

30/11/2020

How warm are non-thermal relics?



東京大学
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IFT-UAM

2011.13458

with G. Ballesteros and M. Pierre

2006.03325

with Y. Mambrini, K. Olive and S. Verner

2004.08404

with Y. Mambrini, K. Olive and K. Kaneta

1806.01865

with M. Amin

1709.01549

with Y. Mambrini, K. Olive and M. Peloso



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CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

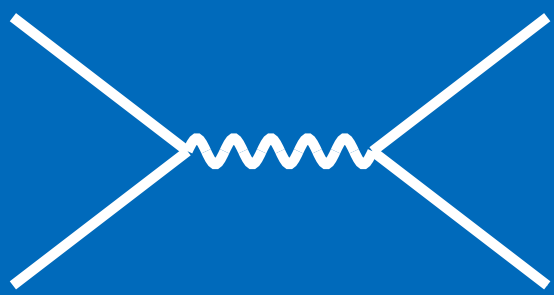
The Goals

1. Reheating



Understand the (perturbative) repopulation of the Universe after the end of inflation (rates, energy densities, temperatures, distributions,..)

2. Freeze-in



Characterize the out-of-equilibrium production of dark matter (from particle/condensate decays, scatterings, misalignment...)

3. Lyman- α



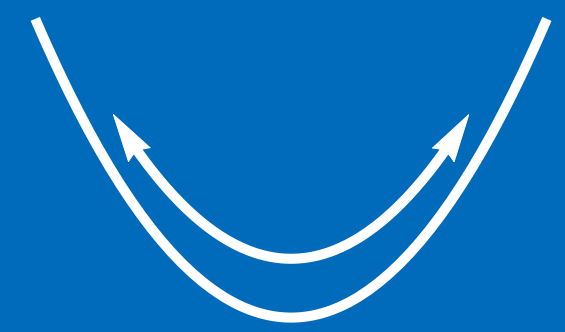
In the absence of a direct signal, can we say more beyond Ω_{DM} ?
Use the Ly- α measurement of the power spectrum to impose constraints

4. The end?

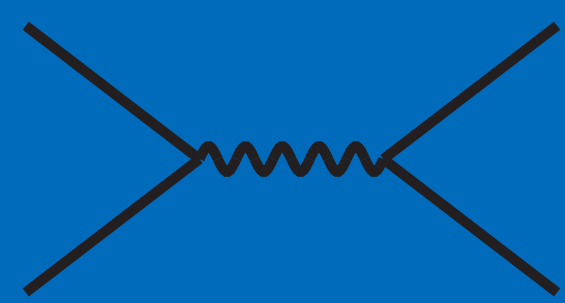


A short summary of what we (don't) understand

1. Reheating



2. Freeze-in



3. Lyman-α



4. The end?

Accelerated expansion can be driven by a slowly rolling scalar field

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - V(\Phi) \right]$$

$$ds^2 = dt^2 - a(t)^2 dx^2$$

$$G_{\mu\nu} = M_P^{-2} T_{\mu\nu}$$

$$\rho_\Phi = \frac{1}{2} \dot{\Phi}^2 + V(\Phi)$$

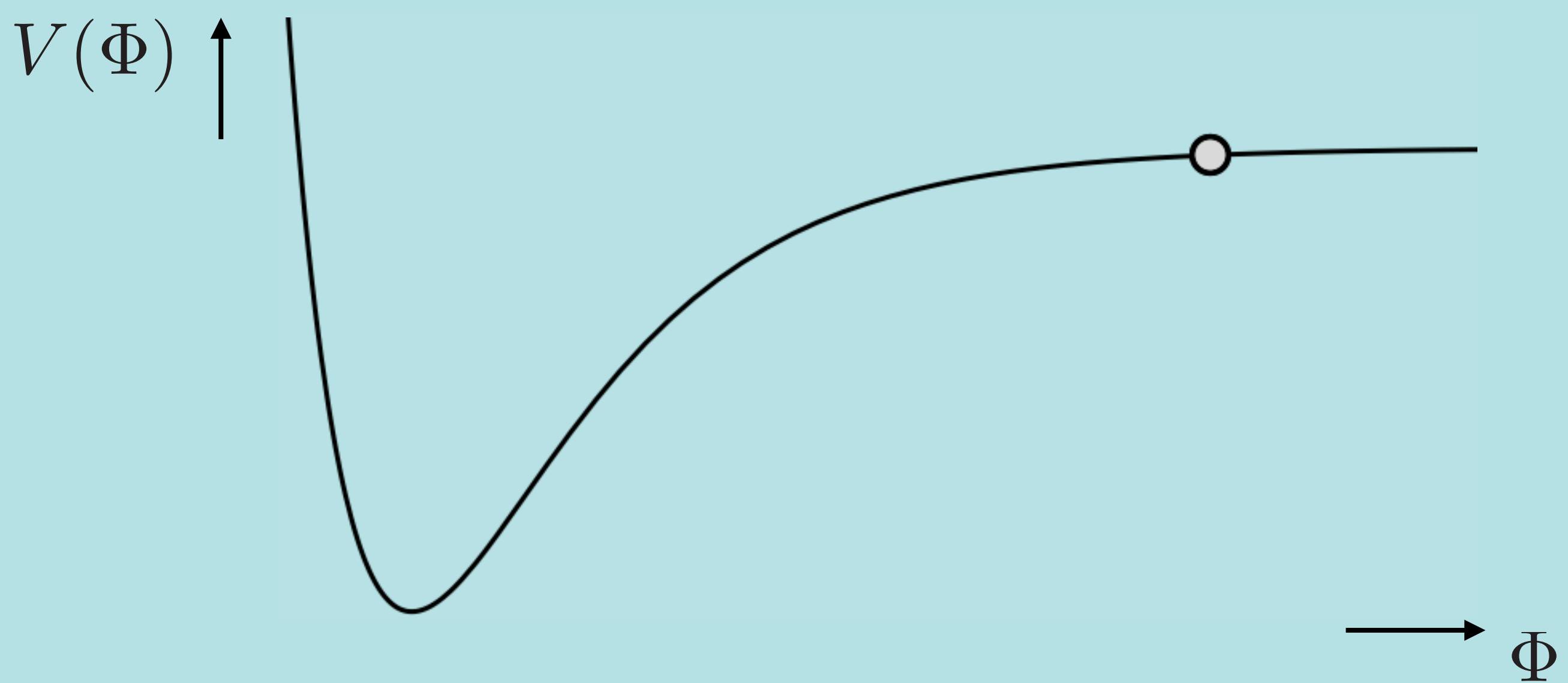
$$P_\Phi = \frac{1}{2} \dot{\Phi}^2 - V(\Phi)$$

$$H \equiv \frac{\dot{a}}{a} = \left(\frac{\rho_\Phi}{3M_P^2} \right)^{1/2}$$

$$\dot{\rho}_\Phi + 3H(\rho_\Phi + P_\Phi) = 0$$

or

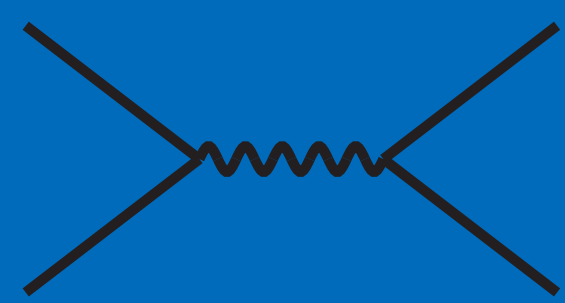
$$\ddot{\Phi} + 3H\dot{\Phi} + V'(\Phi) = 0$$



1. Reheating



2. Freeze-in



3. Lyman- α



4. The end?

Accelerated expansion can be driven by a slowly rolling scalar field

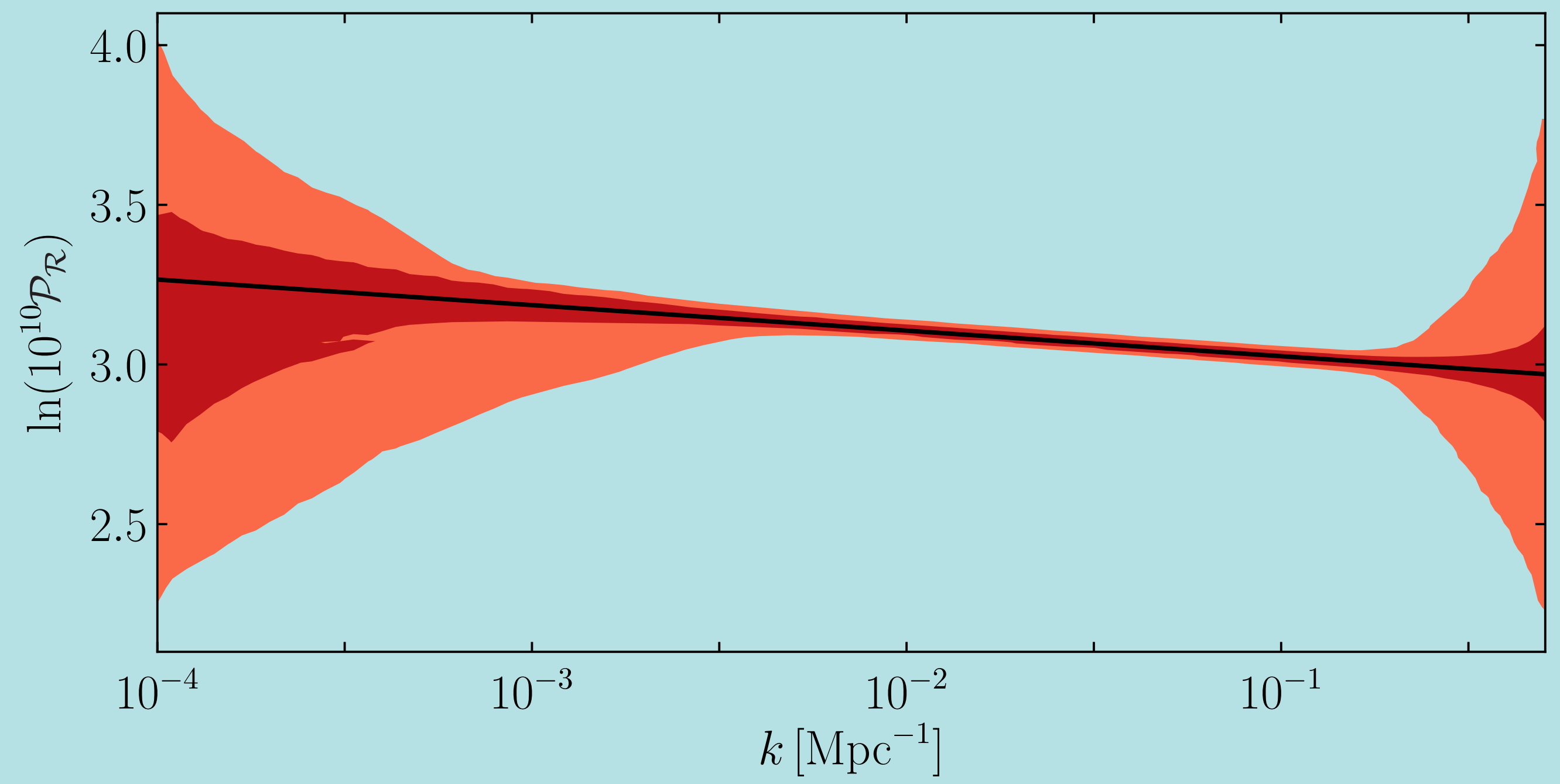
$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - V(\Phi) \right]$$

$$ds^2 = dt^2 - a(t)^2 dx^2$$

$$\Phi \rightarrow \Phi + \delta\Phi$$

$$g \rightarrow g + \delta g$$

Planck TT, TE, EE + lowE + lensing + BK15 (■ 1 σ , ■ 2 σ)



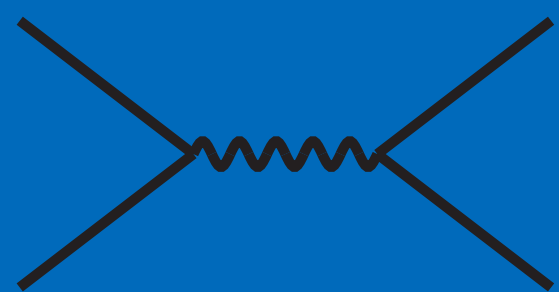
$$\mathcal{P}_{\mathcal{R}} = \frac{H_*^4}{4\pi^2 \dot{\phi}_*^2} \left(\frac{k}{aH} \right)^{n_s - 1}$$

$$\mathcal{P}_{\mathcal{T}} = \frac{2}{\pi^2} H_*^2 \left(\frac{k}{aH} \right)^{n_T}$$

1. Reheating



2. Freeze-in



3. Lyman- α



4. The end?

After inflation ends, reheating begins

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - V(\Phi) \right]$$

$$ds^2 = dt^2 - a(t)^2 dx^2$$

$$G_{\mu\nu} = M_P^{-2} T_{\mu\nu}$$

$$\rho_\Phi = \frac{1}{2} \dot{\Phi}^2 + V(\Phi)$$

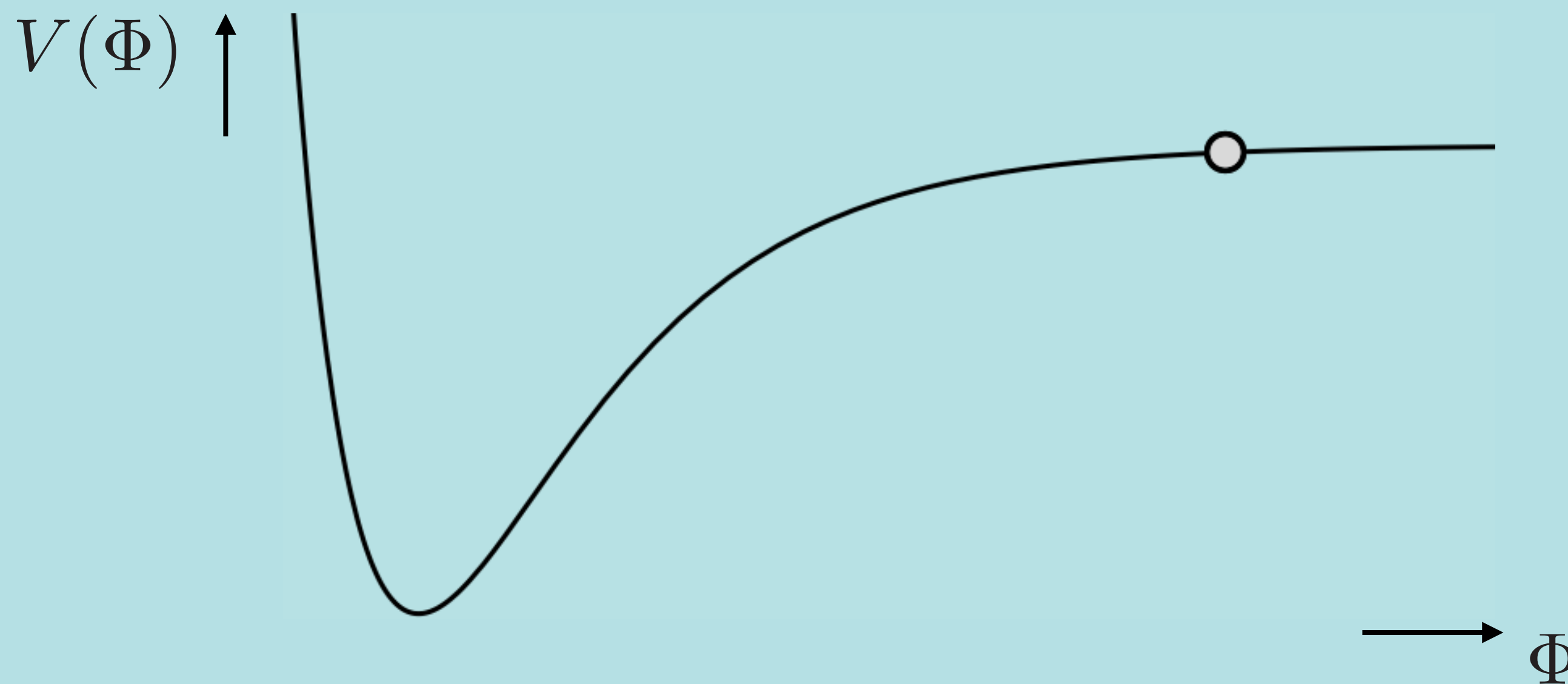
$$P_\Phi = \frac{1}{2} \dot{\Phi}^2 - V(\Phi)$$

$$H \equiv \frac{\dot{a}}{a} = \left(\frac{\rho_\Phi}{3M_P^2} \right)^{1/2}$$

$$\dot{\rho}_\Phi + 3H(\rho_\Phi + P_\Phi) = 0$$

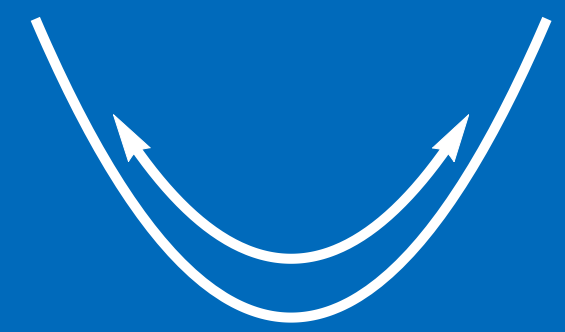
or

$$\ddot{\Phi} + 3H\dot{\Phi} + V'(\Phi) = 0$$

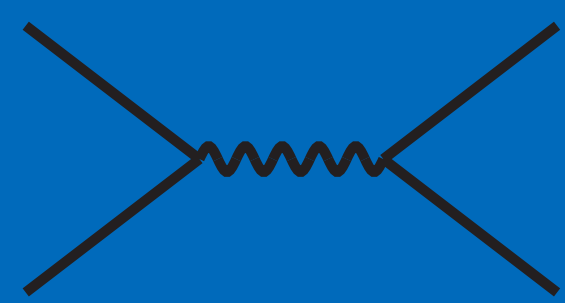


1. Reheating

Reheating depends on the shape of the potential



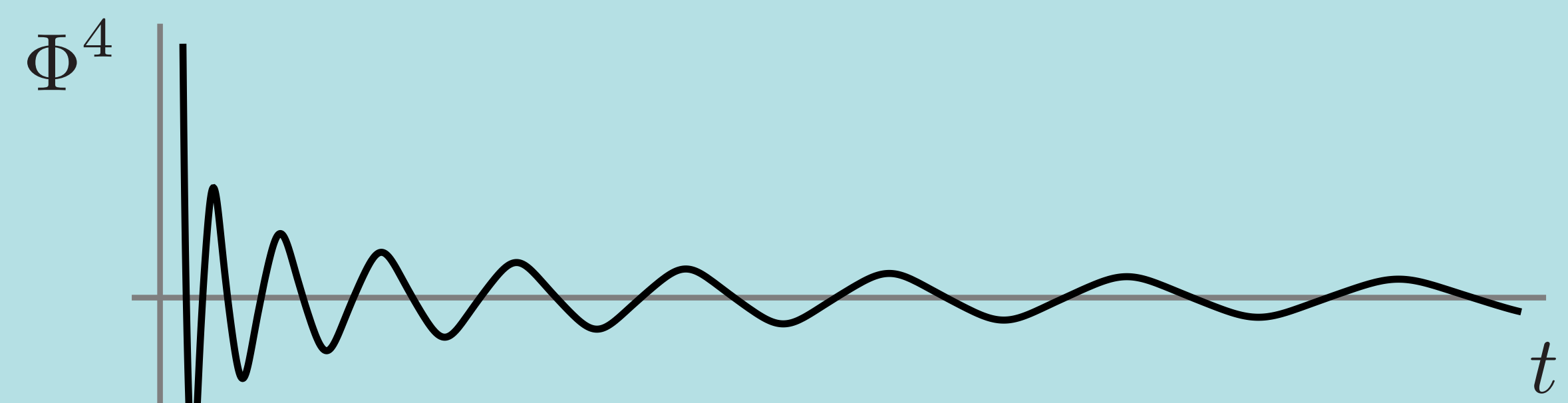
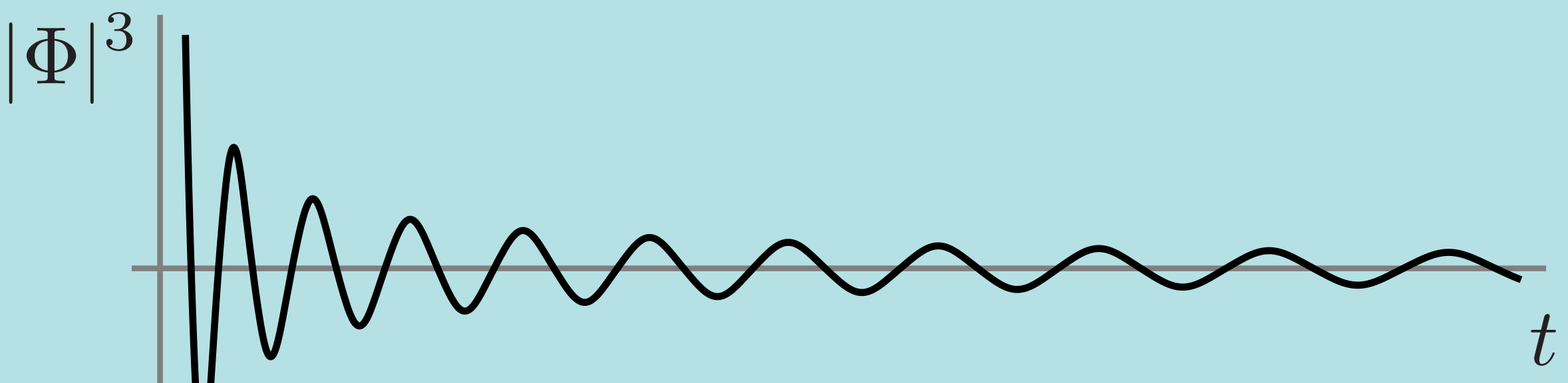
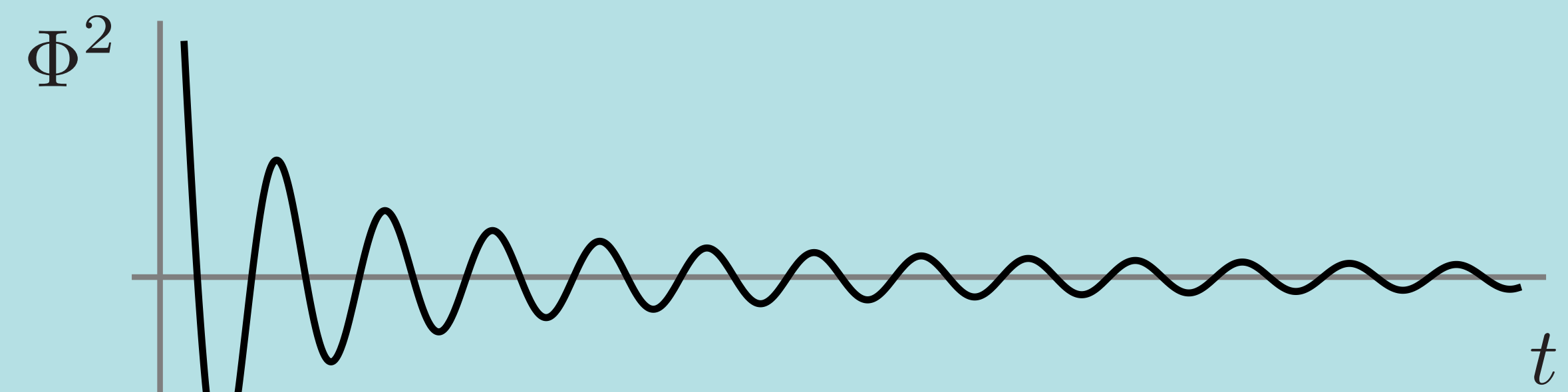
2. Freeze-in



