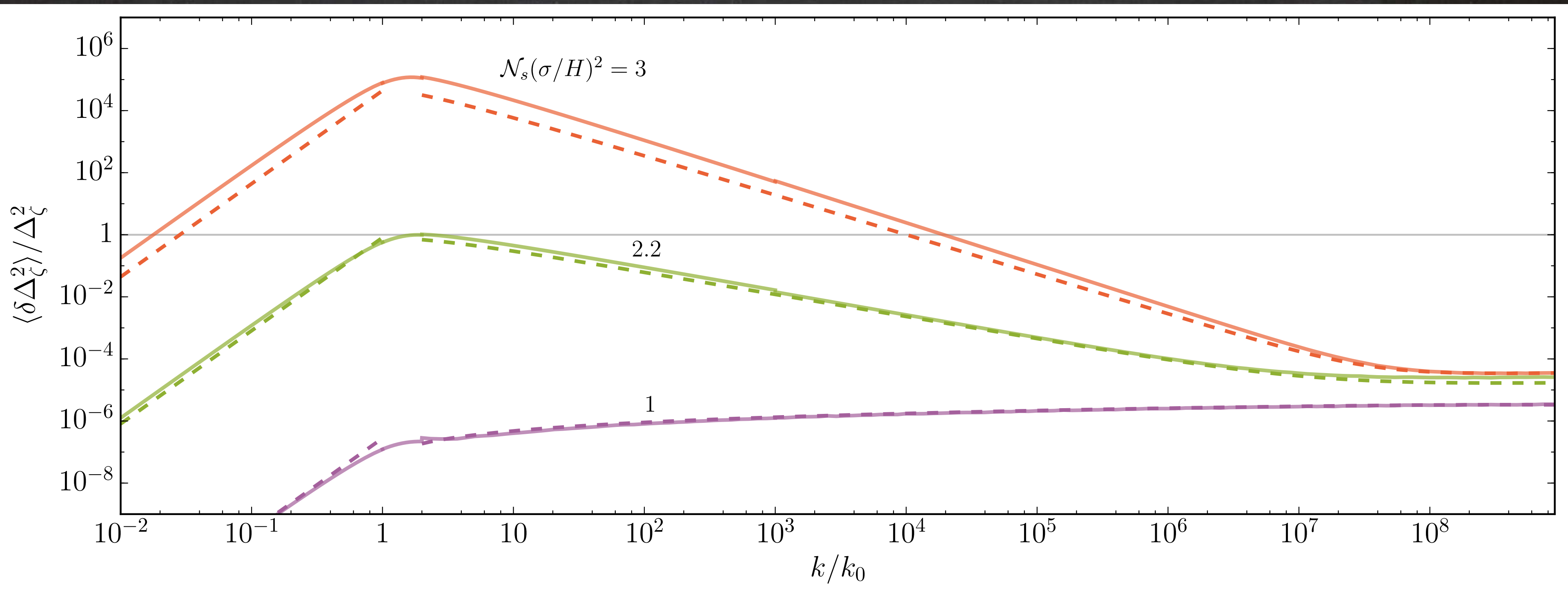
we can compute moments:

$$\boxed{\begin{aligned}\langle\delta\Delta_\zeta^2(k)\rangle \;&=\; 4\pi^2(\Delta_\zeta^2)^2\frac{k^3\sigma^2}{H^6}\sum_i\mathcal{G}_k(\tau,\tau')\int\frac{d^3\mathbf{p}}{(2\pi)^3}f(p,|\mathbf{p}-\mathbf{k}|)\,\langle|\chi_p(\tau_i)|^2|\chi_{|\mathbf{p}-\mathbf{k}|}(\tau_i)|^2\rangle\\ &=\; \frac{16\pi^3}{9}(\Delta_\zeta^2)^2\mathcal{N}_s\left(\frac{\sigma}{H}\right)^2 F\left[k,\,k_0,\,N_{\rm tot},\,\mathcal{N}_s\left(\frac{\sigma}{H}\right)^2\right]\end{aligned}}$$