3. Lyman- α



4. The end?



$$V(\Phi) \propto |\Phi|^k$$



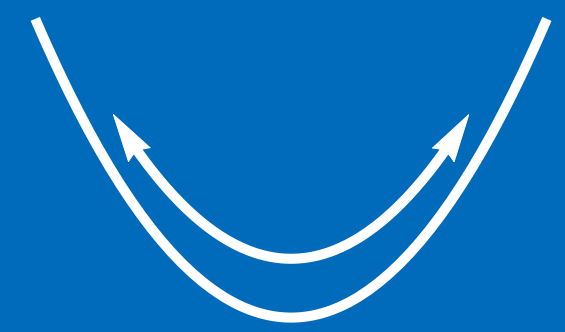
over one oscillation,

$$\langle \dot{\Phi}^2 \rangle \simeq \langle \Phi V'(\Phi) \rangle$$

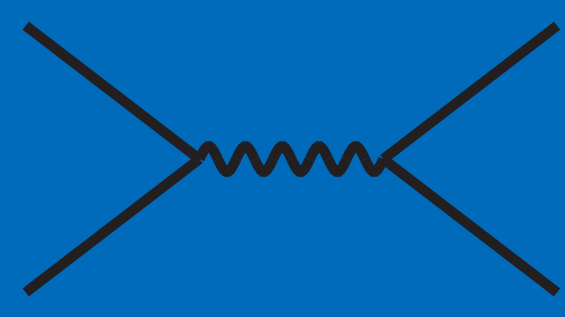


$$\langle P_\Phi \rangle = \frac{k-2}{k+2} \langle \rho_\Phi \rangle$$

1. Reheating



2. Freeze-in

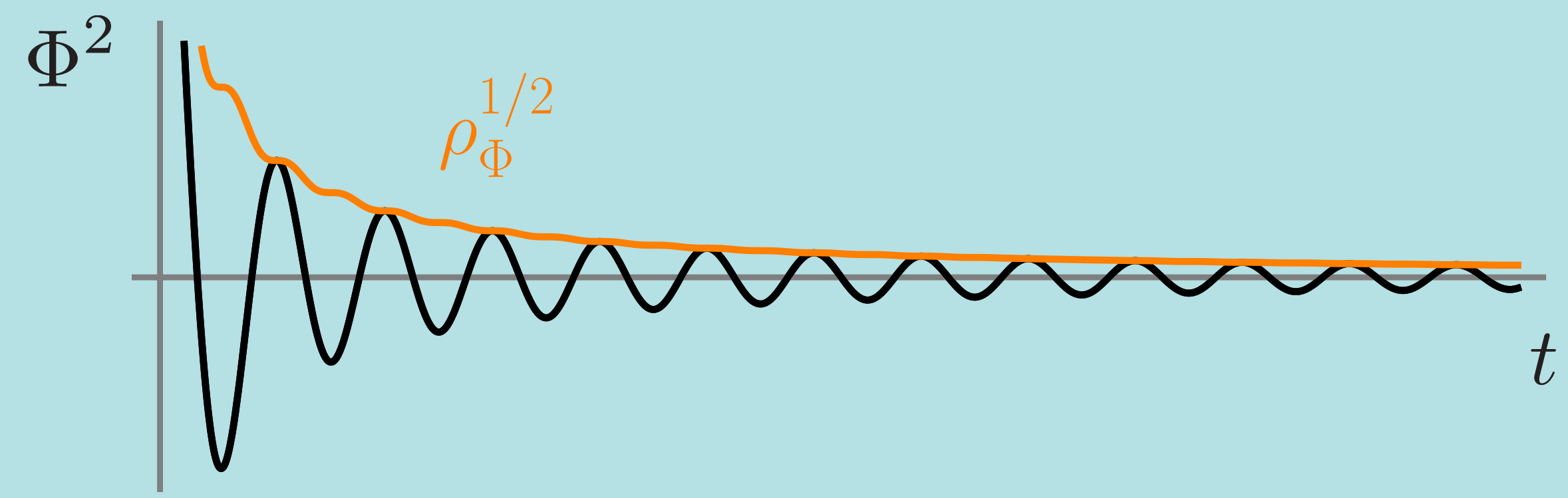


3. Lyman- α

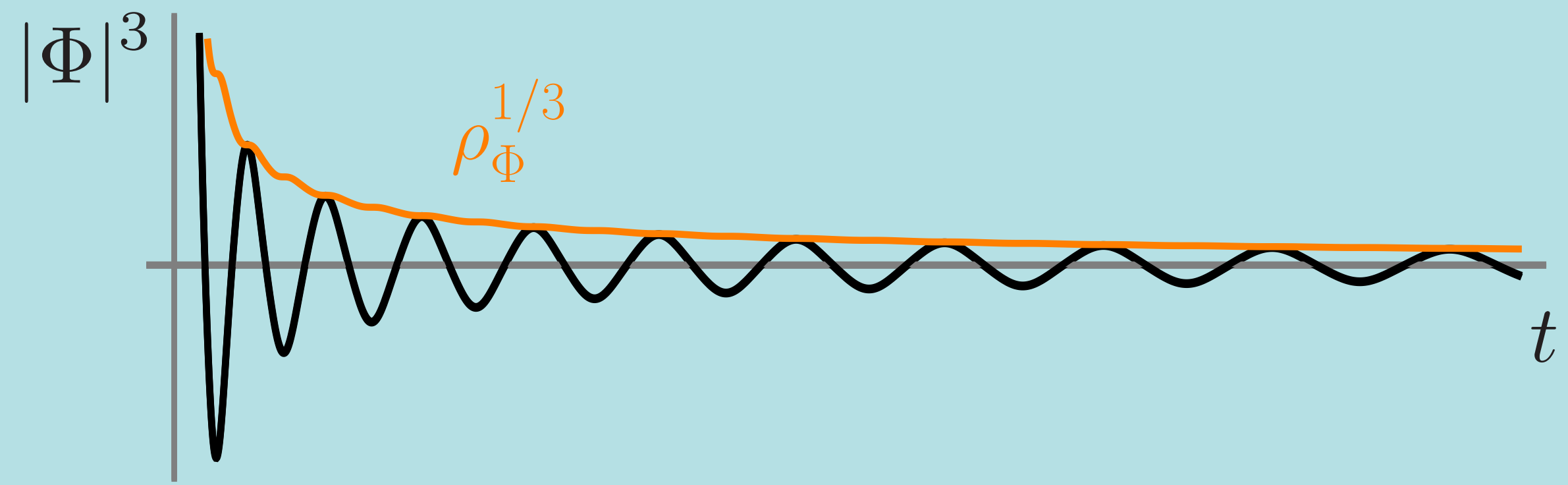


4. The end?

Reheating depends on the shape of the potential

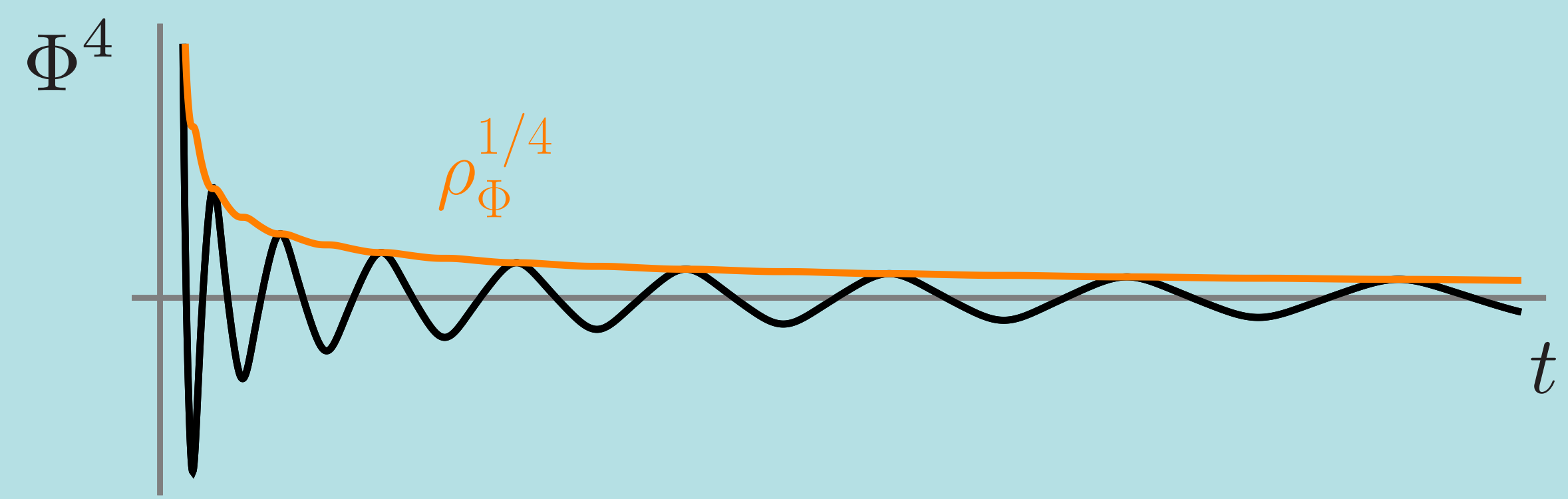


~ matter



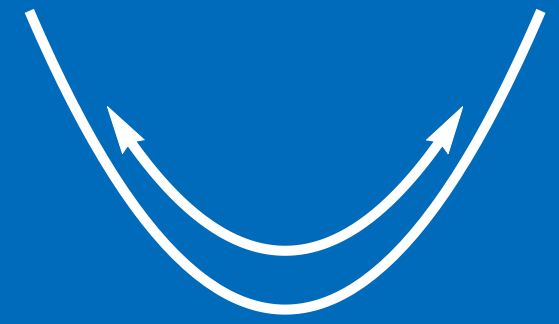
$$\rho_\Phi = \rho_{\text{end}} \left(\frac{a}{a_{\text{end}}} \right)^{-\frac{6k}{k+2}}$$

$$a \propto t^{\frac{k+2}{3k}}$$



~ radiation

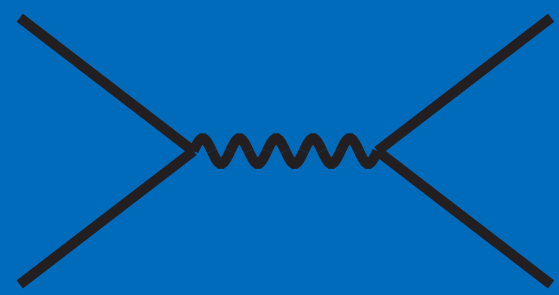
1. Reheating



Particle production in the expanding Universe $\chi + a + b + \dots \longleftrightarrow i + j + \dots$

$$\frac{\partial f_\chi}{\partial t} - H|\mathbf{p}|\frac{\partial f_\chi}{\partial |\mathbf{p}|} = \mathcal{C}[f_\chi(|\mathbf{p}|, t)]$$

2. Freeze-in



$$\begin{aligned} \mathcal{C}[f_\chi] = & -\frac{1}{2p_0} \int \frac{g_a d^3 \mathbf{p}_a}{(2\pi)^3 2p_{a0}} \frac{g_b d^3 \mathbf{p}_b}{(2\pi)^3 2p_{b0}} \dots \frac{g_i d^3 \mathbf{p}_i}{(2\pi)^3 2p_{i0}} \frac{g_j d^3 \mathbf{p}_j}{(2\pi)^3 2p_{j0}} \dots \\ & \times (2\pi)^4 \delta^{(4)}(p_\chi + p_a + p_b + \dots - p_i - p_j - \dots) \\ & \times \left[|\mathcal{M}|_{\chi+a+b+\dots \rightarrow i+j+\dots}^2 f_a f_b \dots f_\chi (1 \pm f_i)(1 \pm f_j) \dots \right. \\ & \left. - |\mathcal{M}|_{i+j+\dots \rightarrow \chi+a+b+\dots}^2 f_i f_j \dots (1 \pm f_a)(1 \pm f_b) \dots (1 \pm f_\chi) \right] \end{aligned}$$

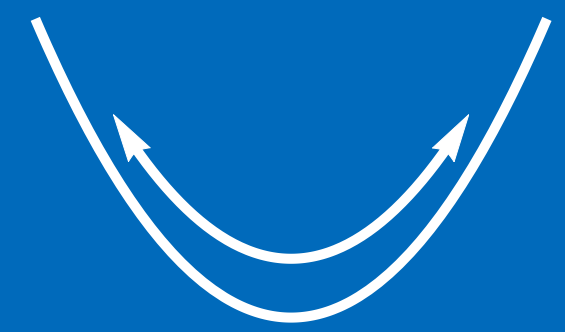
3. Lyman- α



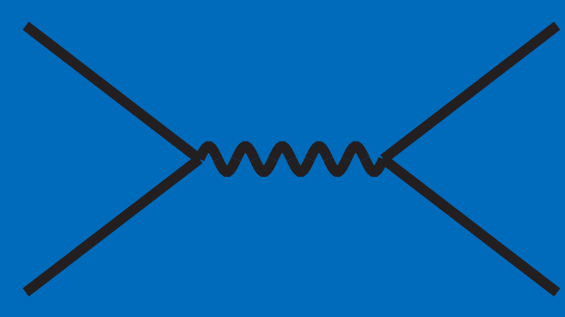
$$n_\chi(t) = \frac{g_\chi}{(2\pi)^3} \int d^3 \mathbf{p} f_\chi(p_0, t) \qquad \rho_\chi(t) = \frac{g_\chi}{(2\pi)^3} \int d^3 \mathbf{p} p_0 f_\chi(p_0, t)$$

4. The end?

1. Reheating



2. Freeze-in



3. Lyman- α



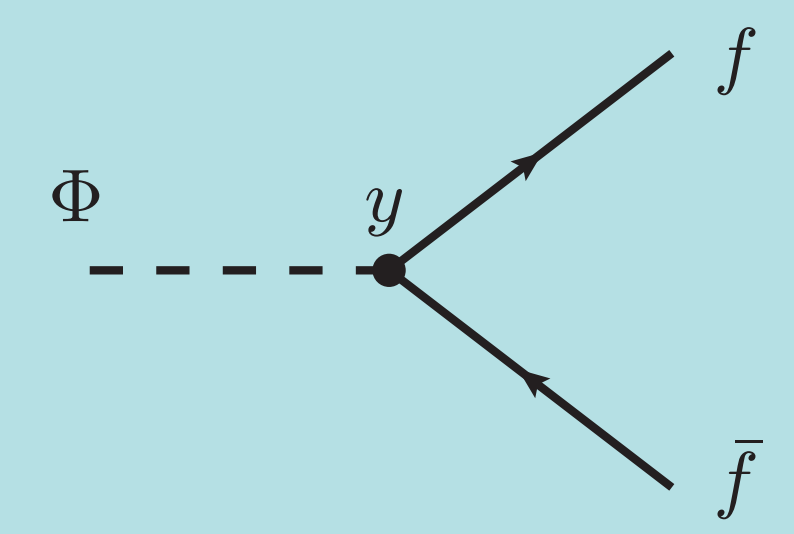
4. The end?

Integrated Boltzmann equations

$$\dot{\rho}_{\Phi} + 3 \left(\frac{2k}{k+2} \right) H \rho_{\Phi} = -\Gamma_{\Phi}(t) \rho_{\Phi}$$

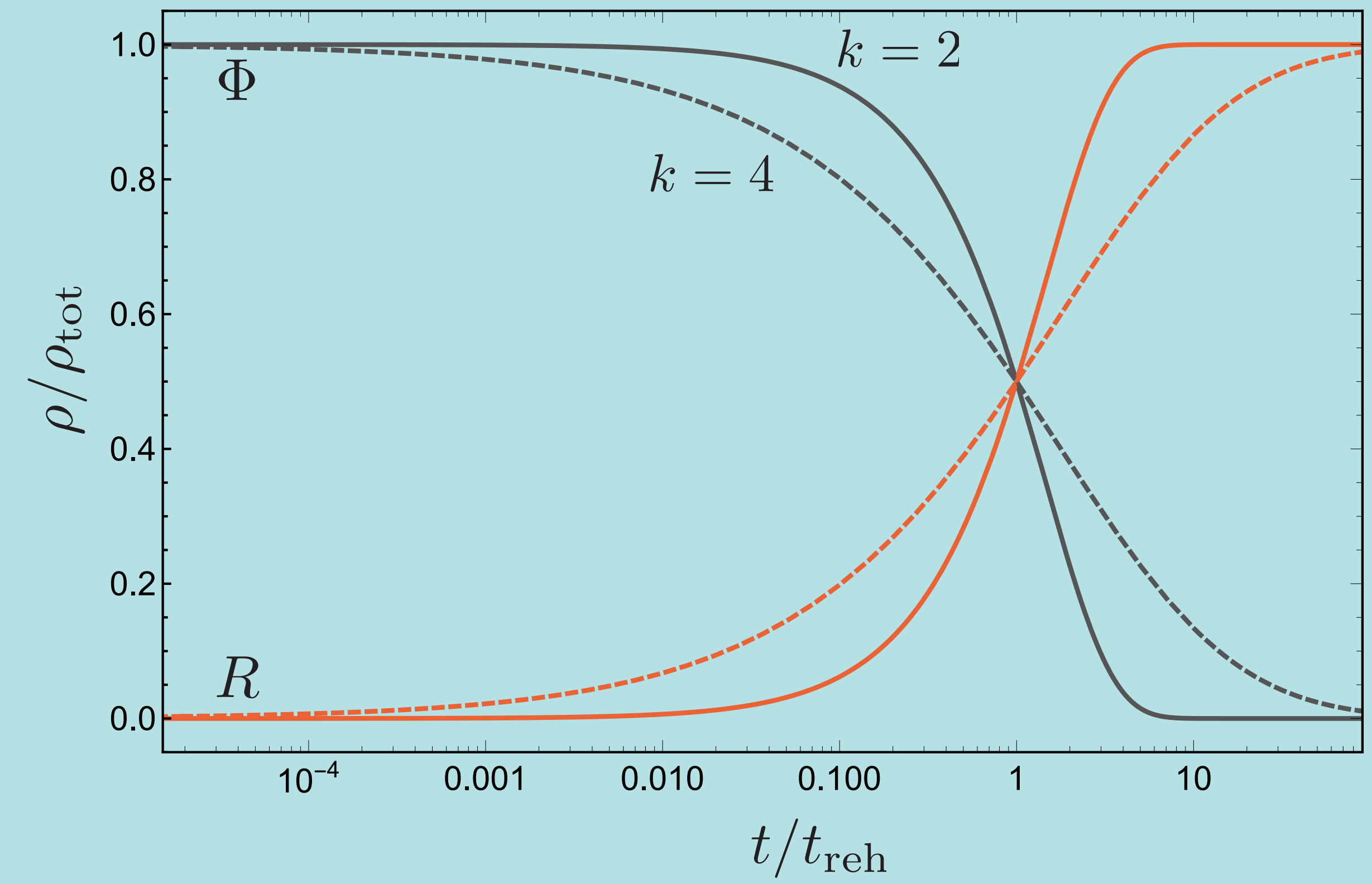
$$\dot{\rho}_R + 4H \rho_R = \Gamma_{\Phi}(t) \rho_{\Phi}$$

$$3M_P^2 H^2 = \rho_{\Phi} + \rho_R$$



$$\Gamma_{\Phi} = \frac{y^2}{8\pi} m_{\Phi}(t),$$

$$m_{\Phi}^2 \equiv \partial_{\phi}^2 V(\Phi) \propto \rho_{\Phi}^{\frac{k-2}{k}}$$



1. Reheating

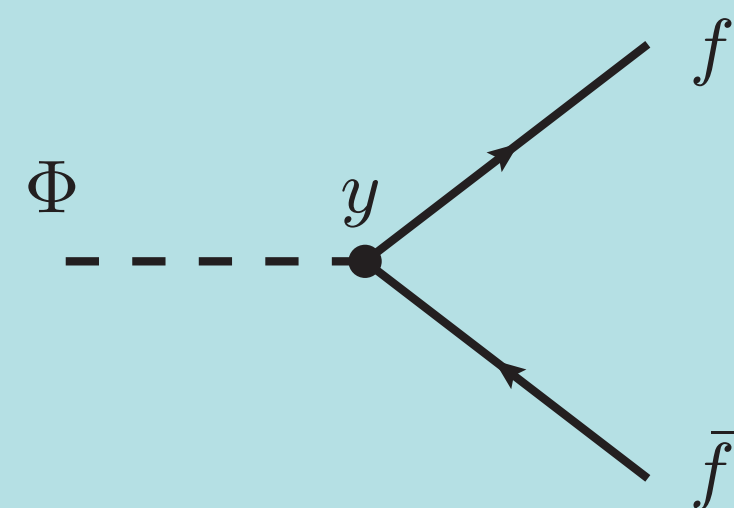


Integrated Boltzmann equations

$$\dot{\rho}_\Phi + 3 \left(\frac{2k}{k+2} \right) H \rho_\Phi = -\Gamma_\Phi(t) \rho_\Phi$$

$$\dot{\rho}_R + 4H \rho_R = \Gamma_\Phi(t) \rho_\Phi$$

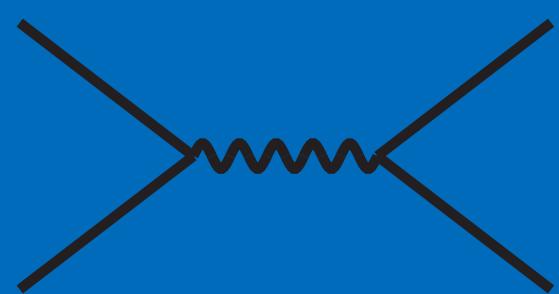
$$3M_P^2 H^2 = \rho_\Phi + \rho_R$$



$$\Gamma_\Phi = \frac{y^2}{8\pi} m_\Phi(t),$$

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2. Freeze-in



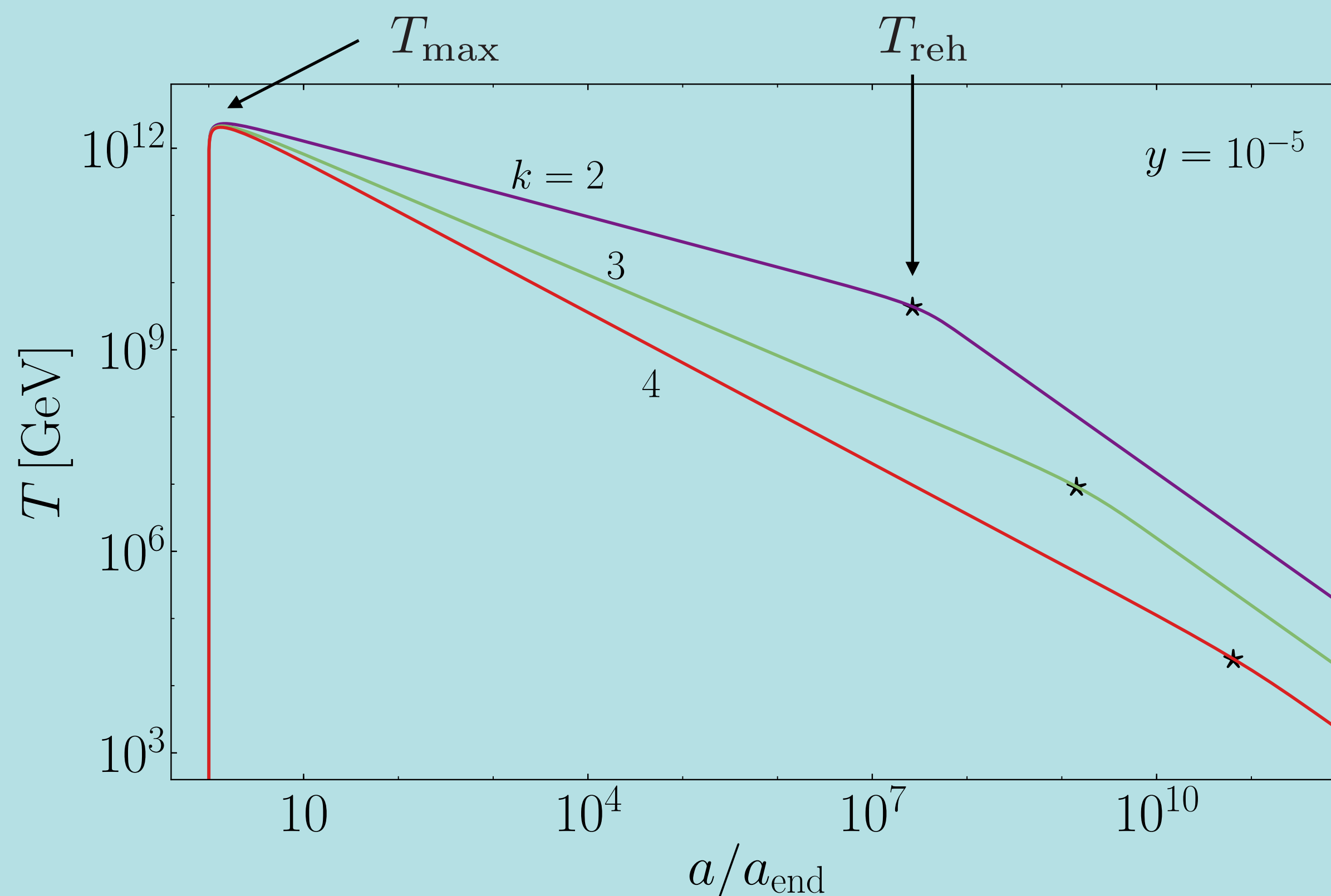
3. Lyman- α



$$T = \left(\frac{30\rho_R}{\pi^2 g_*} \right)^{1/4}$$

$$\propto a^{-\frac{3k-3}{2k+4}}$$

4. The end?



1. Reheating

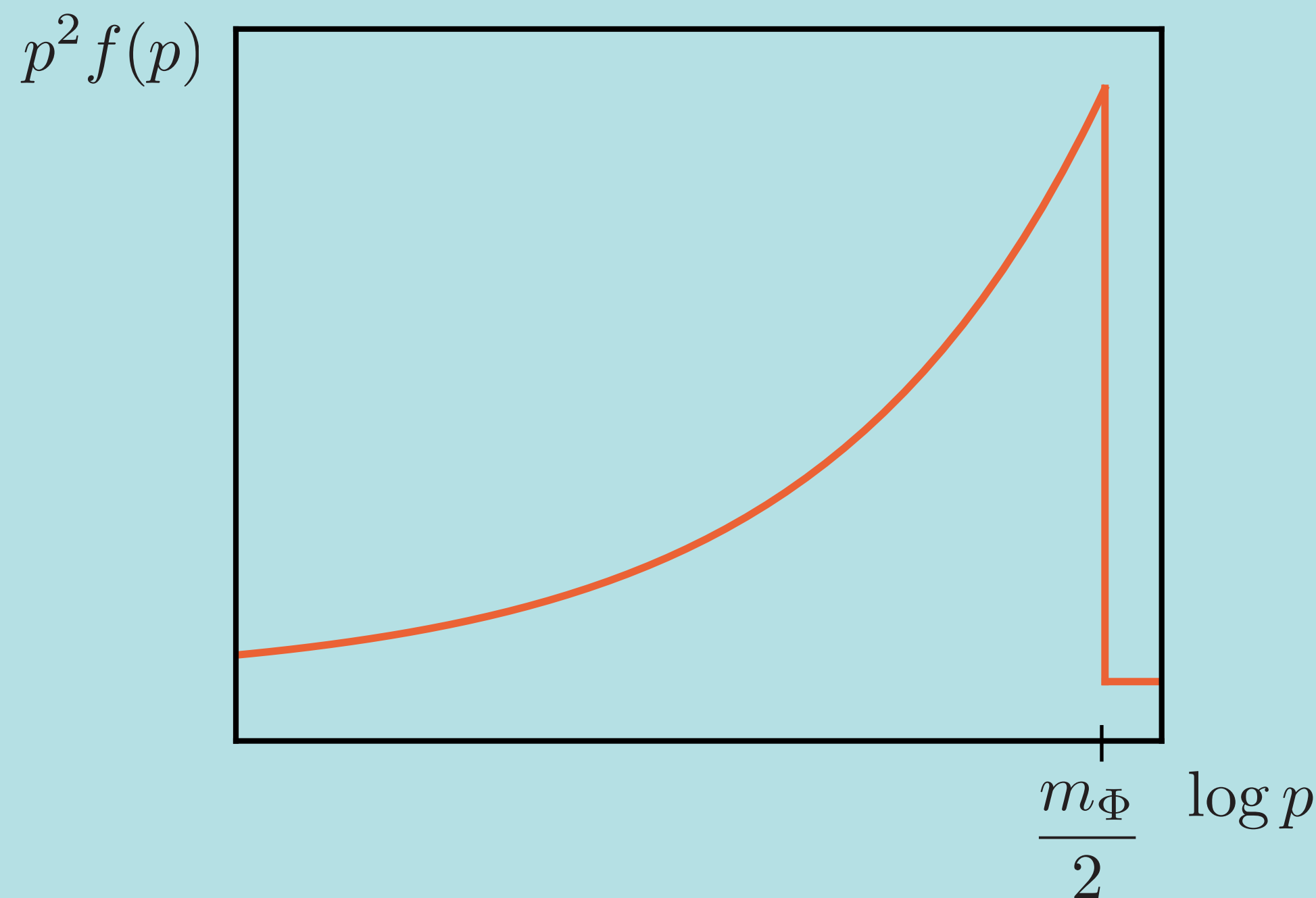
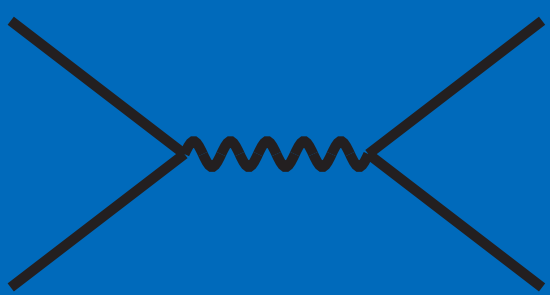
A closer look: thermalization is not instantaneous! (quadratic only)



$$\mathcal{C}[f_\chi(p, t)] = \frac{8\pi^2}{g_\chi m_\Phi^2} n_\Phi \Gamma_{\Phi \rightarrow \chi\psi} \delta(p - m_\Phi/2)$$

2. Freeze-in

$$f_\chi(p, t) = \frac{16\pi^2 \Gamma_{\Phi \rightarrow \chi\psi} n_\Phi(\hat{t})}{g_\chi m_\Phi^3 H(\hat{t})} \theta(t - \hat{t}), \quad \frac{a(t)}{a(\hat{t})} = \frac{m_\Phi}{2p}$$



$$f_\chi(p, t) \simeq \frac{24\pi^2 n_\chi(t)}{g_\chi m_\Phi^3} \left(\frac{m_\Phi}{2p}\right)^{3/2} \theta(m_\Phi/2 - p)$$

$(t \ll t_{\text{reh}})$

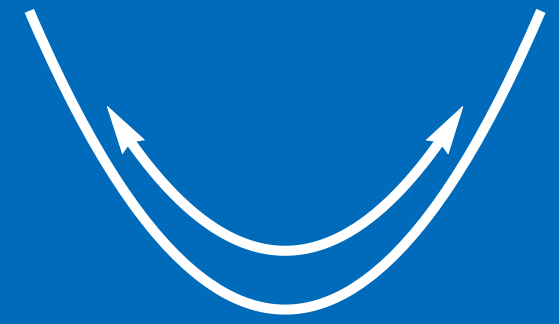
3. Lyman- α



4. The end?

1. Reheating

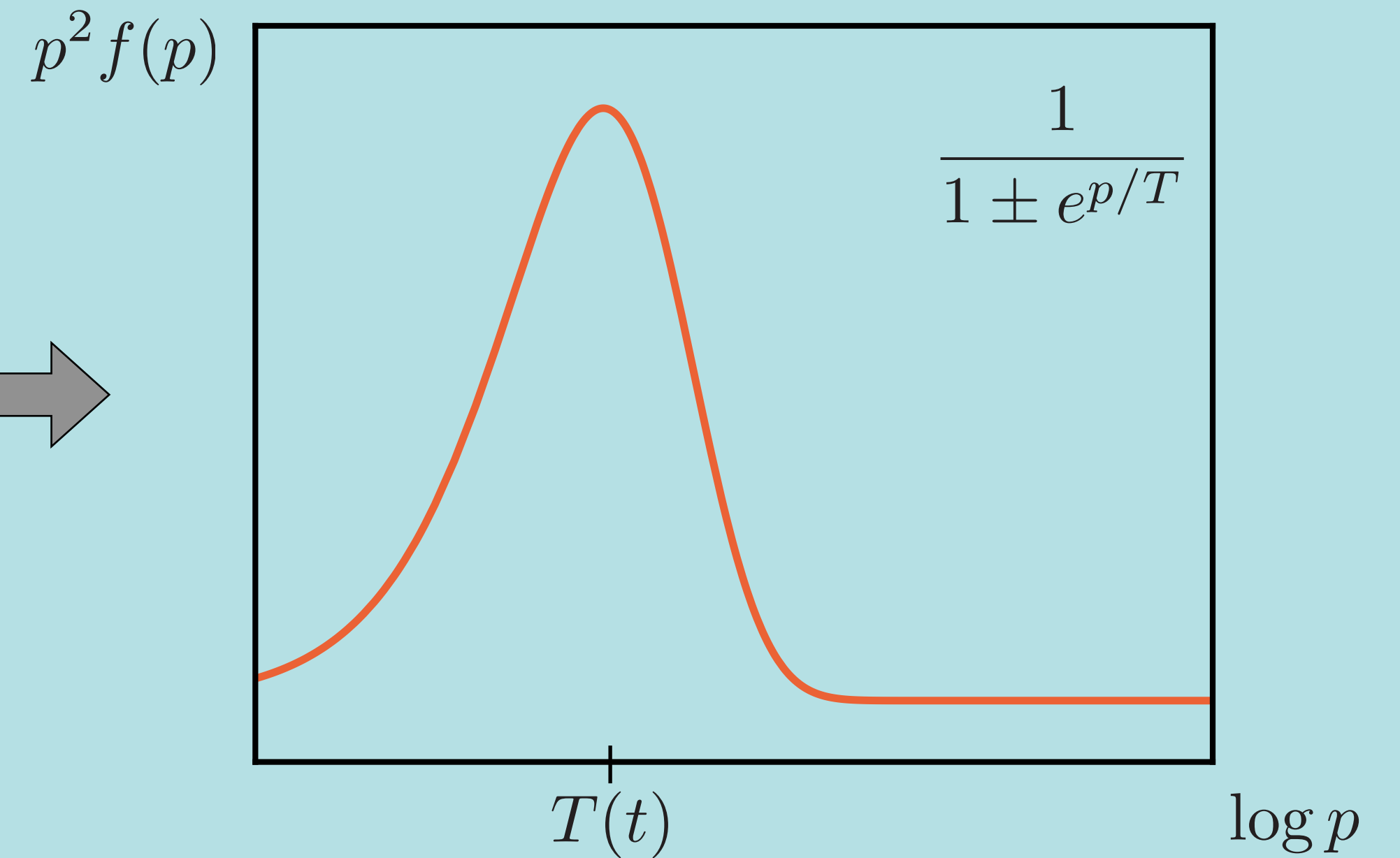
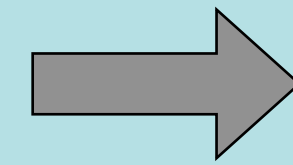
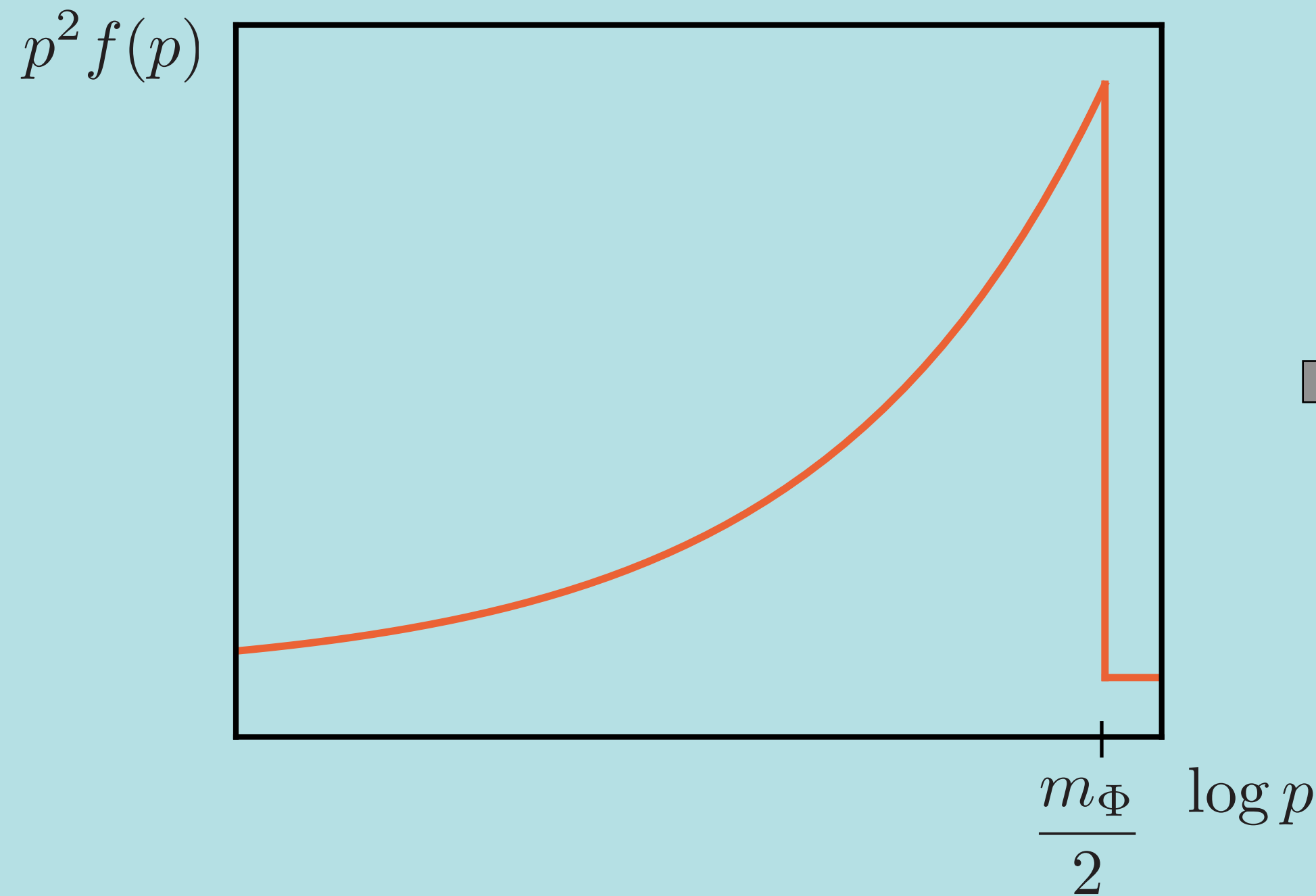
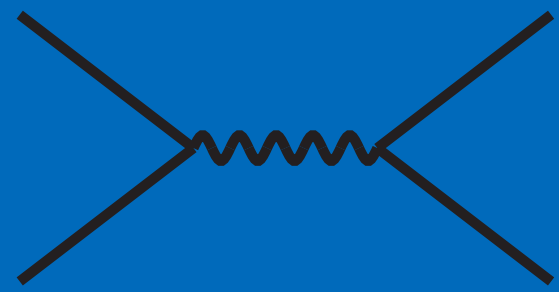
A closer look: thermalization is not instantaneous! (quadratic only)



$$\mathcal{C}[f_\chi(p, t)] = \frac{8\pi^2}{g_\chi m_\Phi^2} n_\Phi \Gamma_{\Phi \rightarrow \chi\psi} \delta(p - m_\Phi/2)$$

2. Freeze-in

$$f_\chi(p, t) = \frac{16\pi^2 \Gamma_{\Phi \rightarrow \chi\psi} n_\Phi(\hat{t})}{g_\chi m_\Phi^3 H(\hat{t})} \theta(t - \hat{t}), \quad \frac{a(t)}{a(\hat{t})} = \frac{m_\Phi}{2p}$$



3. Lyman- α



4. The end?

1. Reheating



$$\frac{\partial f_\chi}{\partial t} - Hp \frac{\partial f_\chi}{\partial p} = \left| \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \right|^2 + \left| \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \right|^2 + \dots$$

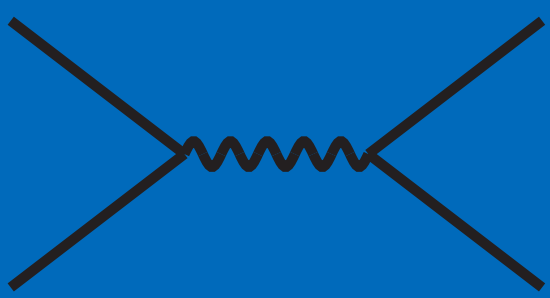
$$\equiv -\mathcal{C}^{2\leftrightarrow 2}[f_\chi] - \mathcal{C}^{\text{"1}\leftrightarrow\text{"2}}[f_\chi] + \dots,$$

2. Freeze-in

K. Harigaya, K. Mukaida, JHEP 05 (2014) 006

K. Mukaida, M. Yamada, JCAP 02 (2016) 003

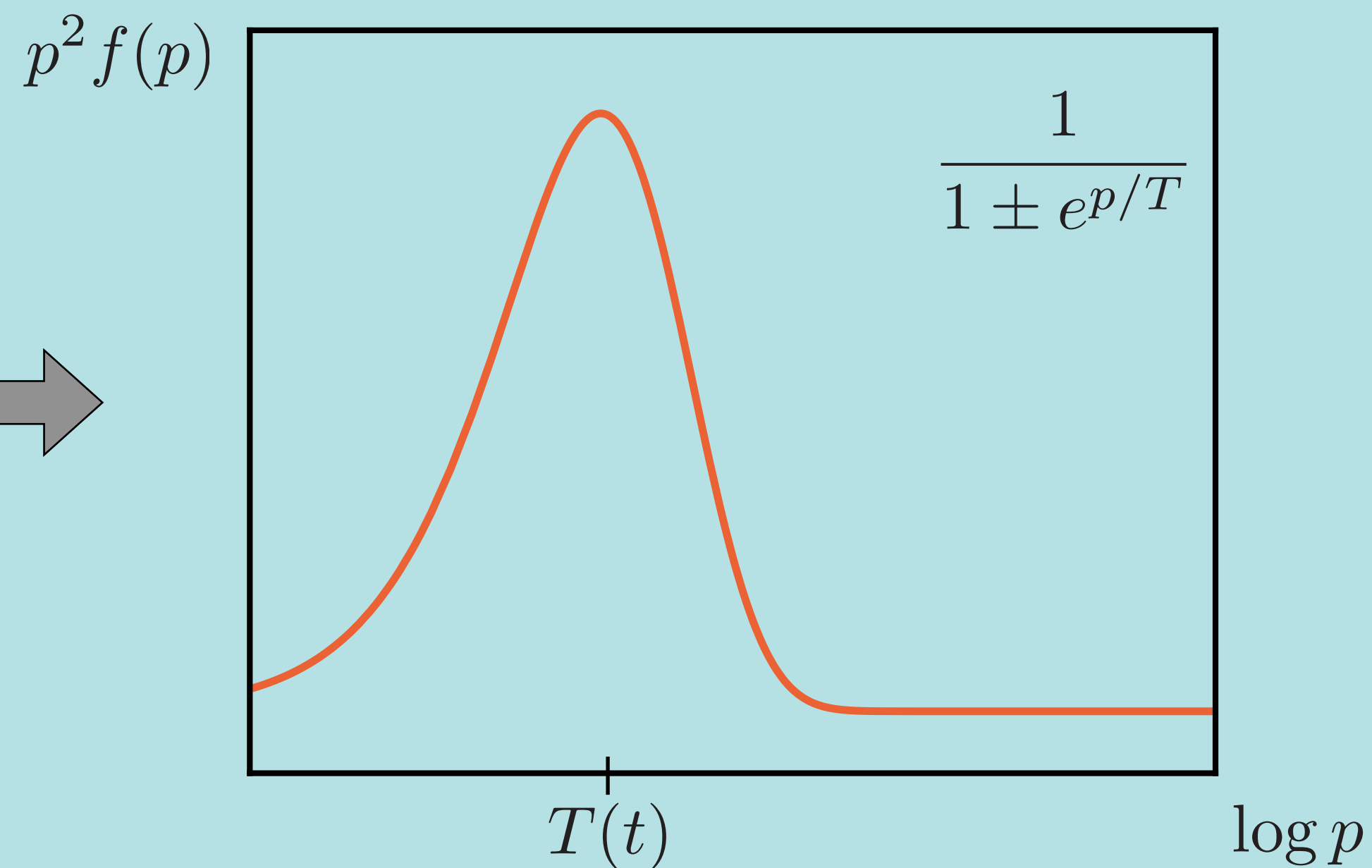
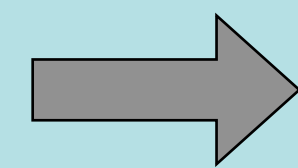
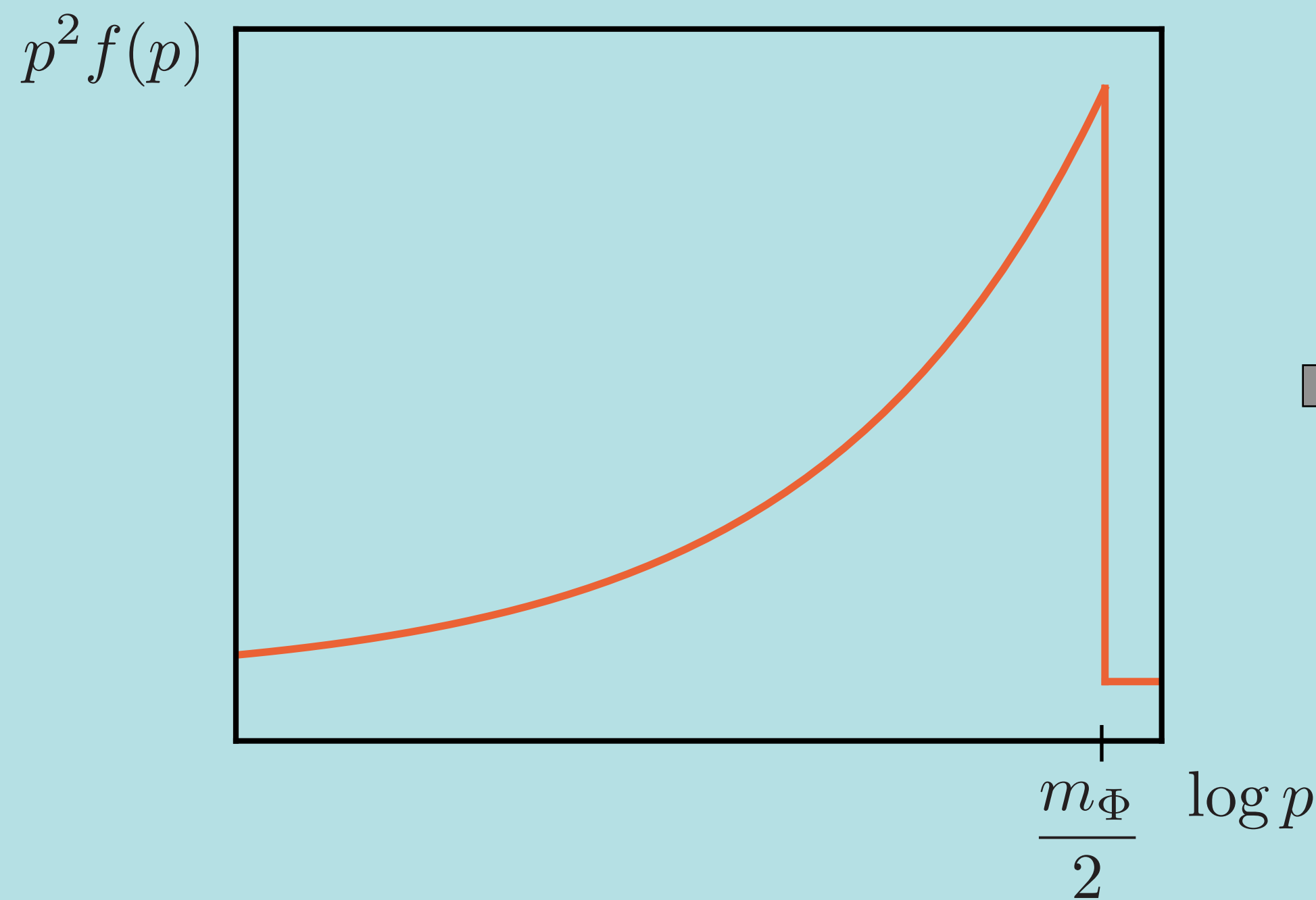
$$\Gamma_\Phi t_{\text{th}} \simeq \alpha_{\text{SM}}^{-16/5} \left(\frac{\Gamma_\Phi m_\Phi^2}{M_P^3} \right)^{2/5}$$