(when scatterings begin)

(e-fold duration of scatterings)

The correction overcomes the $(\Delta_\zeta^2)^2$ suppression for $\mathcal{N}_s\left(\frac{\sigma}{H}\right)^2 \gtrsim \mathcal{O}(1)$



(conformal, $N_{\rm tot} = 20$)

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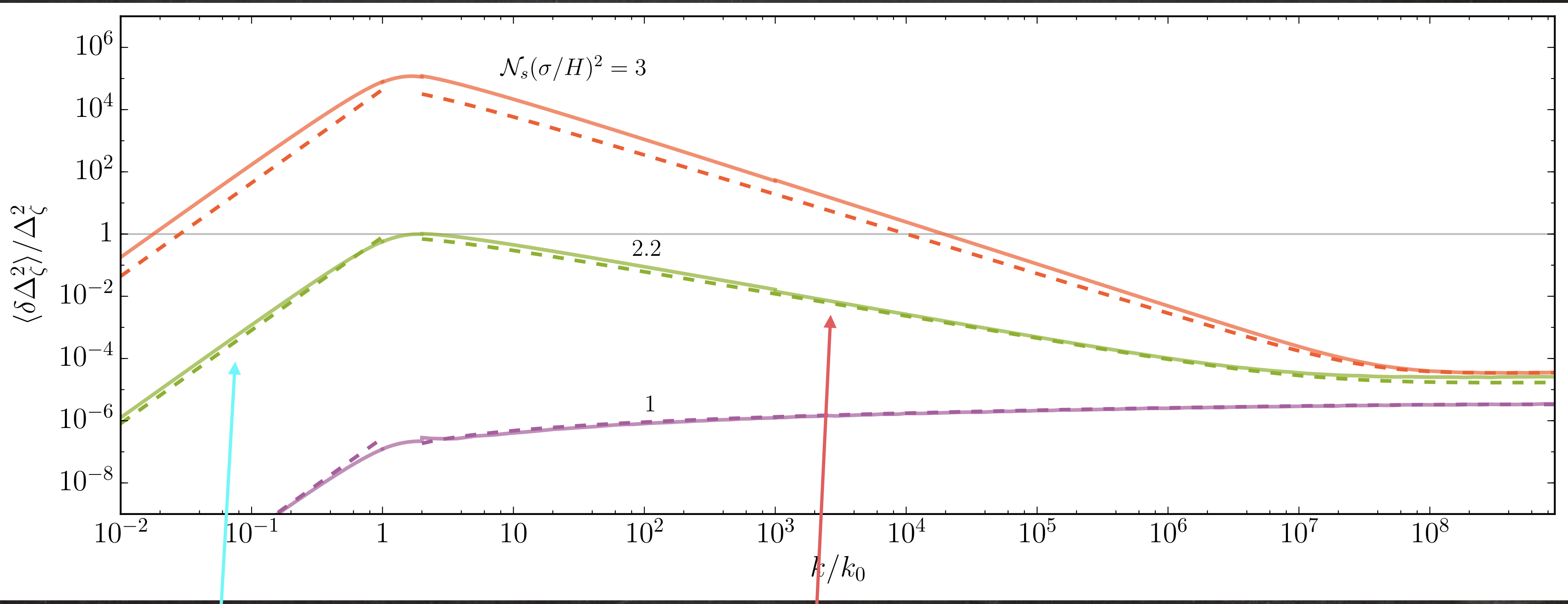
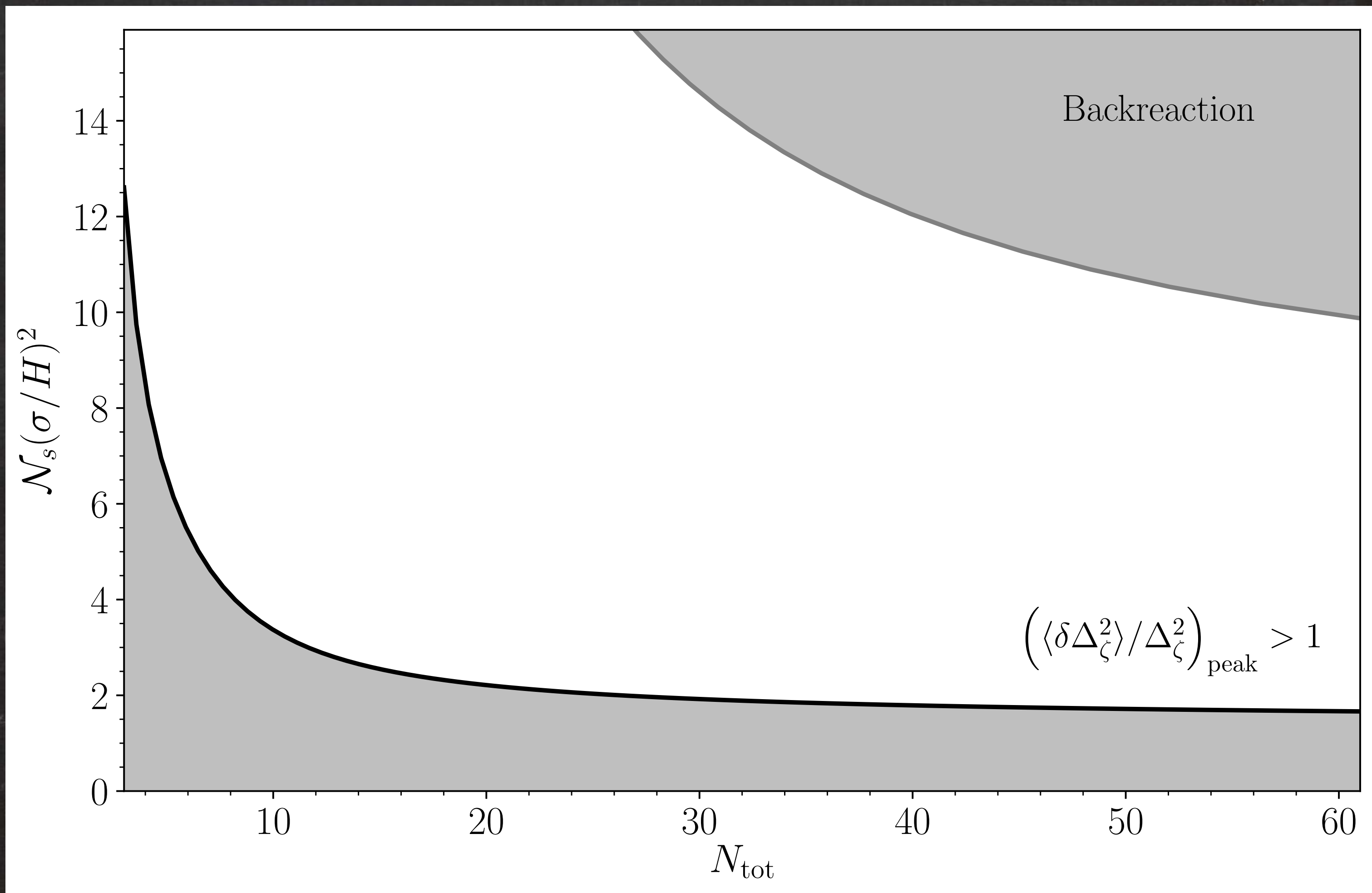