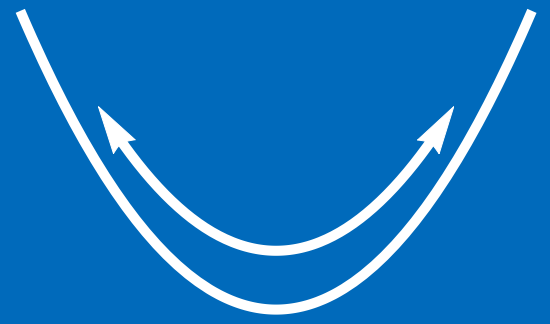
3. Lyman- α



4. The end?

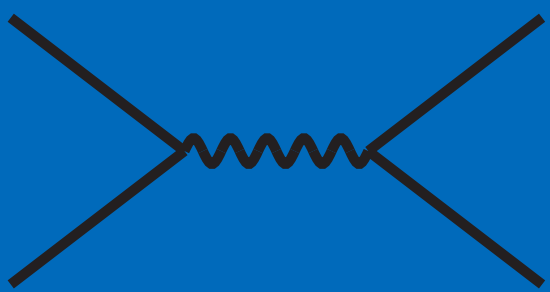


1. Reheating



$$\frac{\partial f_\chi}{\partial t} - H p \frac{\partial f_\chi}{\partial p} = \left| \begin{array}{c} \text{---} \\ | \\ \text{---} \\ \text{---} \\ | \\ \text{---} \end{array} \right|^2 + \left| \begin{array}{c} \text{---} \\ | \\ \text{---} \\ \text{---} \\ | \\ \text{---} \end{array} \right|^2 + \dots$$
$$\equiv -\mathcal{C}^{2\leftrightarrow 2}[f_\chi] - \mathcal{C}^{\text{"1}\leftrightarrow\text{"2}}[f_\chi] + \dots,$$

2. Freeze-in

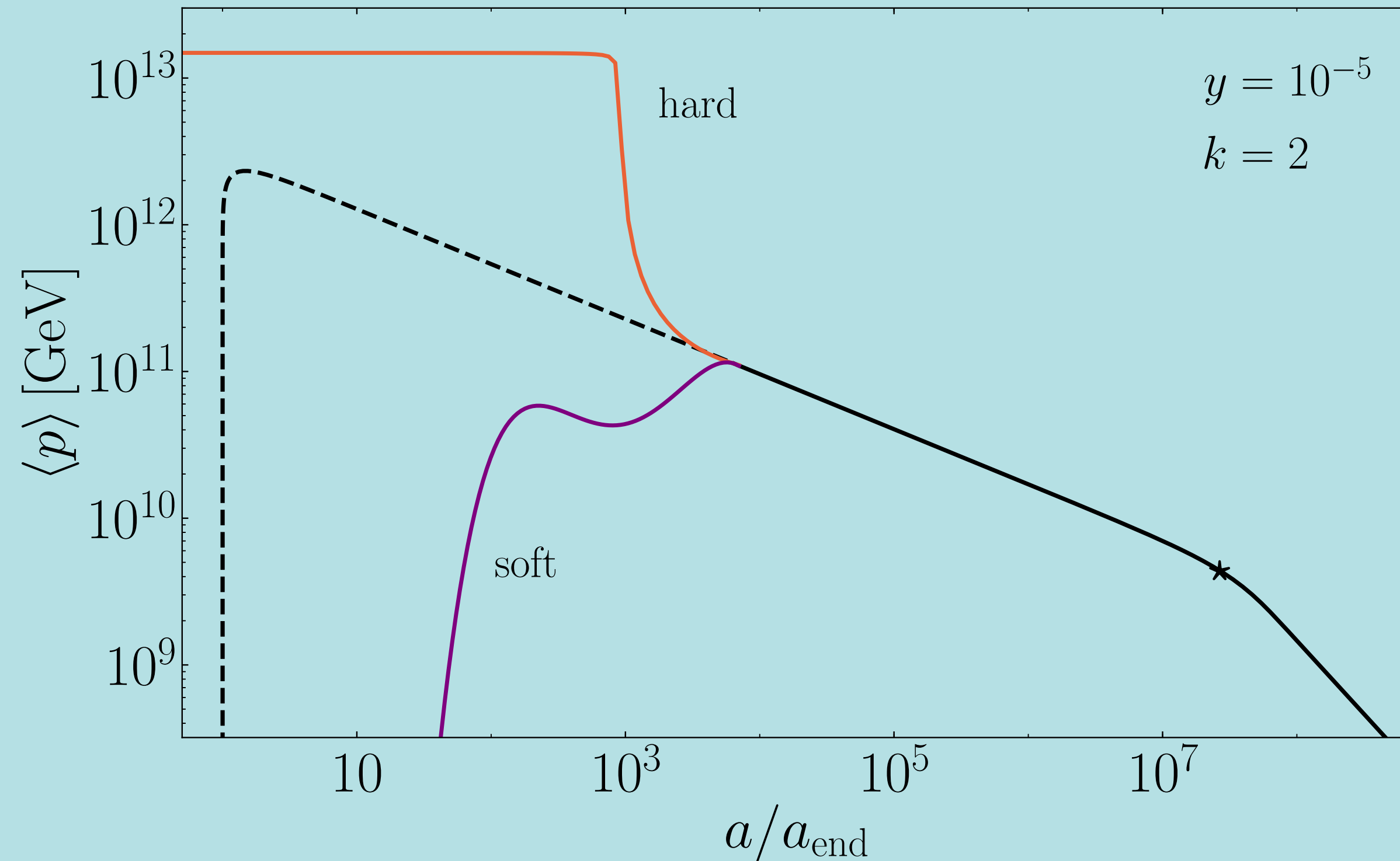


3. Lyman- α



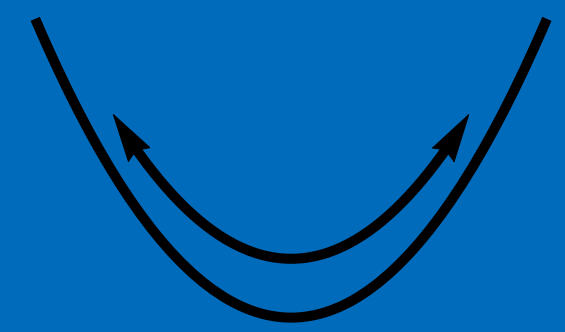
4. The end?

$$\Gamma_\Phi t_{\text{th}} \simeq \alpha_{\text{SM}}^{-16/5} \left(\frac{\Gamma_\Phi m_\Phi^2}{M_P^3} \right)^{2/5}$$

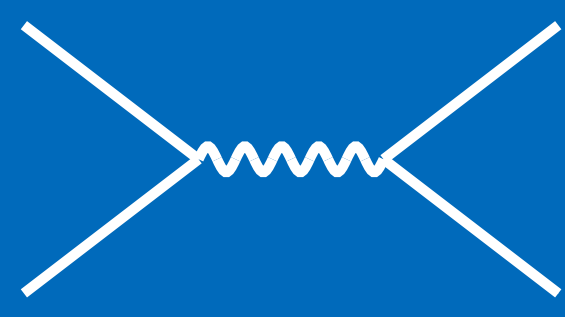


Freeze-in: producing dark matter out-of-equilibrium

1. Reheating



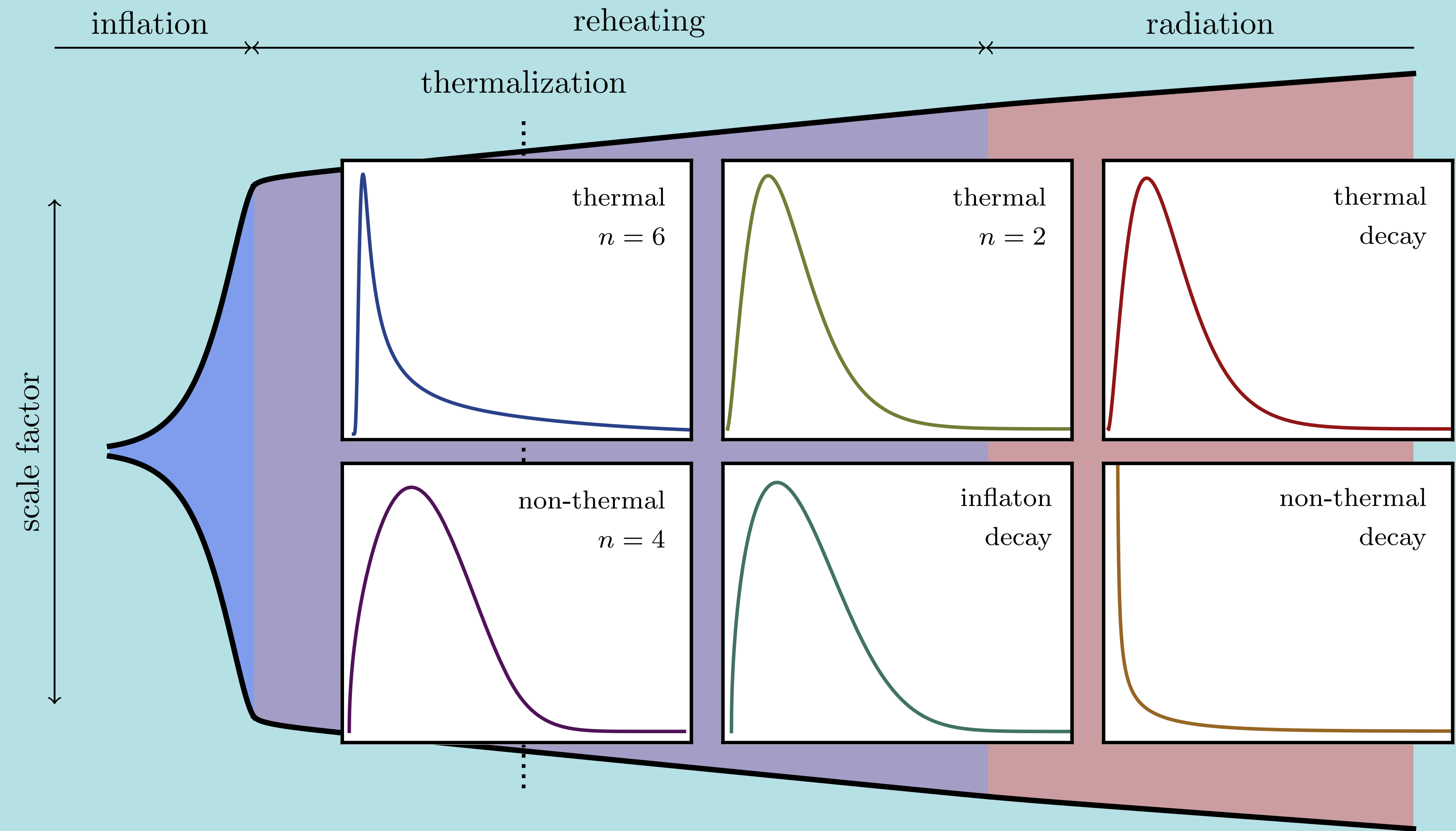
2. Freeze-in



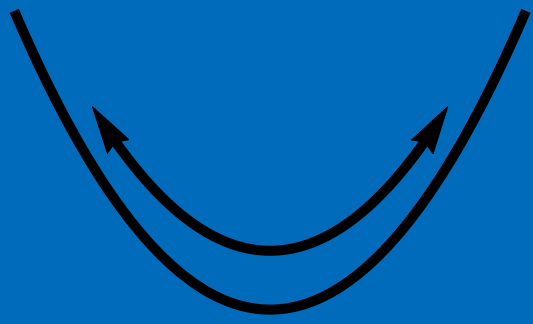
3. Lyman- α



4. The end?



1. Reheating

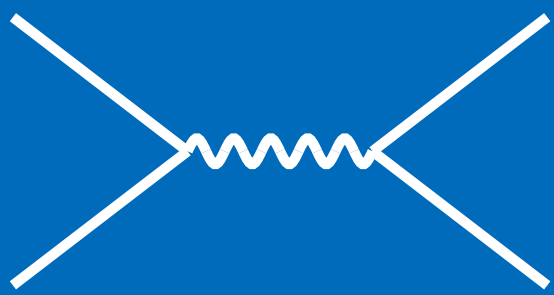


Decay of an oscillating condensate (inflaton, non-stabilized moduli, ...)

Notation. If $f_\chi(p) \equiv f_\chi(p/p_0)$ at decoupling ($t = t_*$), then for $t > t_*$

$$f_\chi \left(\frac{p}{p_0} \frac{a(t)}{a_*} \right) = f_\chi \left(\frac{p a(t)/a_0}{p_0 a_*/a_0} \right) = f_\chi \left(\underbrace{\frac{p_{\text{co}}}{p_0 a_*/a_0}}_{T_* \text{ (or } T_{\text{NCDM}})} \right) \equiv f_\chi(q)$$

2. Freeze-in



3. Lyman- α



4. The end?

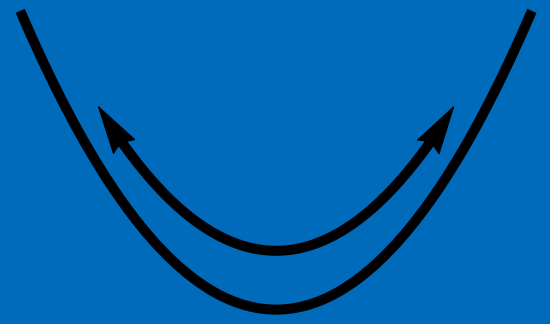
Standard in literature and cosmology codes (CLASS)

C. Ma, E. Bertschinger, *Astrophys. J.* 455 (1995) 7

D. Blas, J. Lesgourgues, T. Tram, *JCAP* 07 (2011) 034

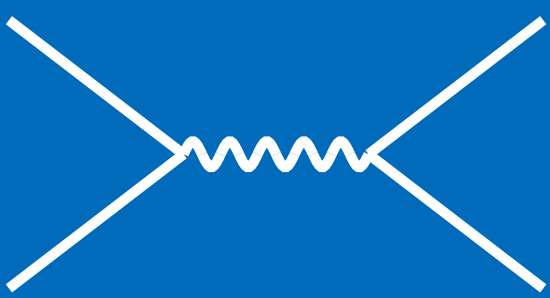
Decay of an oscillating condensate (inflaton, non-stabilized moduli, ...)

1. Reheating



$$f_\chi(p, t) d^3 p = \frac{4\pi^4 \text{Br}_\chi g_{*s}^{\text{reh}}}{5g_\chi} \left(\frac{T_{\text{reh}}}{m_\Phi}\right)^4 \left(\frac{a_0}{a(t)}\right)^3 T_*^3 \bar{f}_R(q) d^3 q, \quad T_* = \left(\frac{g_{*s}^0}{g_{*s}^{\text{reh}}}\right)^{1/3} \frac{m_\Phi}{2T_{\text{reh}}} T_0$$

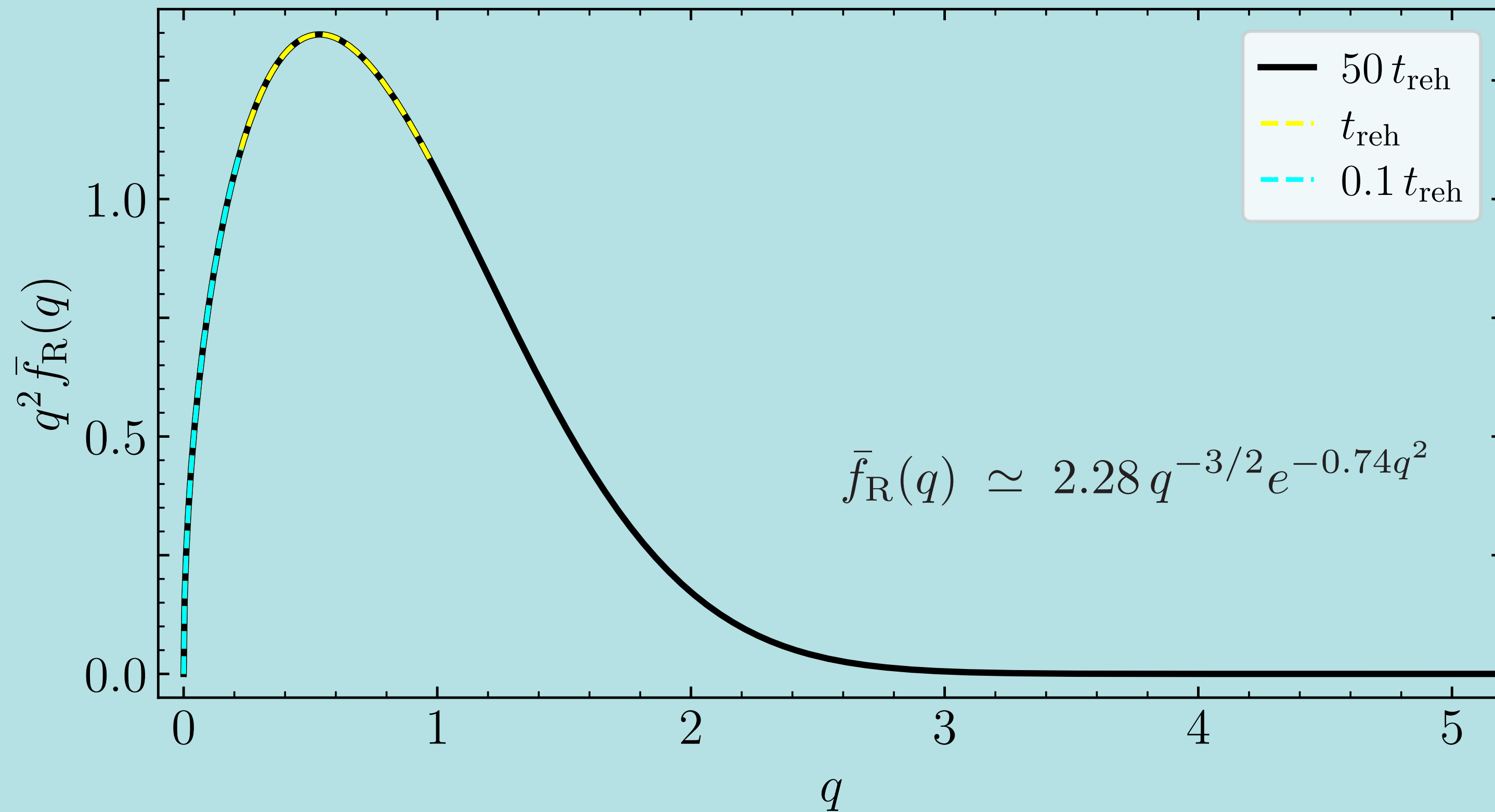
2. Freeze-in



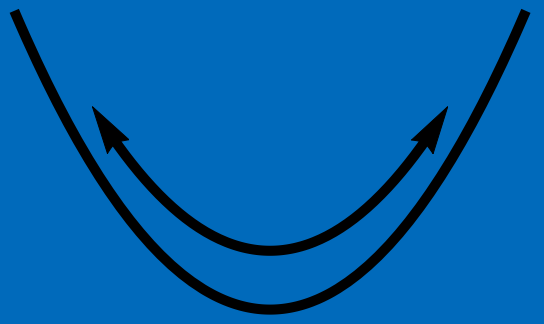
3. Lyman- α



4. The end?



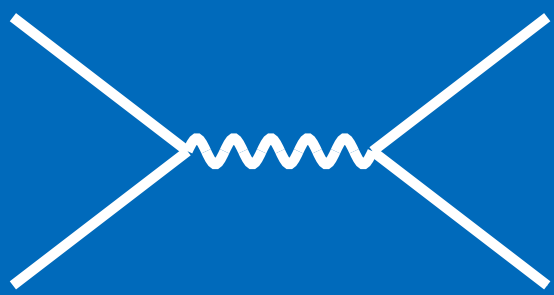
1. Reheating



Decay of an oscillating condensate (inflaton, non-stabilized moduli, ...)

$$f_{\chi}(p, t) d^3 \mathbf{p} = \frac{4\pi^4 \text{Br}_{\chi} g_{*s}^{\text{reh}}}{5g_{\chi}} \left(\frac{T_{\text{reh}}}{m_{\Phi}} \right)^4 \left(\frac{a_0}{a(t)} \right)^3 T_*^3 \bar{f}_{\text{R}}(q) d^3 \mathbf{q}, \quad T_* = \left(\frac{g_{*s}^0}{g_{*s}^{\text{reh}}} \right)^{1/3} \frac{m_{\Phi}}{2T_{\text{reh}}} T_0$$

2. Freeze-in



$$n_{\chi}(t) \simeq 0.70\pi^2 \text{Br}_{\chi} g_{*s}^{\text{reh}} \left(\frac{T_{\text{reh}}}{m_{\Phi}} \right)^4 \left(\frac{a_0}{a(t)} \right)^3 T_*^3$$

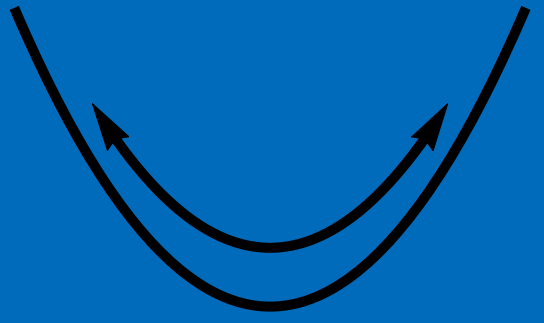
3. Lyman- α



$$\Omega_{\chi} h^2 \simeq 0.1 \left(\frac{\text{Br}_{\chi}}{5.5 \times 10^{-4}} \right) \left(\frac{m_{\text{DM}}}{1 \text{ MeV}} \right) \left(\frac{T_{\text{reh}}}{10^{10} \text{ GeV}} \right) \left(\frac{3 \times 10^{13} \text{ GeV}}{m_{\Phi}} \right)$$

4. The end?

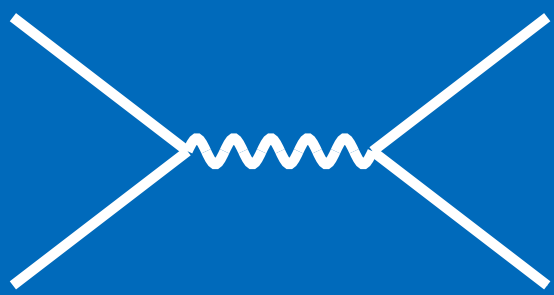
1. Reheating



Out-of-equilibrium decay chain $\Phi \rightarrow A \rightarrow \chi$

$$\Omega_\chi h^2 \simeq 0.1 \left(\frac{\text{Br}_\chi}{5.5 \times 10^{-4}} \right) \left(\frac{m_{\text{DM}}}{1 \text{ MeV}} \right) \left(\frac{T_{\text{reh}}}{10^{10} \text{ GeV}} \right) \left(\frac{3 \times 10^{13} \text{ GeV}}{m_\Phi} \right)$$

2. Freeze-in



$$f_\chi(p, t) d^3\mathbf{p} = \frac{24\pi^3 \sqrt{10g_{*s}^{\text{reh}}} \text{Br}_\chi \text{Br}_A \Gamma_A M_P}{5g_A m_A^2} \left(\frac{T_{\text{reh}}}{m_\phi} \right)^2 \mathcal{F}(q, \mathcal{R}) \left(\frac{a_0}{a(t)} \right)^3 T_*^3 d^3\mathbf{q}$$

3. Lyman- α



$$\mathcal{F}(q, \mathcal{R}) = q^{-2} \int_0^{\mathcal{R}} dy y^2 \int_{|q - \frac{y^2}{q}|}^{\infty} \frac{z dz}{\sqrt{q^2 + 4y^2}} \bar{f}_R(z) \simeq \begin{cases} \bar{f}_{\text{D,NR}}(q), & \mathcal{R} \gg 1, \\ \frac{\mathcal{R}^3}{3} \bar{f}_{\text{D,R}}(q), & \mathcal{R} \ll 1. \end{cases}$$

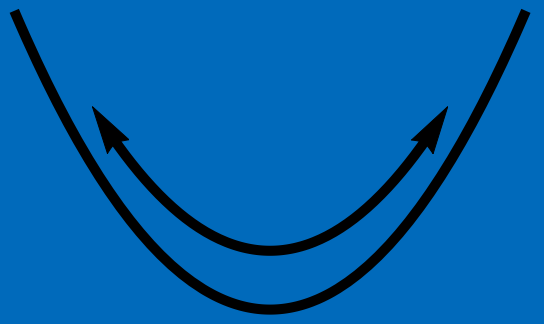
$$\mathcal{R} \equiv \left(\frac{g_{*s}^{\text{reh}}}{g_{*s}^{\text{dec}}} \right)^{1/3} \frac{m_A T_{\text{reh}}}{m_\phi T_{\text{dec}}}$$

(for thermal decays, see also K. Petraki and A. Kusenko, Phys. Rev. D 77 (2008) 065014;

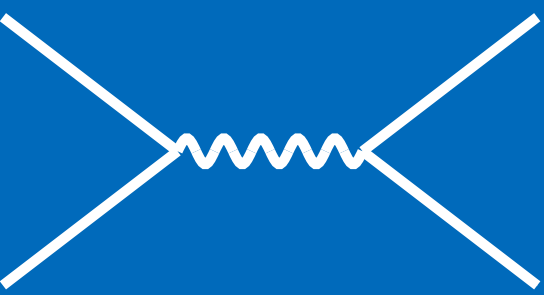
K. J. Bae, A. Kamada, S. P. Liew and K. Yanagi, JCAP 01 (2018) 054)

4. The end?

1. Reheating



2. Freeze-in

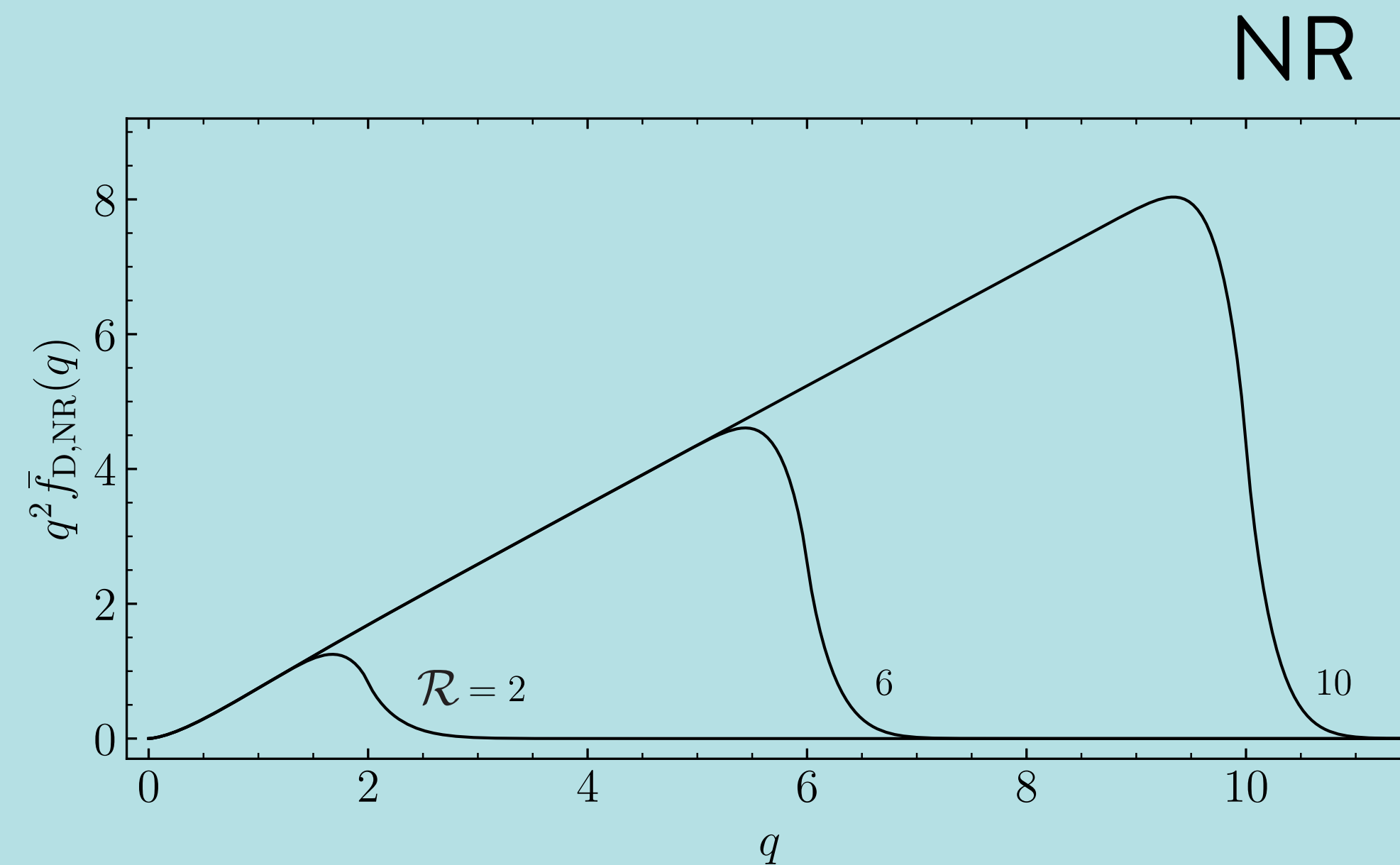
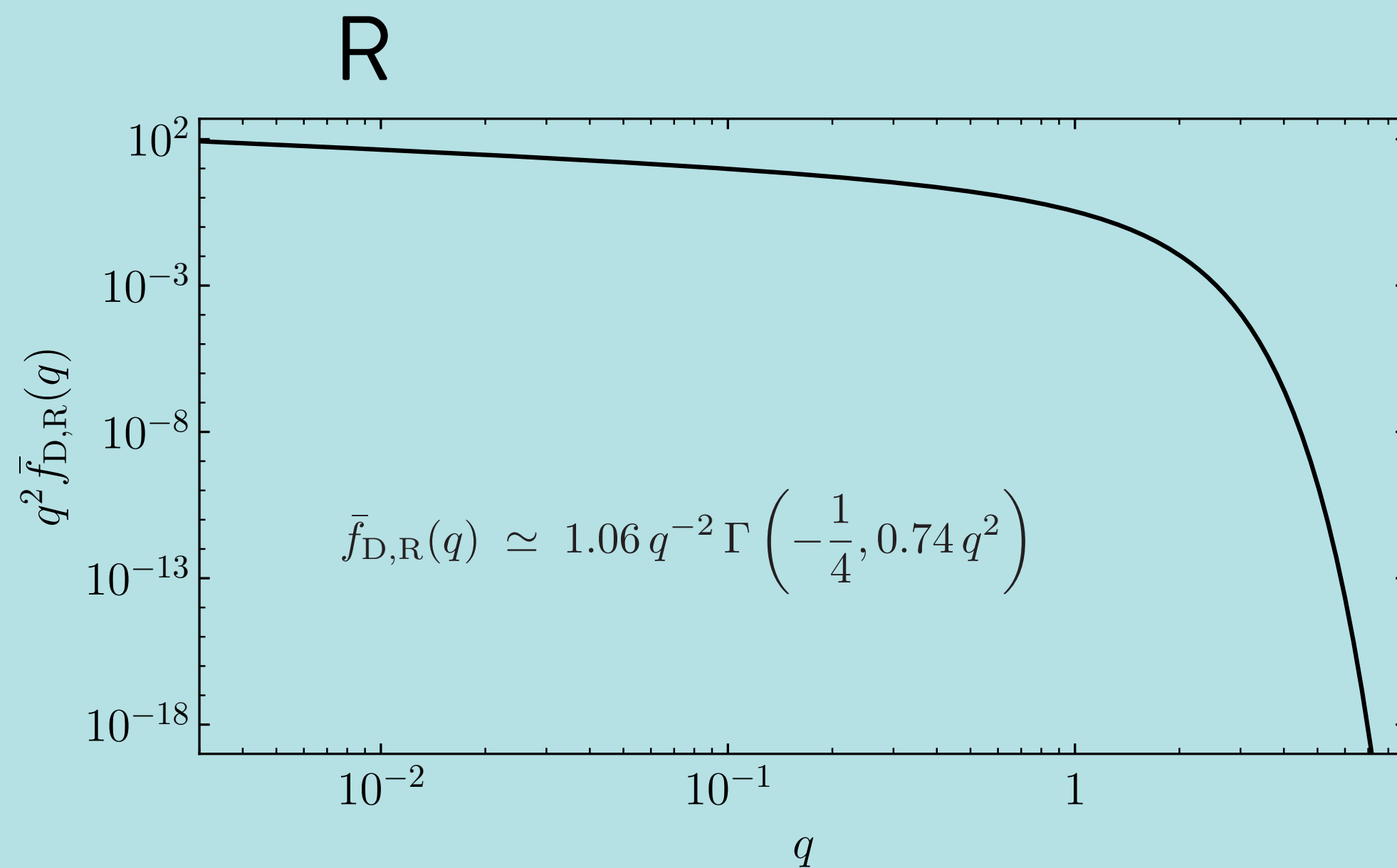
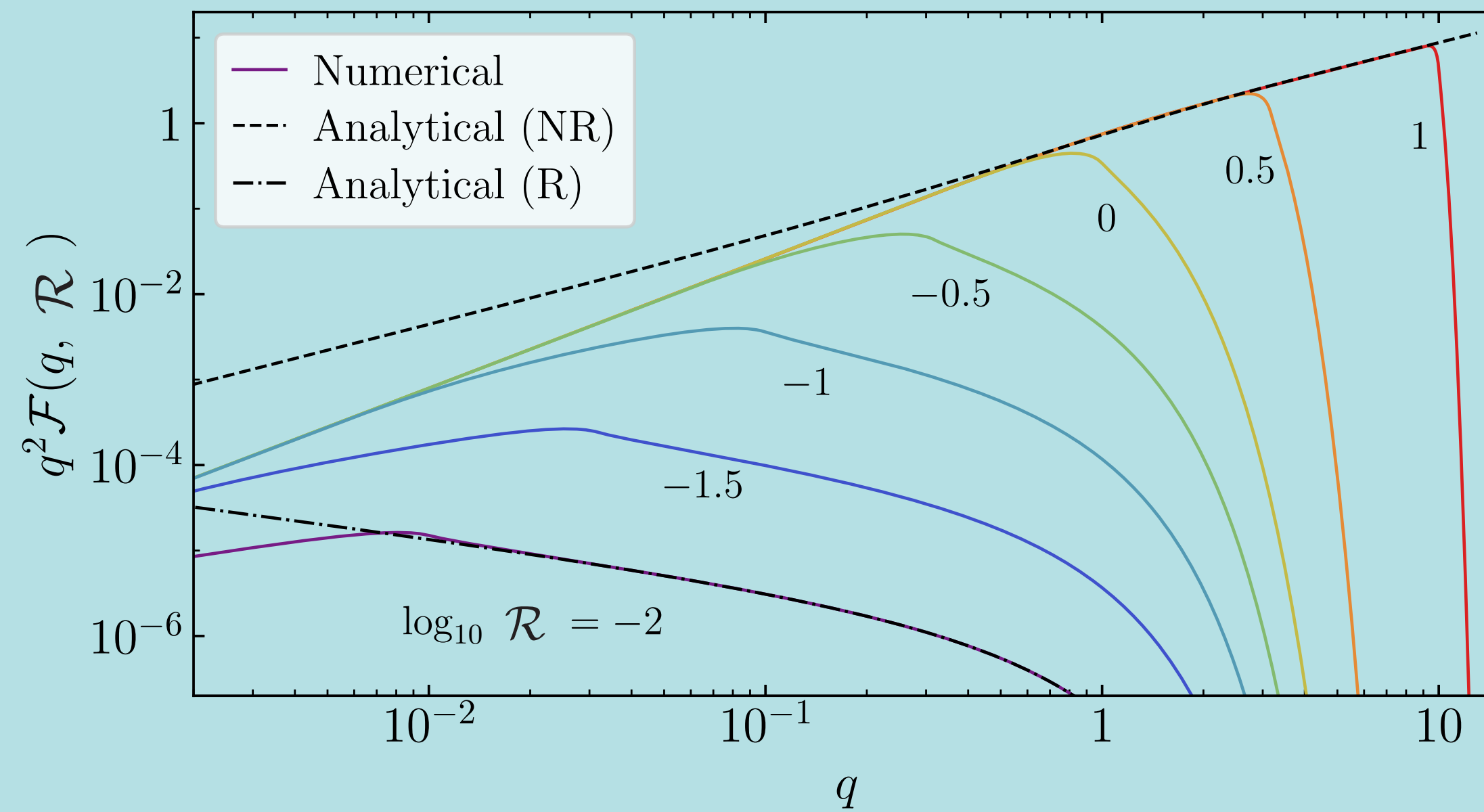


3. Lyman- α

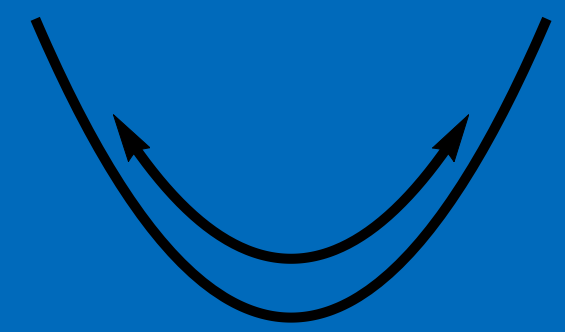


4. The end?

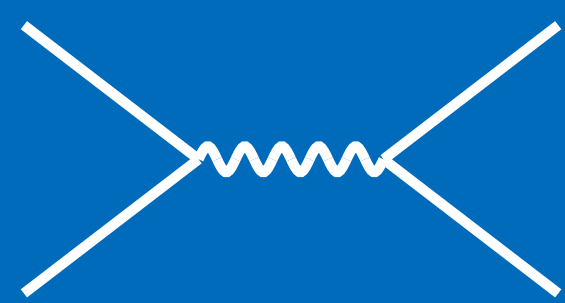
Out-of-equilibrium decay chain $\Phi \rightarrow A \rightarrow \chi$



1. Reheating



2. Freeze-in

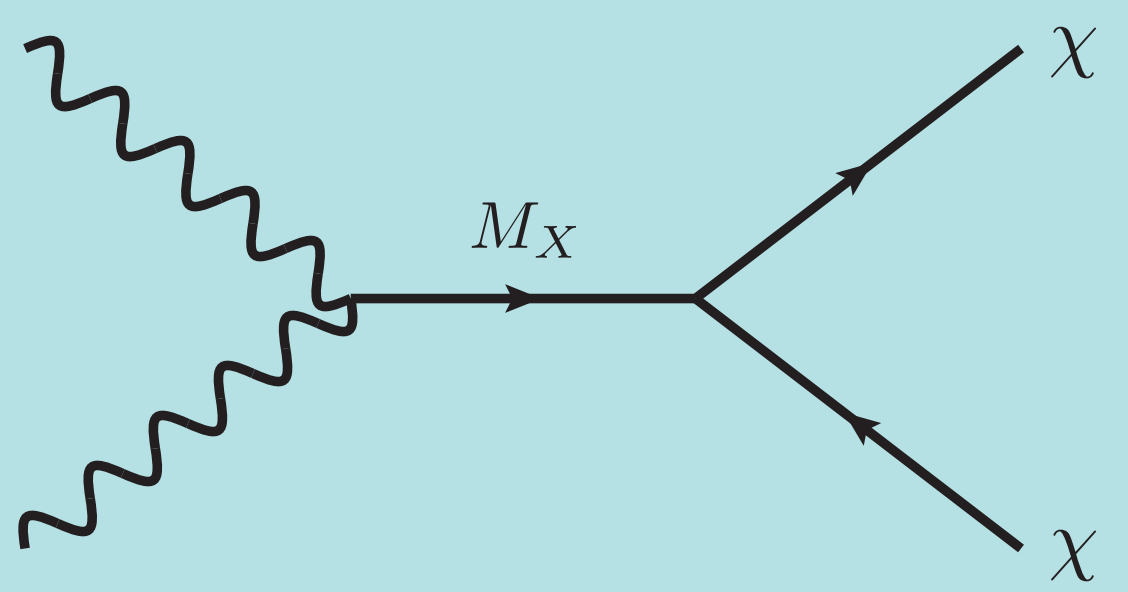


3. Lyman-α

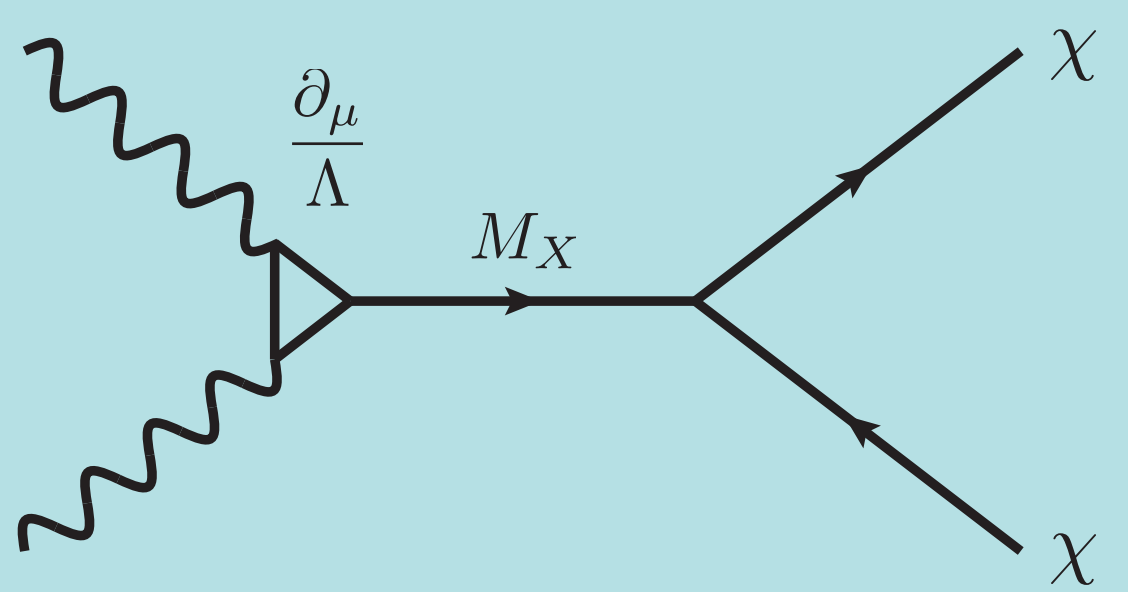


4. The end?

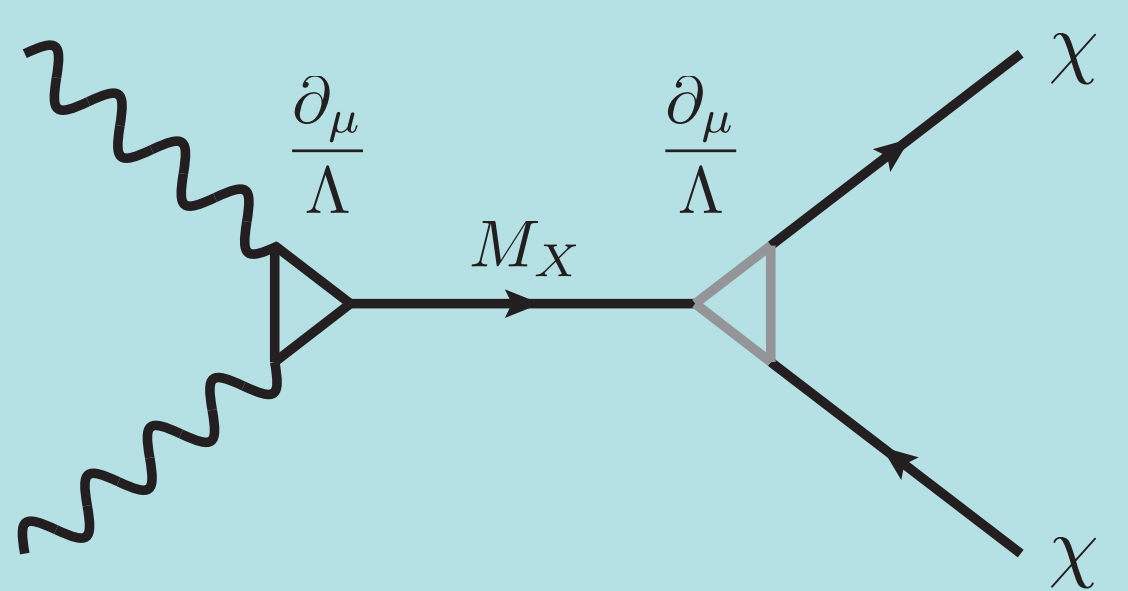
Freezing-in through scatterings



$$\sigma(s) = \frac{s}{M_X^4}$$



$$\sigma(s) = \frac{s^2}{\Lambda^2 M_X^4}$$



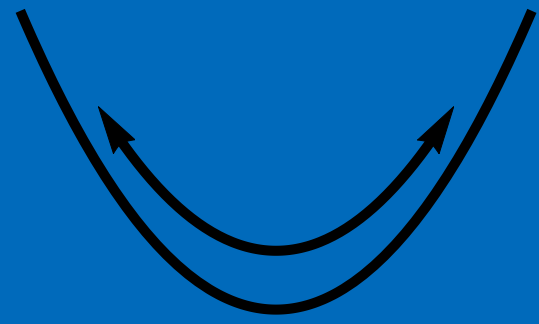
$$\sigma(s) = \frac{s^3}{\Lambda^4 M_X^4}$$

Production at $t \leq t_{\text{reh}}$ if

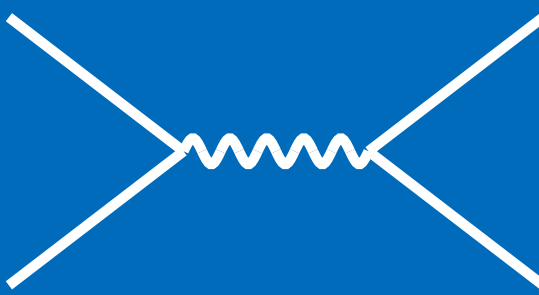
$$\sigma(s) = \frac{s^{n/2}}{\Lambda^{n+2}}$$

with $n > -1$

1. Reheating



2. Freeze-in



3. Lyman- α

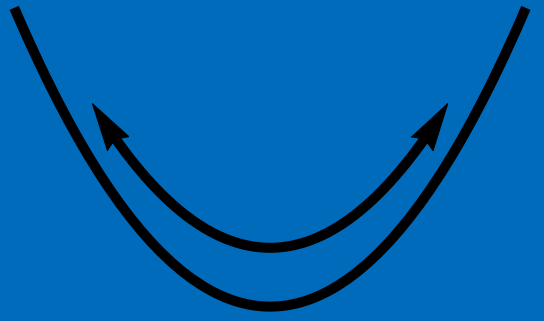


4. The end?

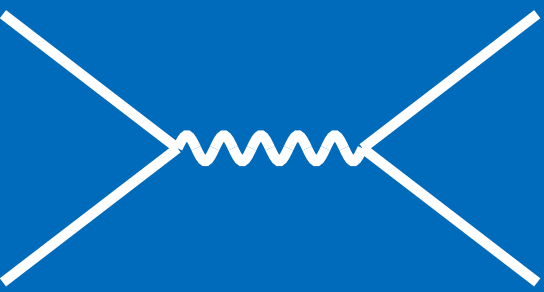
Freezing-in through scatterings (examples)