(conformal, $N_{\rm tot} = 20$)

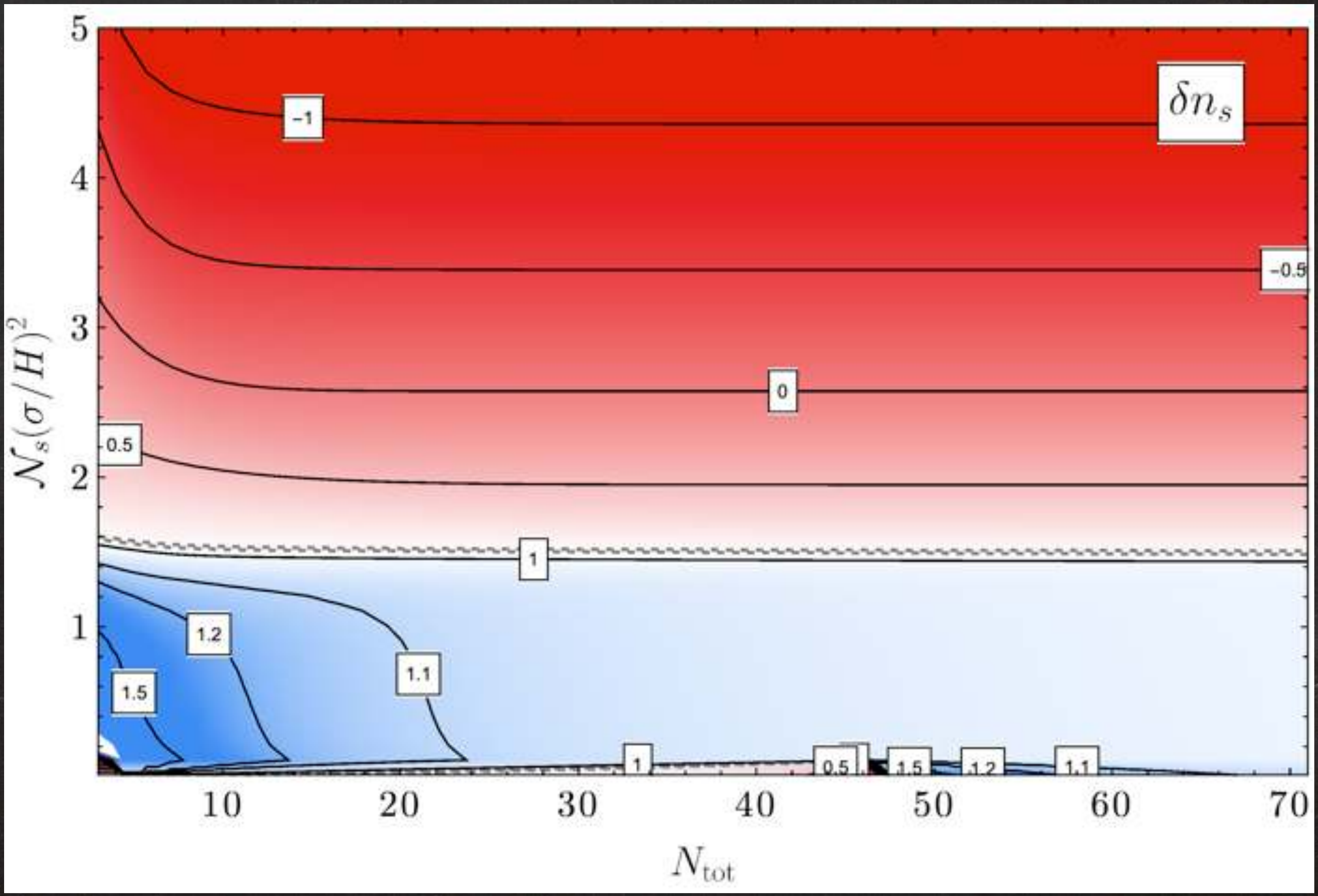
$\delta n_s \simeq 4$

$n_s \simeq f\left[N_{\rm tot}, \mathcal{N}_s\left(\frac{\sigma}{H}\right)^2\right]$

Available parameter space:



Tilt correction:



More caveats:

- The power spectrum calculation is *very* technical. An order of magnitude estimate is not available
- How to proceed then with the N-point function?
- Lognormality of χ may lead to a heavily skewed distribution for π. How representative of the typical member of the ensemble is the mean?
- Unclear how important dissipation effects ($\pi \to \chi$) are
- Reliance on "adiabatic regularization"
- Interpretation of the "feature" in the power spectrum
- Gravitational waves?