- $n = 0$: Low scale susy gravitino, $\sigma \propto M_P^{-2}$. Axino, $\sigma \propto f_a^{-2}$
V. Rychkov, A. Strumia, Phys. Rev. D 75 (2007) 075011 ; A. Strumia, JHEP 06 (2010) 036
- $n = 2$: $SO(10) \rightarrow SU(4) \times SU(2)_L \times U(1)_R \rightarrow$ SM (heavy DM), $\sigma \propto s/M_{\text{int}}^4$
Light spin-2 mediator (light DM), $\sigma \propto s/M_P^4$
Y. Mambrini et al., Phys. Rev. Lett. 110 (2013) 241306; N. Bernal et al., Phys. Rev. D 97 (2018) 115020
- $n = 4$: Non-susy spin-3/2 DM + sterile neutrino, $\sigma(s) \propto (s/m_{3/2}m_R M_P)^2$
- $n = 6$: High scale susy gravitino (heavy DM), $\sigma \propto s^3/(m_{3/2}M_P)^4$
Heavy spin-2 mediator (light DM), $\sigma \propto s^3/(m_{\tilde{h}}M_P)^4$
K. Benakli et al., Phys. Rev. D 95 (2017) 095002; N. Bernal et al., Phys. Rev. D 97 (2018) 115020
- $n > 6$: Vector non-Abelian DM with heavy Z'
G. Bhattacharyya, M. Dutra, Y. Mambrini, M. Pierre, Phys. Rev. D 98 (2018) 035038

1. Reheating



2. Freeze-in



3. Lyman- α



4. The end?

Thermal freeze-in

$n < 6$:

$$\Omega_{\chi}^{(n)} h^2 \simeq \frac{g_A g_B g_{\psi} g_{\chi} \sqrt{c} 2^{n+3} \Gamma(\frac{n}{2} + 3)^2 \zeta(\frac{n}{2} + 3)^2 \mathcal{S}(n)}{(6-n)(n+4)} \left(\frac{106.75}{g_{*s}^{\text{reh}}}\right)^{3/2} \left(\frac{T_{\text{reh}}}{\Lambda}\right)^{n+1} \left(\frac{10^{16} \text{ GeV}}{\Lambda}\right) \left(\frac{m_{\text{DM}}}{1 \text{ keV}}\right)$$

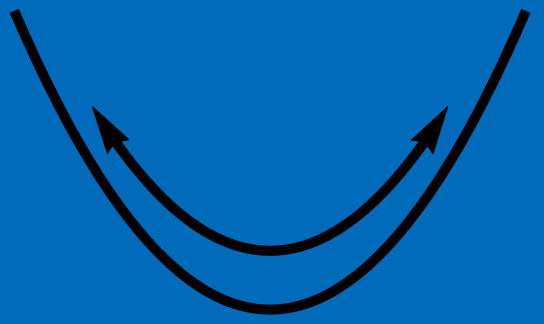
$n = 6$:

$$\Omega_{\chi}^{(6)} h^2 = g_A g_B g_{\psi} g_{\chi} \sqrt{c} \mathcal{S}(6) \left(\frac{106.75}{g_{*s}^{\text{reh}}}\right)^{3/2} \left(\frac{m_{\text{DM}}}{1.2 \text{ keV}}\right) \left(\frac{T_{\text{reh}}}{10^6 \text{ GeV}}\right)^7 \left(\frac{10^8 \text{ GeV}}{\Lambda}\right)^8 \ln\left(\frac{T_{\text{max}}}{T_{\text{reh}}}\right)$$

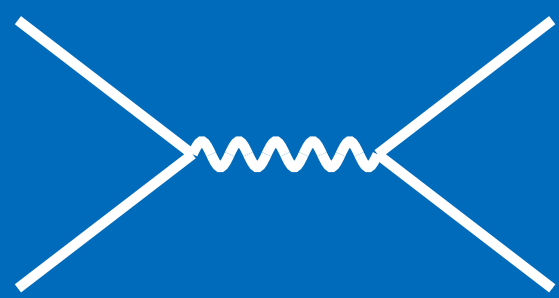
$n > 6$:

$$\Omega_{\chi}^{(n)} h^2 \simeq \frac{g_A g_B g_{\psi} g_{\chi} \sqrt{c} 2^{n+3} \Gamma(\frac{n+4}{2}) \Gamma(\frac{n+6}{2})}{n-6} \left(\frac{106.75}{g_{*s}^{\text{reh}}}\right)^{3/2} \times \left(\frac{T_{\text{max}}}{\Lambda}\right)^{n+1} \left(\frac{T_{\text{reh}}}{T_{\text{max}}}\right)^7 \left(\frac{10^{16} \text{ GeV}}{\Lambda}\right) \left(\frac{m_{\text{DM}}}{1.8 \text{ keV}}\right)$$

1. Reheating



2. Freeze-in



3. Lyman- α



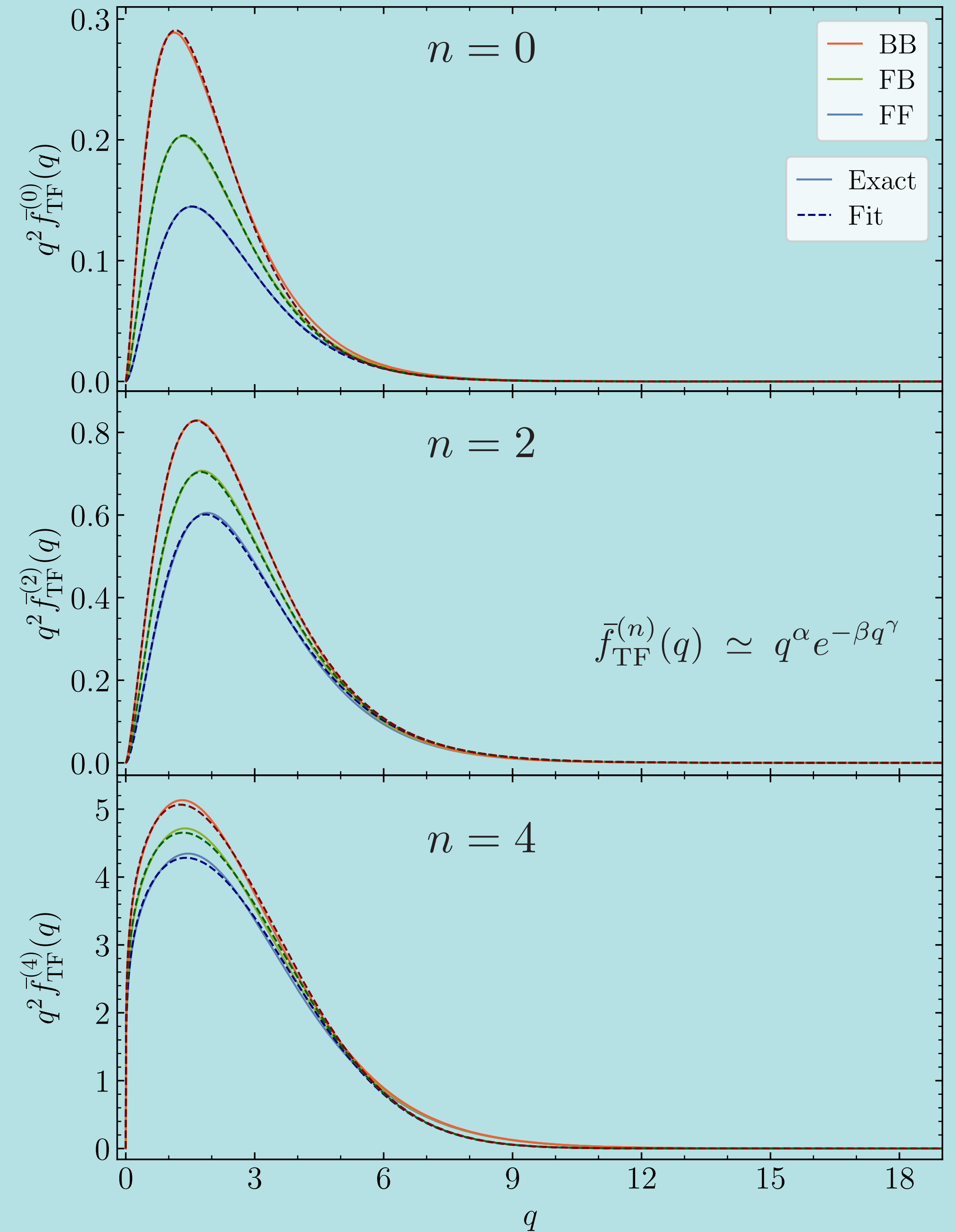
4. The end?

Thermal freeze-in

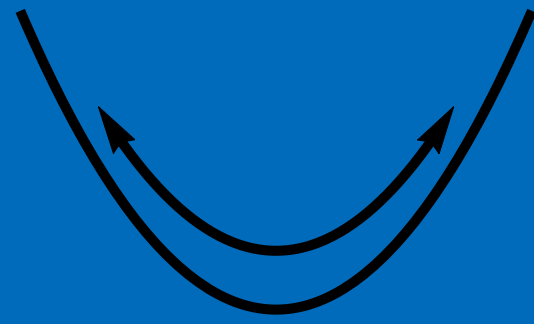
$n < 6$:

$$f_\chi(p, t) d^3\mathbf{p} \simeq \frac{3 \cdot 2^{n+6} \Gamma(\frac{n+4}{2}) g_A g_B g_\psi M_P T_{\text{reh}}^{n+1}}{5(2\pi)^3 \Lambda^{n+2}} \times \left(\frac{6c}{g_{*s}^{\text{reh}}}\right)^{1/2} \left(\frac{a_0}{a(t)}\right)^3 T_*^3 \bar{f}_{\text{TF}}^{(n)}(q) d^3\mathbf{q}$$

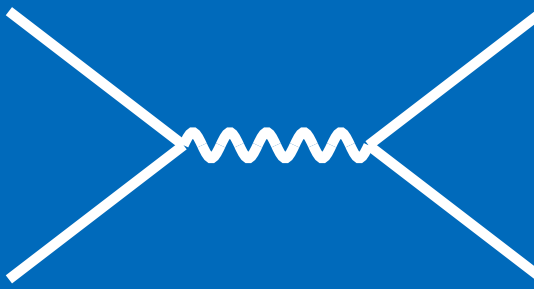
$$T_* = \left(\frac{g_{*s}^0}{g_{*s}^{\text{reh}}}\right)^{1/3} T_0$$



1. Reheating



2. Freeze-in



3. Lyman- α

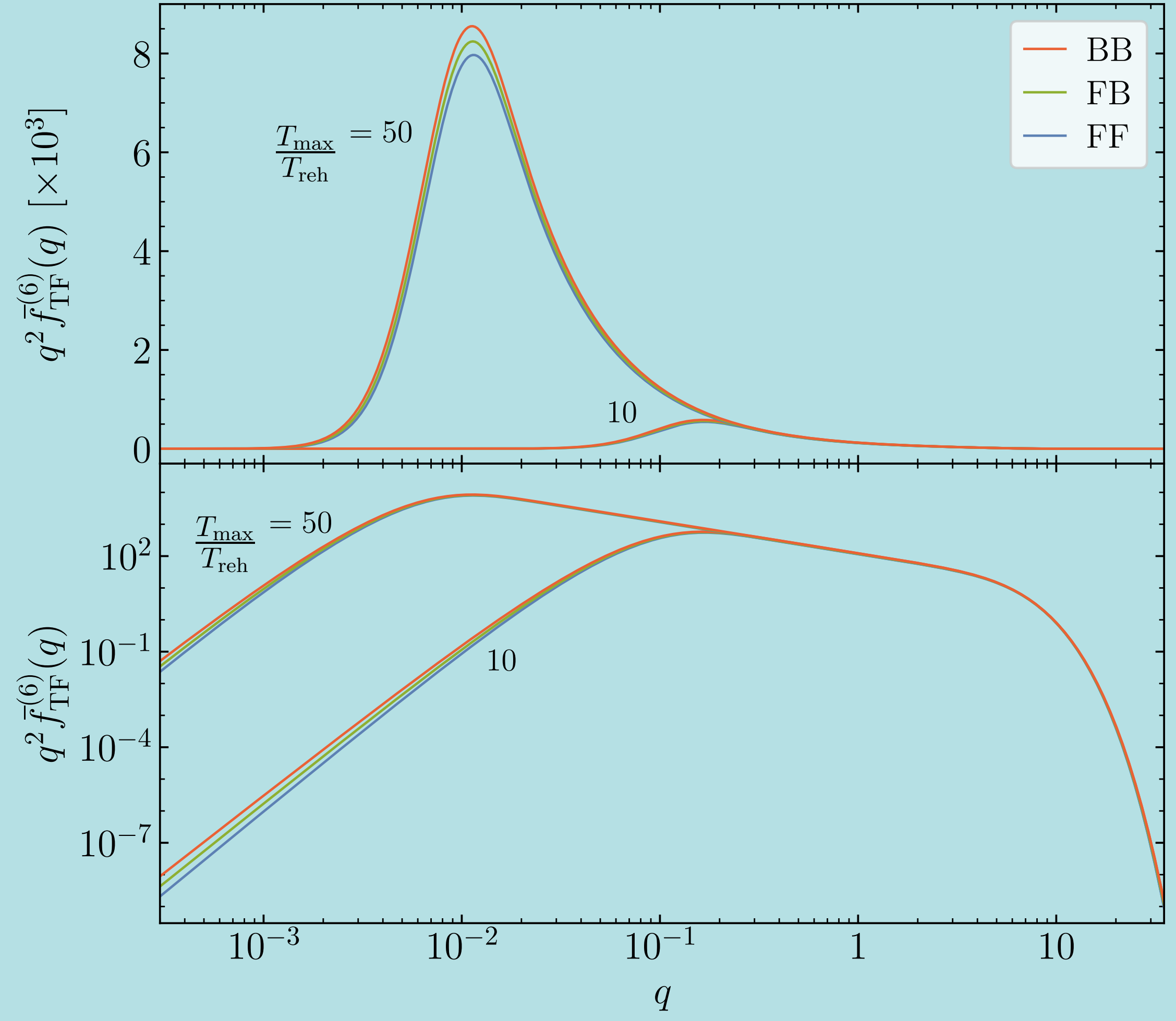


4. The end?

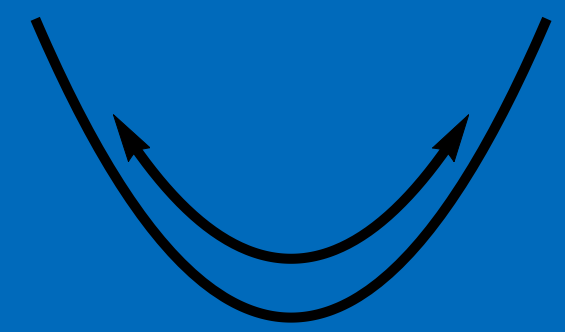
Thermal freeze-in

$n \geq 6$:

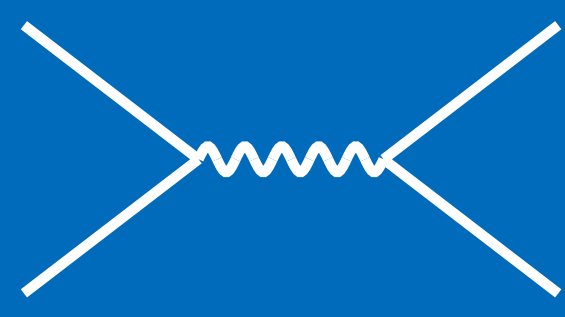
$$\bar{f}_{\text{TF}}^{(n)}(q) \simeq q^{\frac{3}{5}(1-n)} \left[\Gamma \left(\frac{11}{10}n - \frac{3}{5}, q \right) - \Gamma \left(\frac{11}{10}n - \frac{3}{5}, q \left(\frac{T_{\text{max}}}{T_{\text{reh}}} \right)^{5/3} \right) \right]$$



1. Reheating



2. Freeze-in



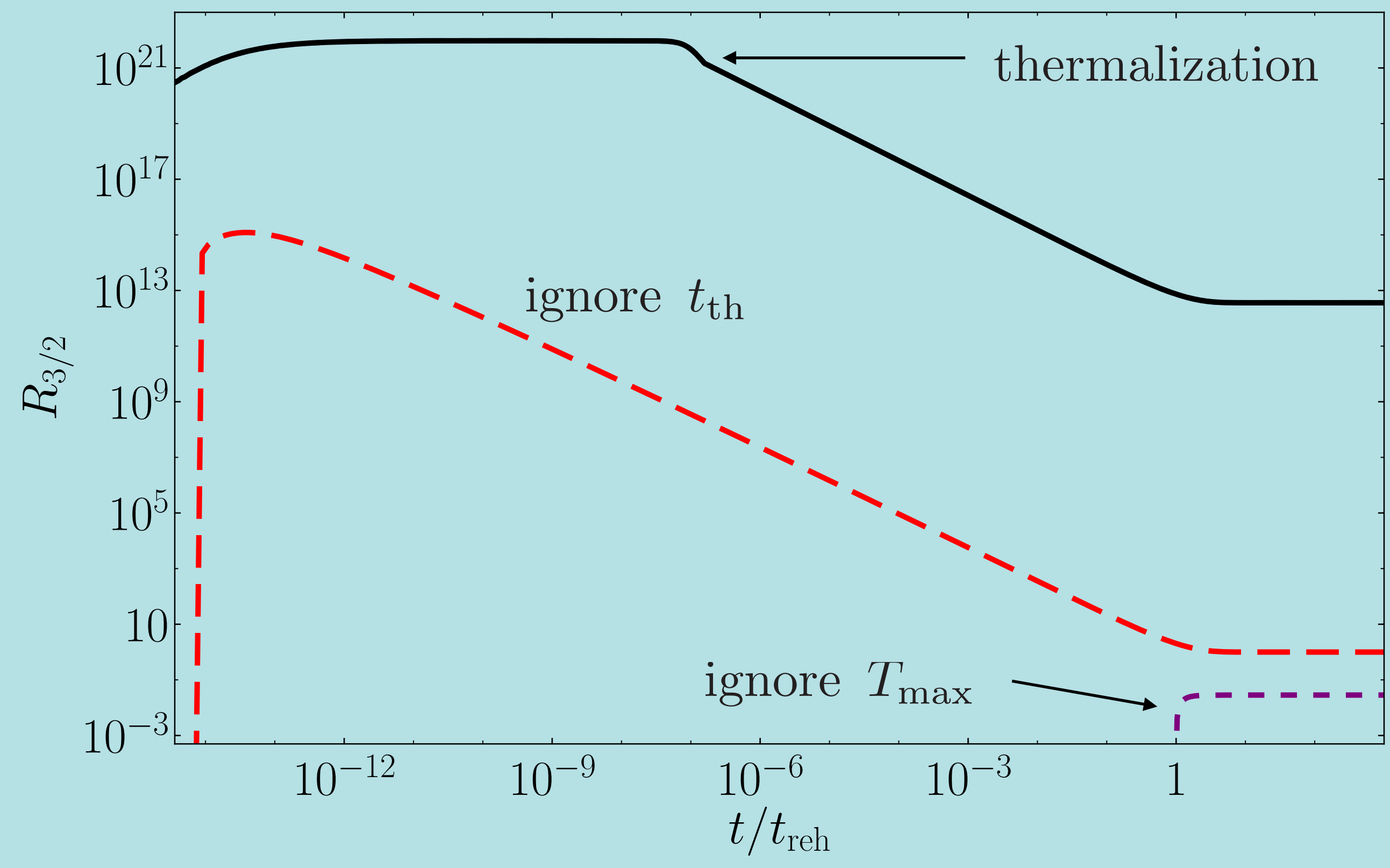
3. Lyman- α



4. The end?

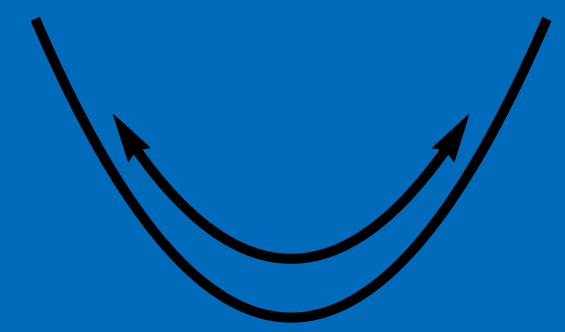
Non-thermal freeze-in

High scale supersymmetry breaking: $m_{\text{susy}} \gg m_{\Phi} \gg m_{3/2}$ ($n = 6$)

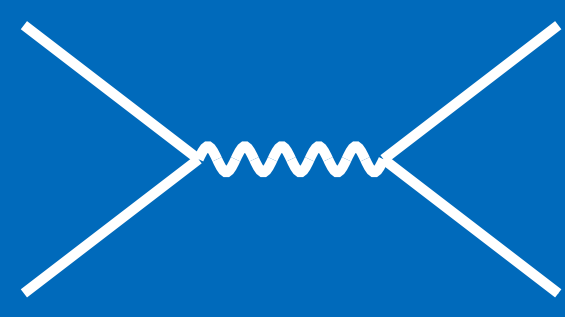


$$R_{3/2} \equiv \frac{n_{3/2}^{\text{NT}}}{n_{3/2}^{\text{T}}}$$

1. Reheating



2. Freeze-in

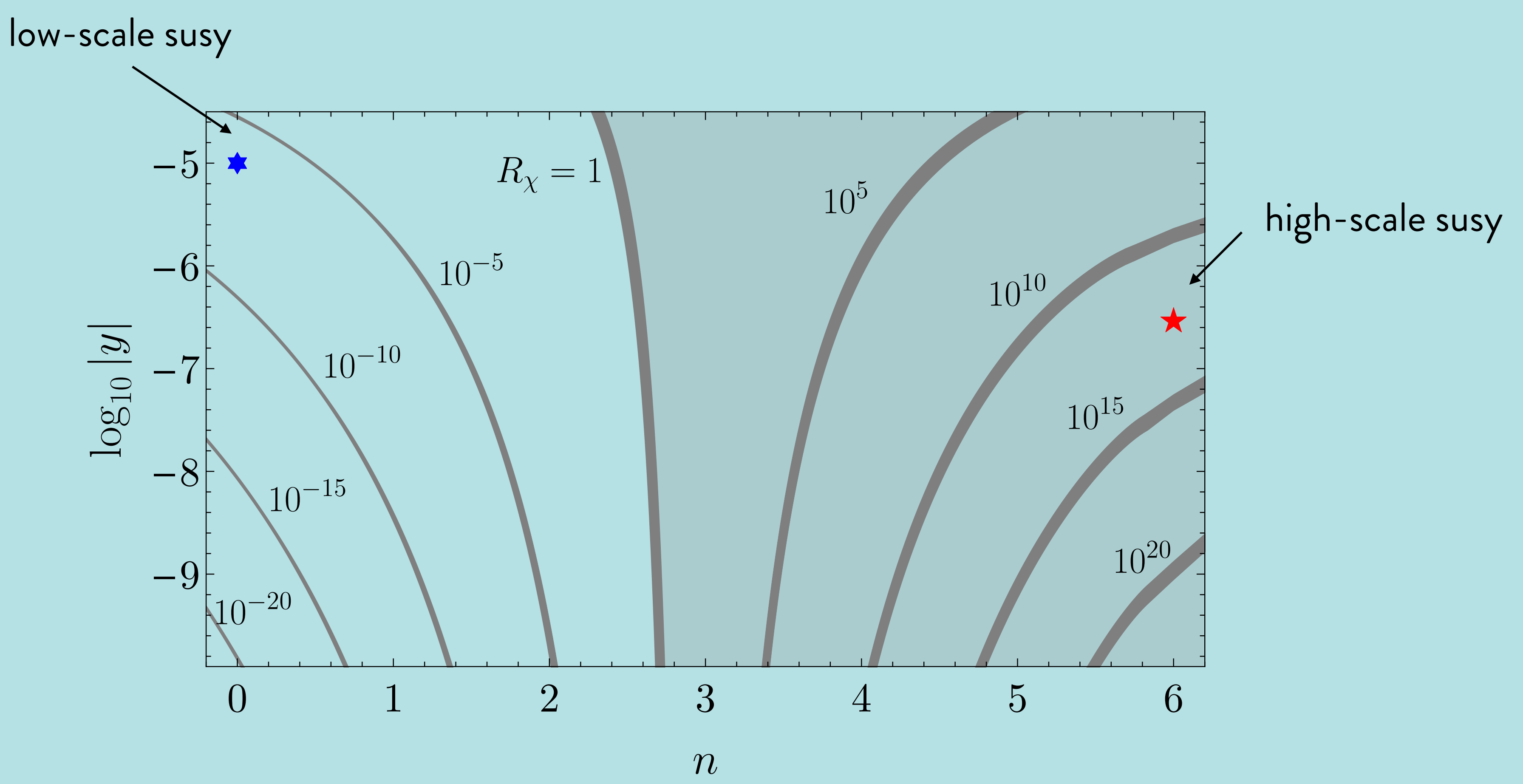


3. Lyman- α



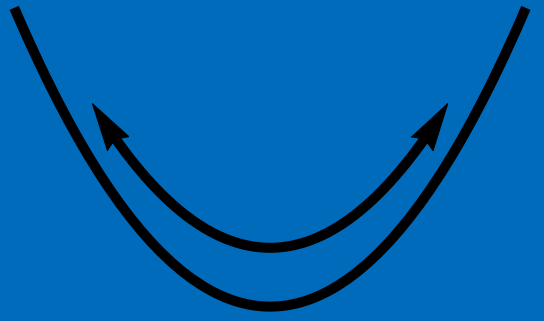
4. The end?

Non-thermal freeze-in



$$\Omega_\chi \propto \frac{m_\Phi^{n-2} M_P T_{\text{reh}}^3}{M^{n+2}} (\Gamma_\Phi t_{\text{th}})$$

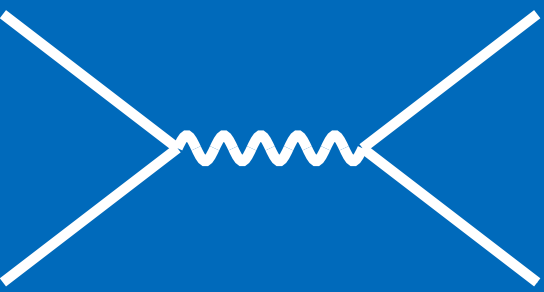
1. Reheating



Non-thermal freeze-in ($n = 4$)

$$f_\chi(p, t) d^3\mathbf{p} \simeq \frac{256\pi^2 g_\psi}{15015\Lambda^6} \left(\frac{\pi^2 c g_{*s}^{\text{reh}}}{24}\right)^{13/10} \left(\frac{\alpha_{\text{SM}}^{16} T_{\text{reh}}^{26} M_P^{13}}{m_\Phi^9}\right)^{1/5} \left(\frac{a_0}{a(t)}\right)^3 T_*^3 \bar{f}_{\text{NF}}^{(4)}(q) d^3\mathbf{q}$$

2. Freeze-in

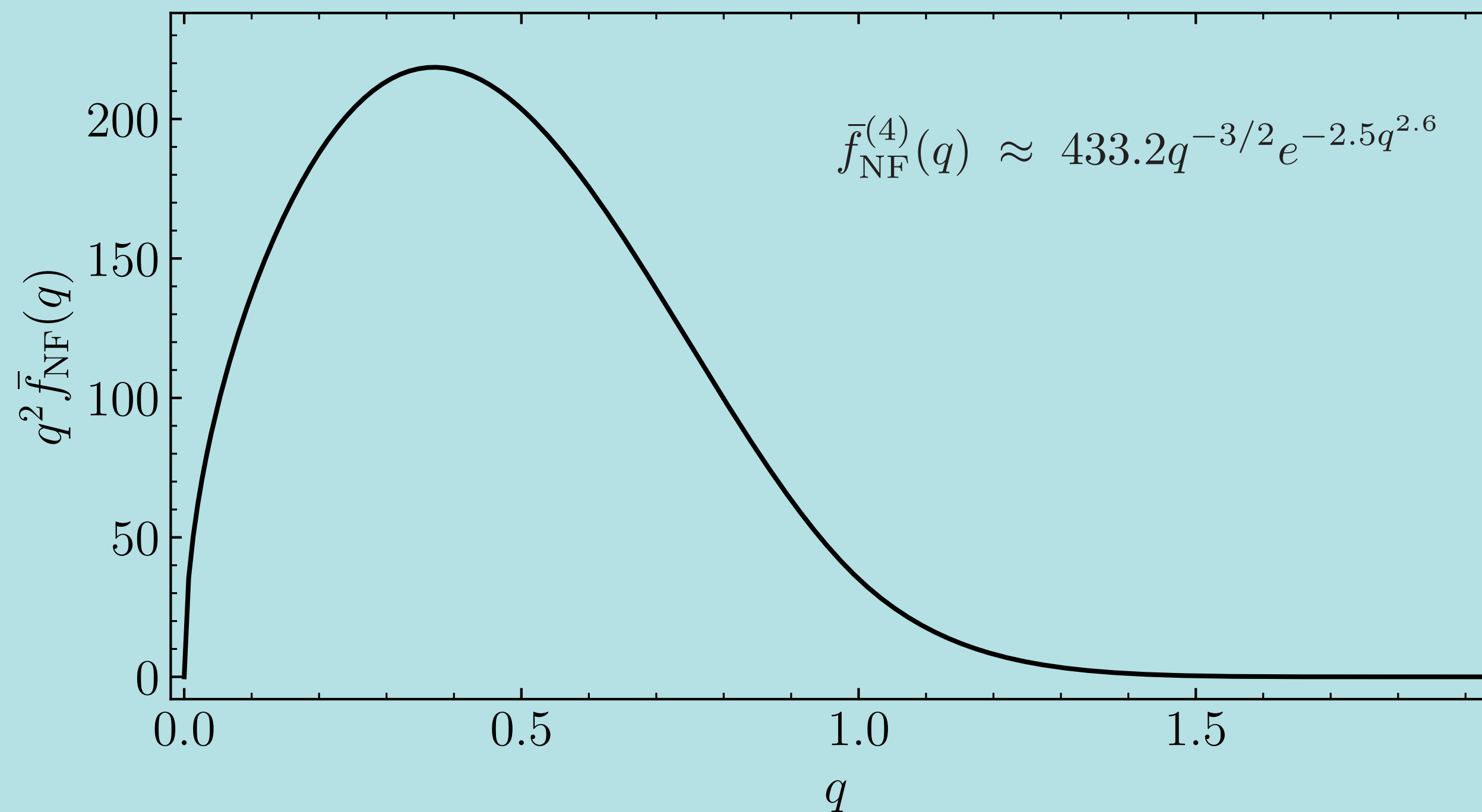


$$T_* = \frac{\alpha_{\text{SM}}^{-32/15}}{2} \left(\frac{g_{*s}^0}{g_{*s}^{\text{reh}}}\right)^{1/3} \left(\frac{\pi^2 c g_{*s}^{\text{reh}}}{24}\right)^{2/15} \left(\frac{m_\Phi}{T_{\text{reh}}}\right)^{7/15} \left(\frac{m_\Phi}{M_P}\right)^{16/15} T_0$$

3. Lyman- α

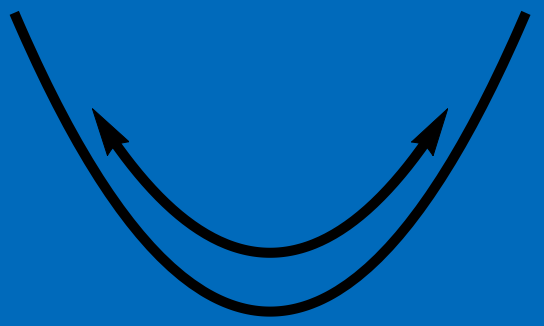


4. The end?

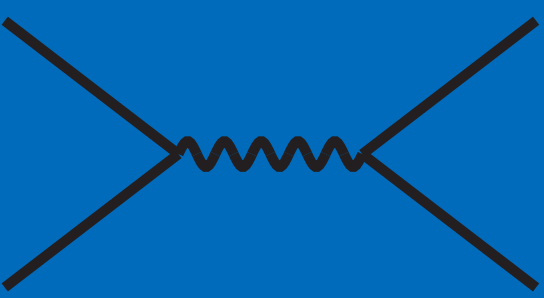


The Lyman- α constraint on Warm Dark Matter

1. Reheating



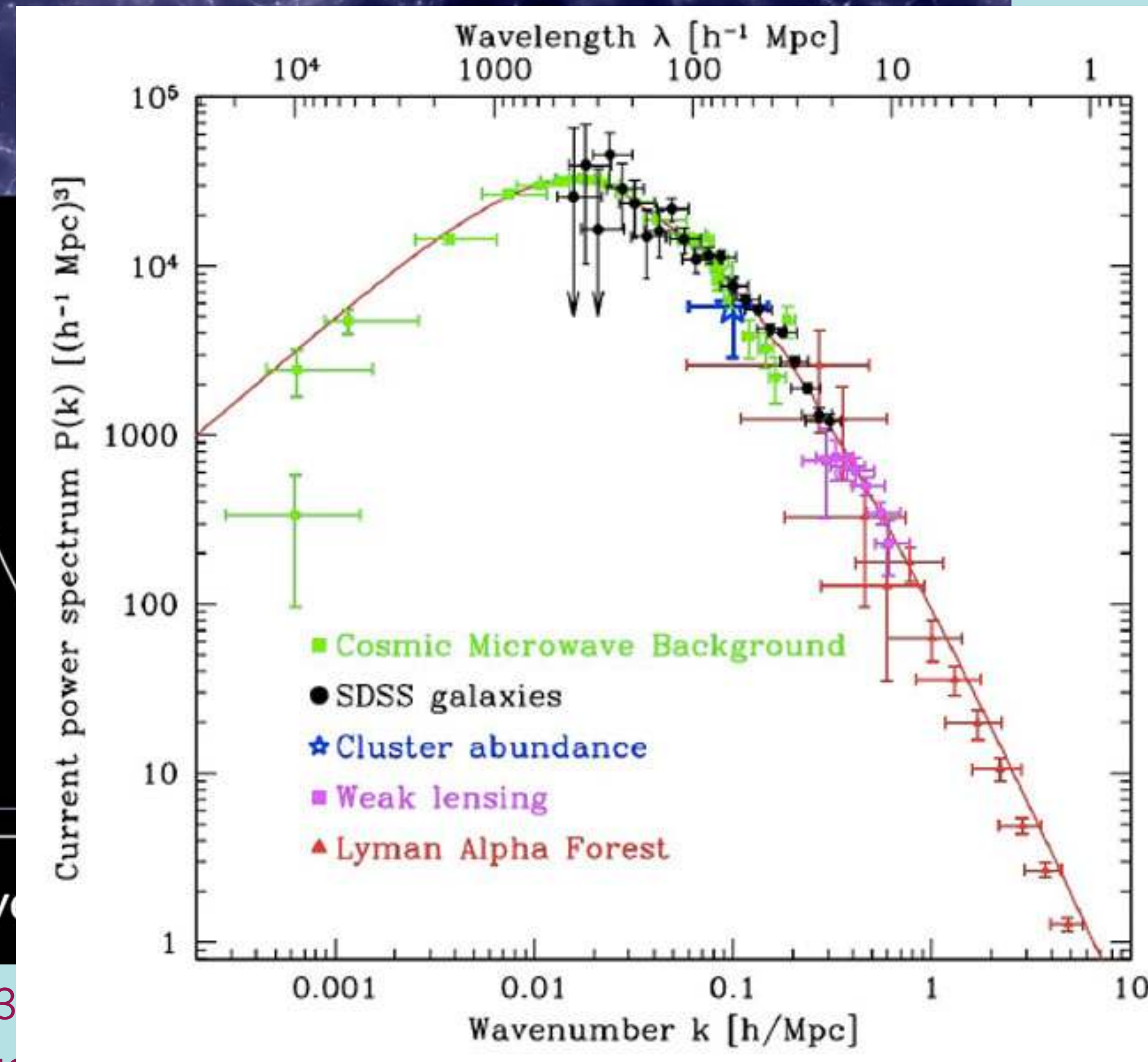
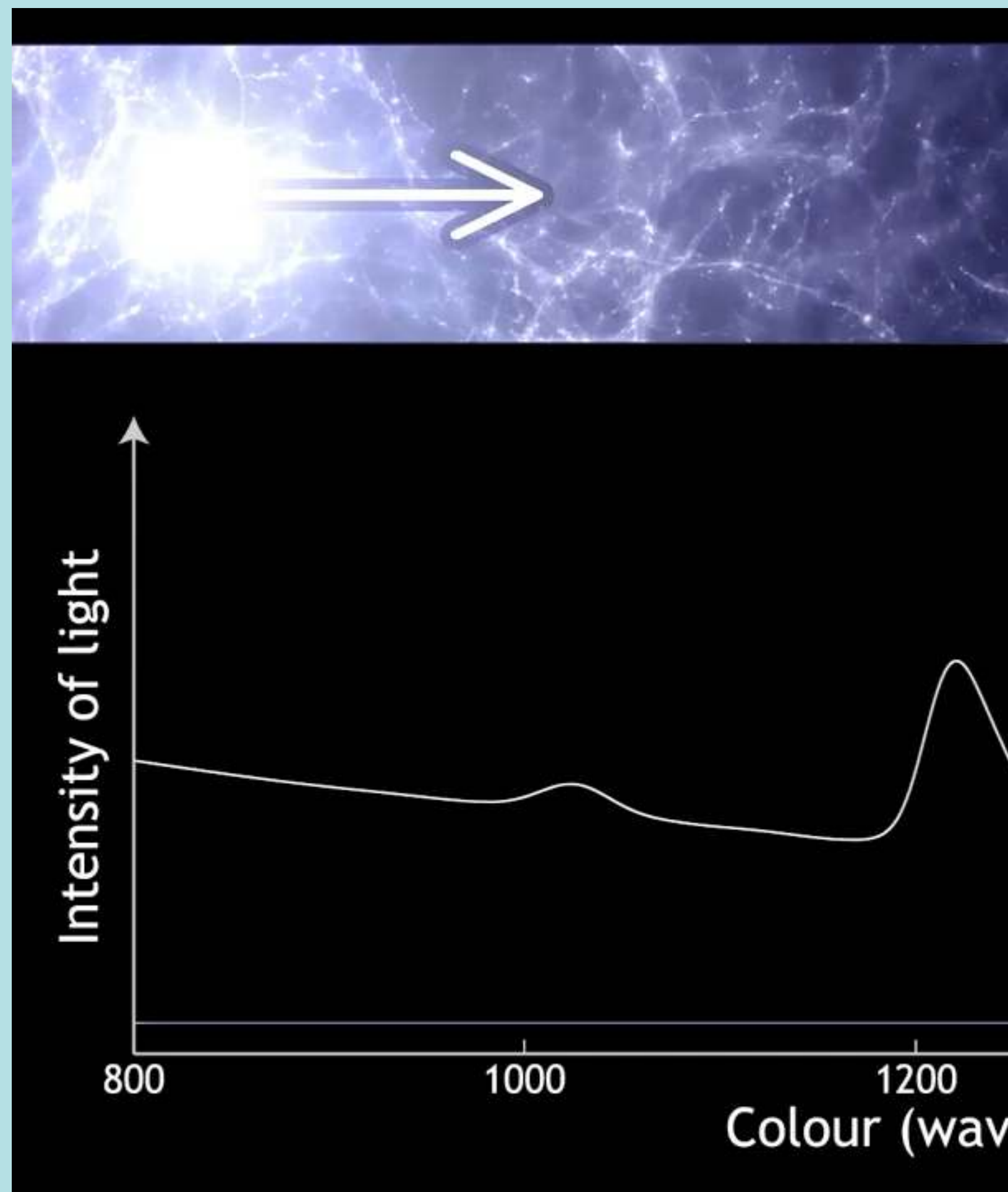
2. Freeze-in



3. Lyman- α



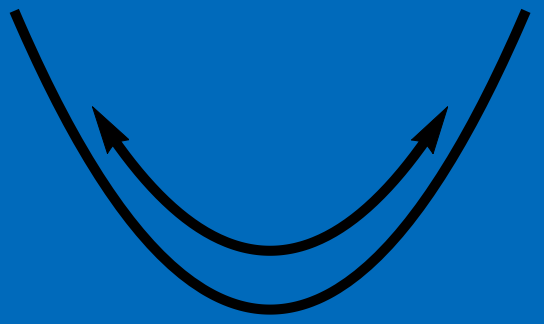
4. The end?



V. Iršič, et al., *Mon.Not.Roy.Astron.Soc.* 466 (2017) 4, 433

N. Palanque-Delabrouille et al., *Astron. Astrophys.* 559 (2013) A85; S. Chabanier et al., *JCAP* 07 (2019) 017

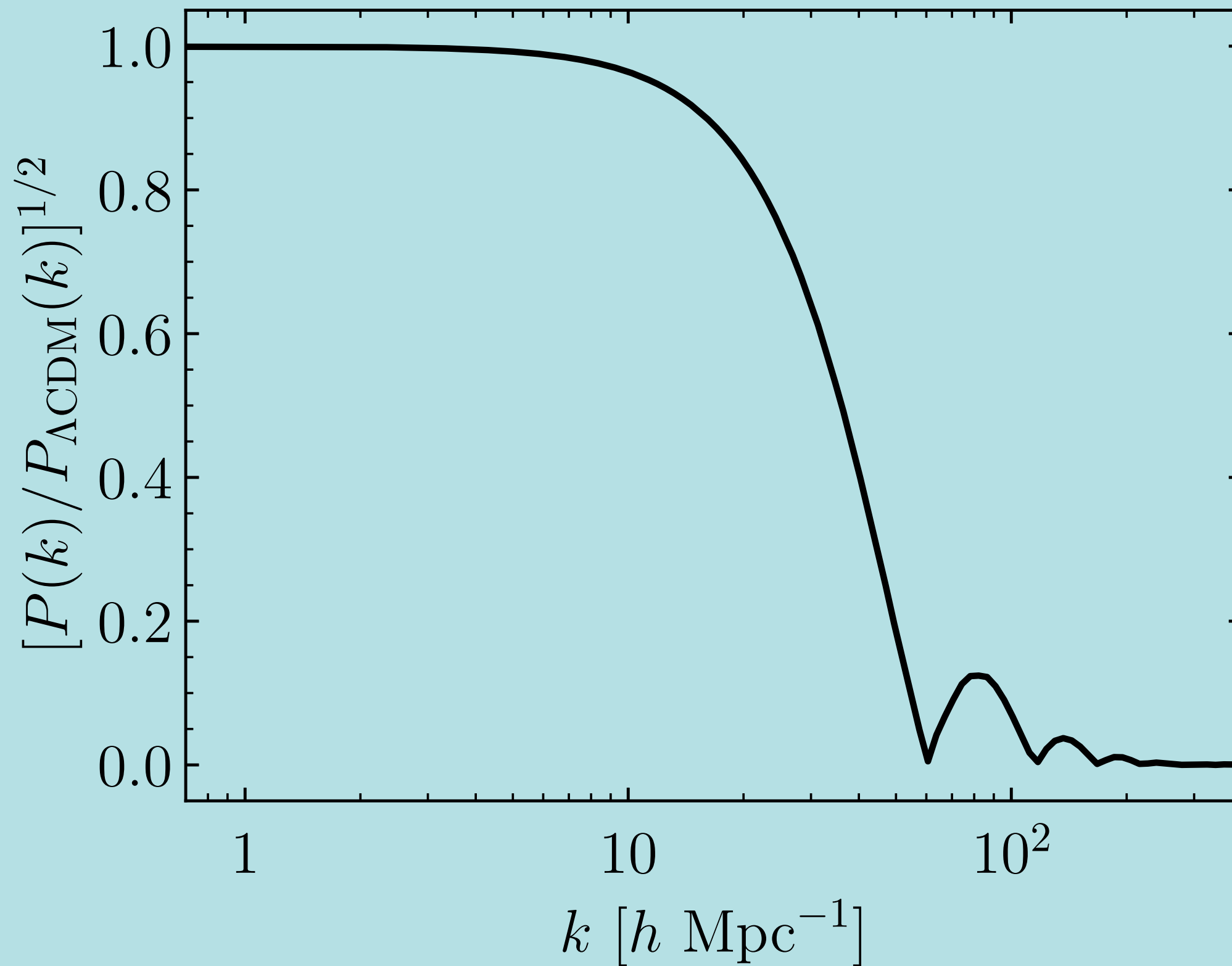
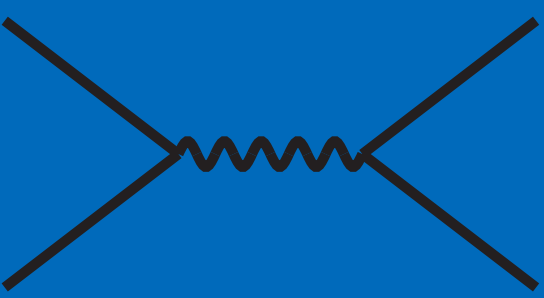
1. Reheating



The perturbed Boltzmann equation

$$f(\mathbf{x}, \mathbf{p}, \tau) = f_0(|\mathbf{p}|, \tau)[1 + \Psi(\mathbf{x}, \mathbf{p}, \tau)] \longrightarrow \ddot{\delta} + \mathcal{H}\dot{\delta} - \frac{3}{2}\mathcal{H}^2 \left(1 - w \frac{10}{9} \frac{k^2}{\mathcal{H}^2}\right) \delta = 0$$

2. Freeze-in



3. Lyman- α



$$k_{\text{FS}}(a) = \frac{9}{10} \frac{\mathcal{H}^2}{w}$$

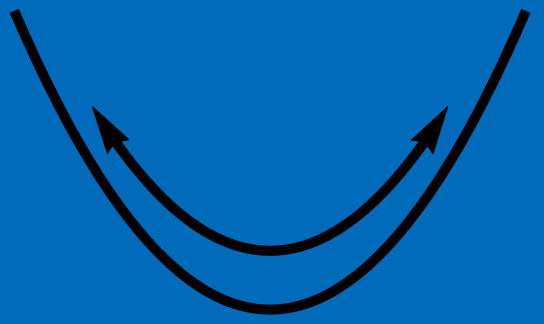
$$k_H(a) \equiv \left[\int_0^a \frac{1}{k_{\text{FS}}(\tilde{a})} \frac{d\tilde{a}}{\tilde{a}} \right]^{-1}$$

$$w \simeq \frac{\delta P}{\delta \rho} = \frac{T_*^2}{3m_{\text{DM}}^2} \frac{\langle q^2 \rangle}{a^2}$$

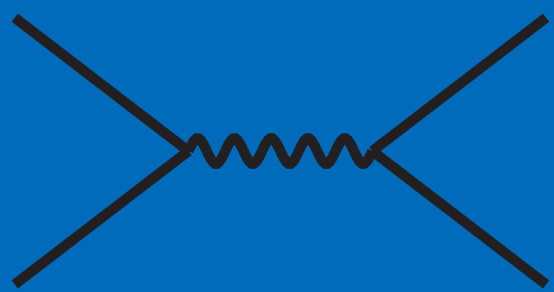
J. Lesgourgues, T. Tram, JCAP 09 (2011) 032

4. The end?

1. Reheating



2. Freeze-in

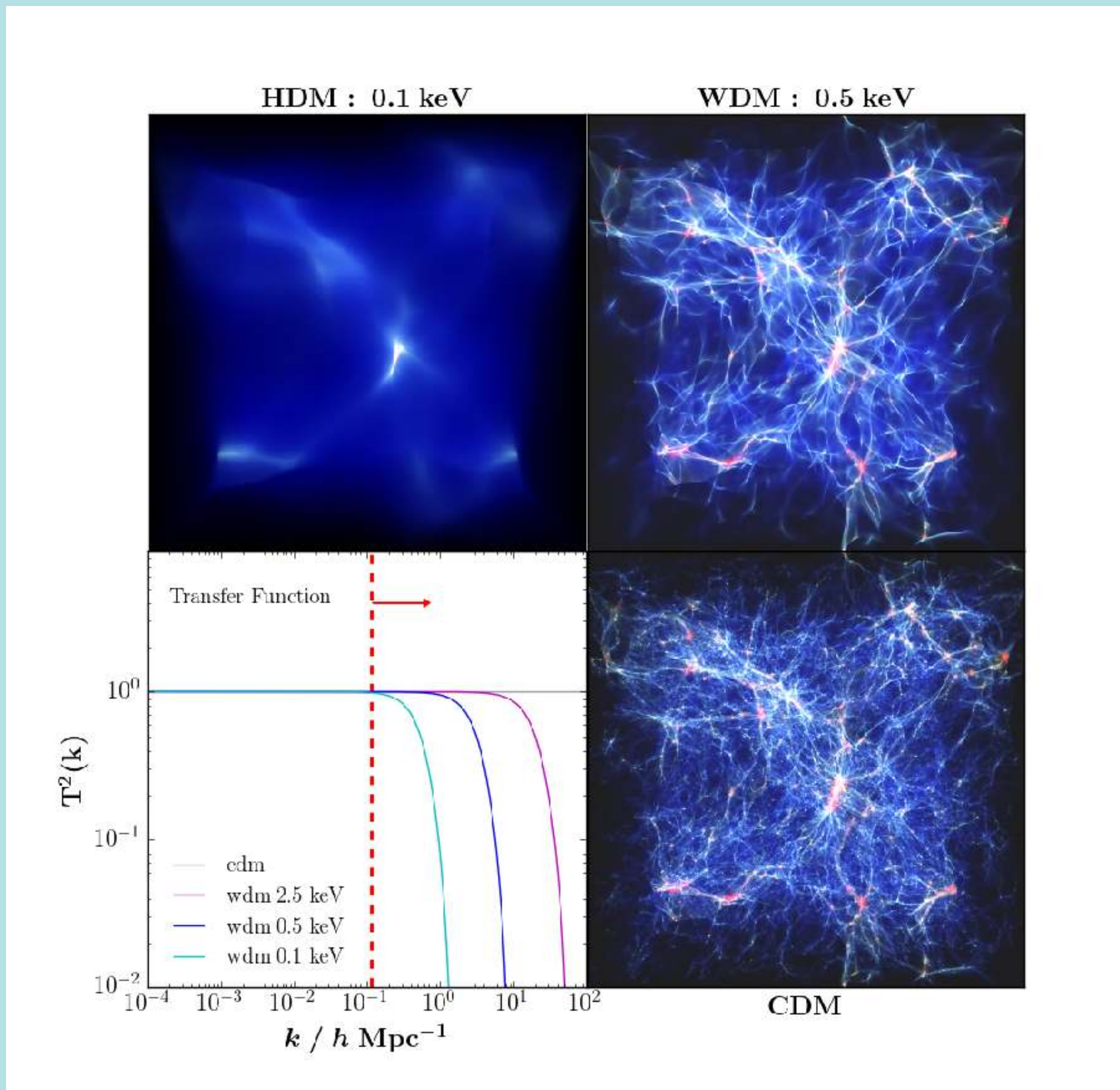


3. Lyman- α



4. The end?

The perturbed Boltzmann equation



$$\ddot{\delta} + \mathcal{H}\dot{\delta} - \frac{3}{2}\mathcal{H}^2 \left(1 - w \frac{10}{9} \frac{k^2}{\mathcal{H}^2} \right) \delta = 0$$

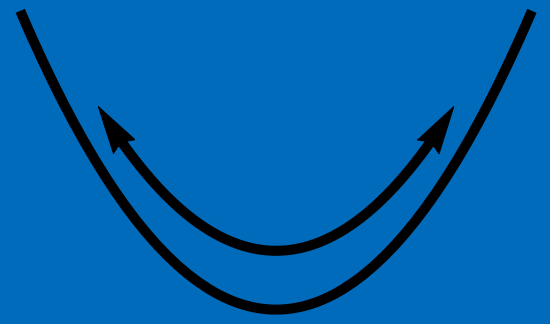
$$k_{\text{FS}}(a) = \frac{9}{10} \frac{\mathcal{H}^2}{w}$$

$$k_H(a) \equiv \left[\int_0^a \frac{1}{k_{\text{FS}}(\tilde{a})} \frac{d\tilde{a}}{\tilde{a}} \right]^{-1}$$

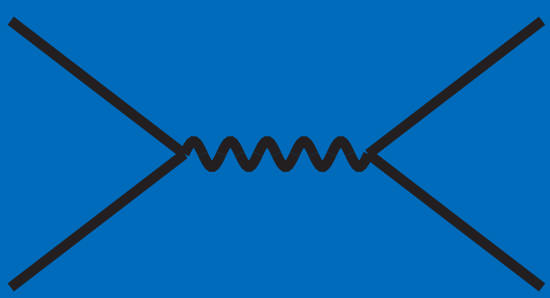
J. Baur et al., JCAP 08 (2016) 012

R. Murgia et al., JCAP 11 (2017) 046

1. Reheating



2. Freeze-in



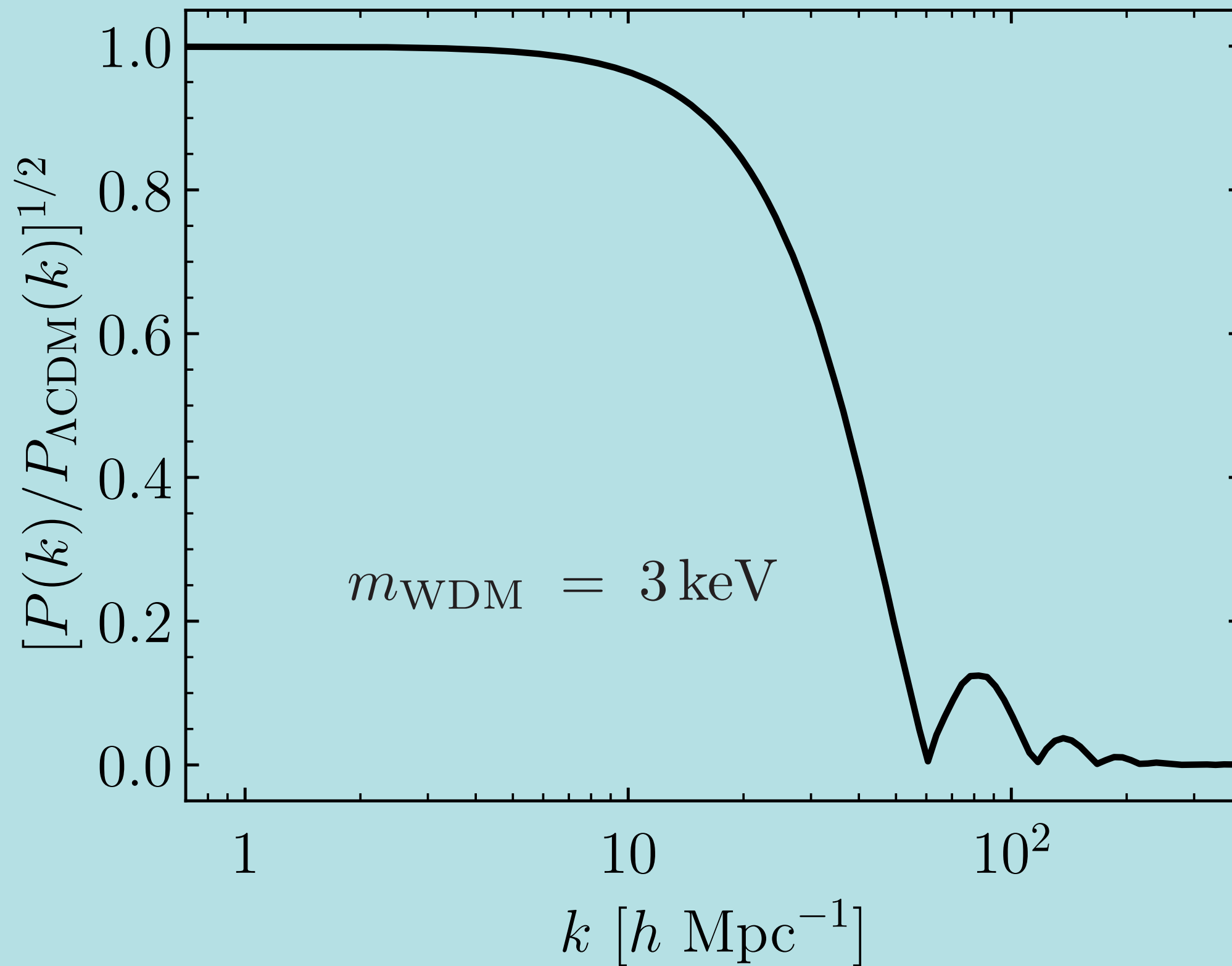
3. Lyman- α



4. The end?

The perturbed Boltzmann equation

$$f(\mathbf{x}, \mathbf{p}, \tau) = f_0(|\mathbf{p}|, \tau)[1 + \Psi(\mathbf{x}, \mathbf{p}, \tau)] \longrightarrow \ddot{\delta} + \mathcal{H}\dot{\delta} - \frac{3}{2}\mathcal{H}^2 \left(1 - w \frac{10}{9} \frac{k^2}{\mathcal{H}^2}\right) \delta = 0$$



$$k_H(a=1) \simeq 3.5 h \text{ Mpc}^{-1}$$

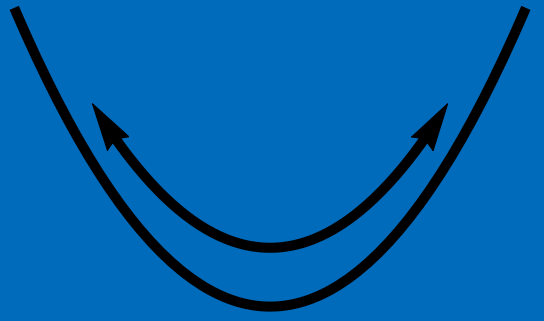
$$w_{\text{WDM}}(a) \simeq 6 \times 10^{-15} a^{-2} \left(\frac{\text{keV}}{m_{\text{WDM}}}\right)^{8/3}$$

$$k_{\text{FS}}(a) = \frac{9}{10} \frac{\mathcal{H}^2}{w}$$

$$k_H(a) \equiv \left[\int_0^a \frac{1}{k_{\text{FS}}(\tilde{a})} \frac{d\tilde{a}}{\tilde{a}} \right]^{-1}$$

$$w \simeq \frac{\delta P}{\delta \rho} = \frac{T_*^2}{3m_{\text{DM}}^2} \frac{\langle q^2 \rangle}{a^2}$$

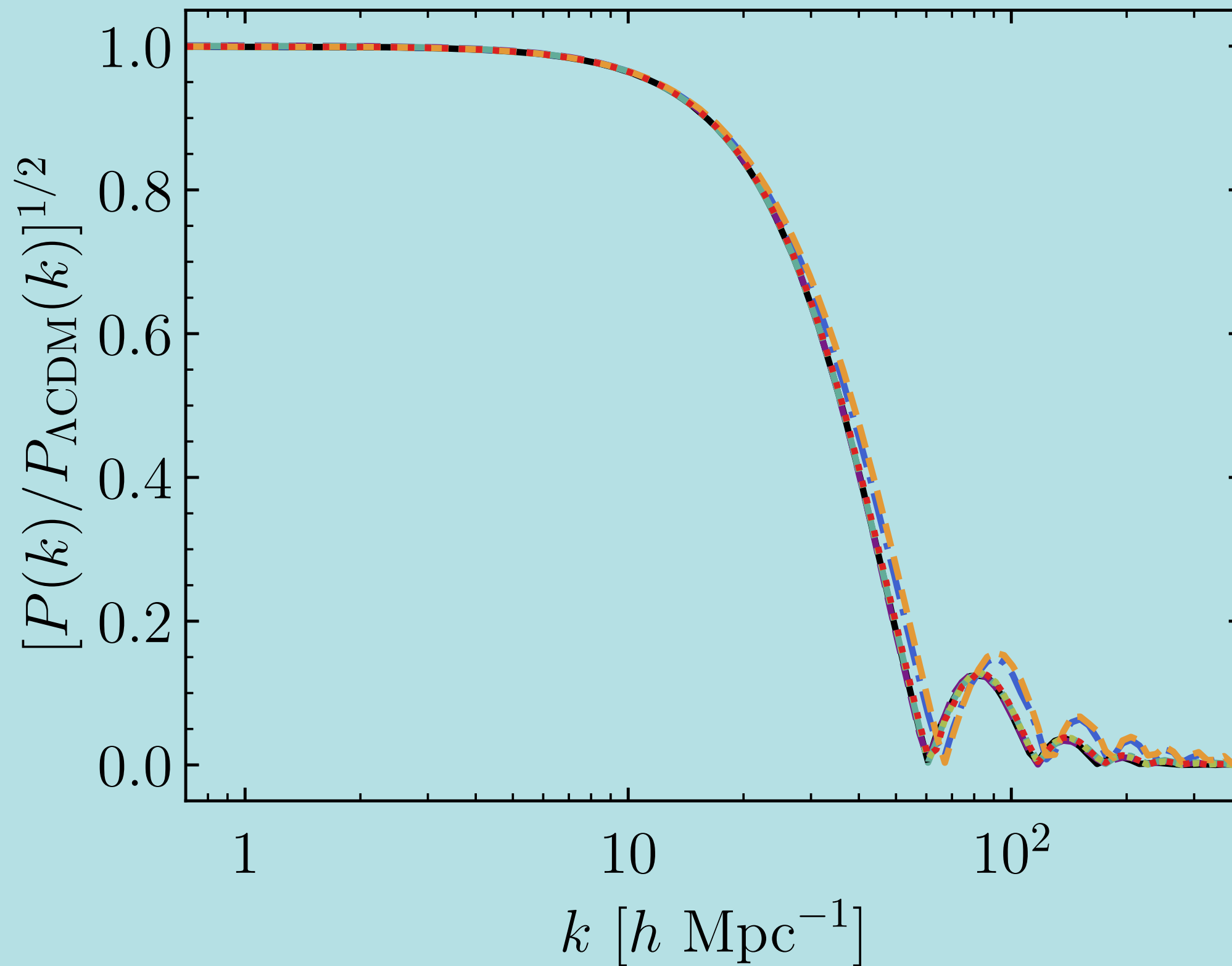
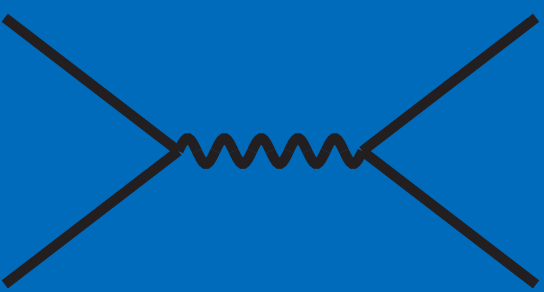
1. Reheating



The perturbed Boltzmann equation

$$f(\mathbf{x}, \mathbf{p}, \tau) = f_0(|\mathbf{p}|, \tau)[1 + \Psi(\mathbf{x}, \mathbf{p}, \tau)] \longrightarrow \ddot{\delta} + \mathcal{H}\dot{\delta} - \frac{3}{2}\mathcal{H}^2 \left(1 - w \frac{10}{9} \frac{k^2}{\mathcal{H}^2}\right) \delta = 0$$

2. Freeze-in



$$w(m_{\text{DM}}) = w_{\text{WDM}}(m_{\text{WDM}})$$

$$k_{\text{FS}}(a) = \frac{9}{10} \frac{\mathcal{H}^2}{w}$$

$$k_H(a) \equiv \left[\int_0^a \frac{1}{k_{\text{FS}}(\tilde{a})} \frac{d\tilde{a}}{\tilde{a}} \right]^{-1}$$

$$w \simeq \frac{\delta P}{\delta \rho} = \frac{T_*^2}{3m_{\text{DM}}^2} \frac{\langle q^2 \rangle}{a^2}$$

$$m_{\text{DM}} = m_{\text{WDM}} \left(\frac{T_*}{T_{\text{WDM}}} \right) \sqrt{\frac{\langle q^2 \rangle}{\langle q^2 \rangle_{\text{WDM}}}}$$

3. Lyman-α



4. The end?

Constraints on dark matter from condensate decay (inflaton, moduli)

1. Reheating



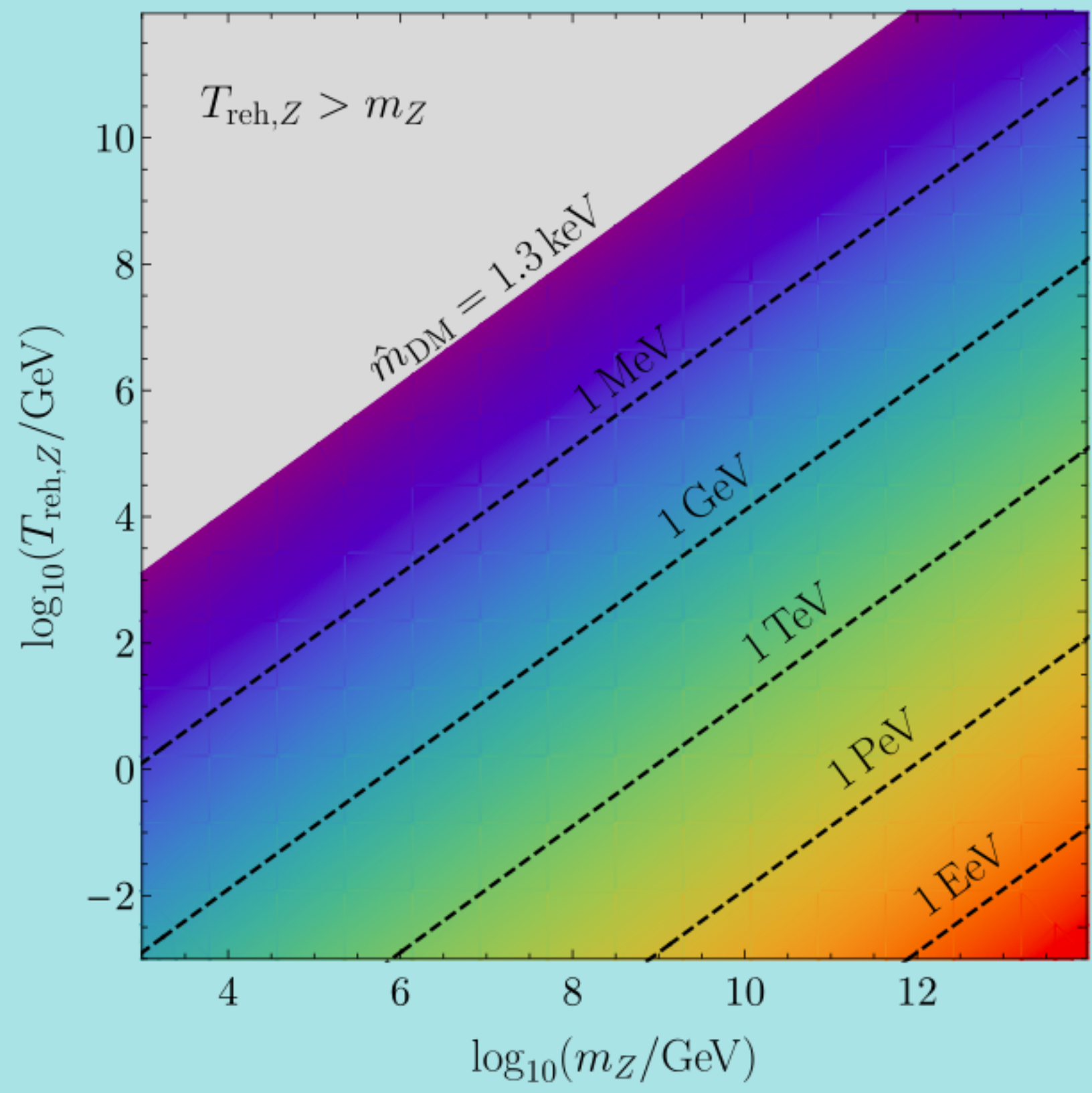
2. Freeze-in



3. Lyman- α



4. The end?

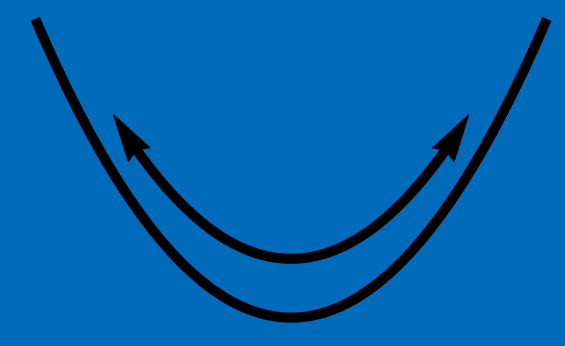


$$m_{\text{DM}} > 3.78 \text{ MeV} \left(\frac{m_{\text{WDM}}}{3 \text{ keV}} \right)^{4/3} \left(\frac{106.75}{g_{*s}^{\text{reh}}} \right)^{1/3} \times \left(\frac{m_{\Phi}}{3 \times 10^{13} \text{ GeV}} \right) \left(\frac{10^{10} \text{ GeV}}{T_{\text{reh}}} \right)$$

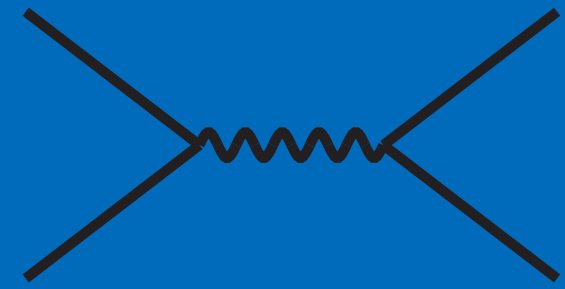
$$\text{Br}_{\chi} \lesssim 1.5 \times 10^{-4} \left(\frac{g_{*s}^{\text{reh}}}{106.5} \right)^{1/3} \left(\frac{3 \text{ keV}}{m_{\text{WDM}}} \right)^{4/3}$$

(absolute)

1. Reheating



2. Freeze-in



3. Lyman-α



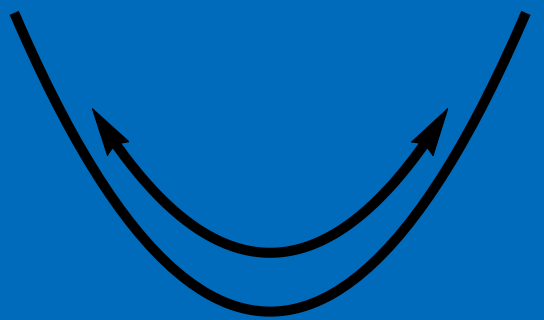
4. The end?

Constraints on dark matter from chain decay

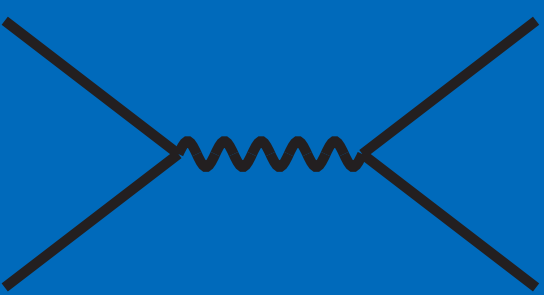


$$m_{\text{DM}} > \left(\frac{m_{\text{WDM}}}{3 \text{ keV}}\right)^{4/3} \times \begin{cases} 1.23 \text{ MeV} \left(\frac{106.75}{g_{*s}^{\text{reh}}}\right)^{1/3} \left(\frac{m_{\Phi}}{3 \times 10^{13} \text{ GeV}}\right) \left(\frac{10^{10} \text{ GeV}}{T_{\text{reh}}}\right), & \text{R} \\ 2.7 \text{ MeV} \left(\frac{106.75}{g_{*s}^{\text{dec}}}\right)^{1/3} \left(\frac{m_A}{3 \times 10^{13} \text{ GeV}}\right) \left(\frac{10^{10} \text{ GeV}}{T_{\text{dec}}}\right), & \text{NR} \end{cases}$$

1. Reheating



2. Freeze-in



3. Lyman- α

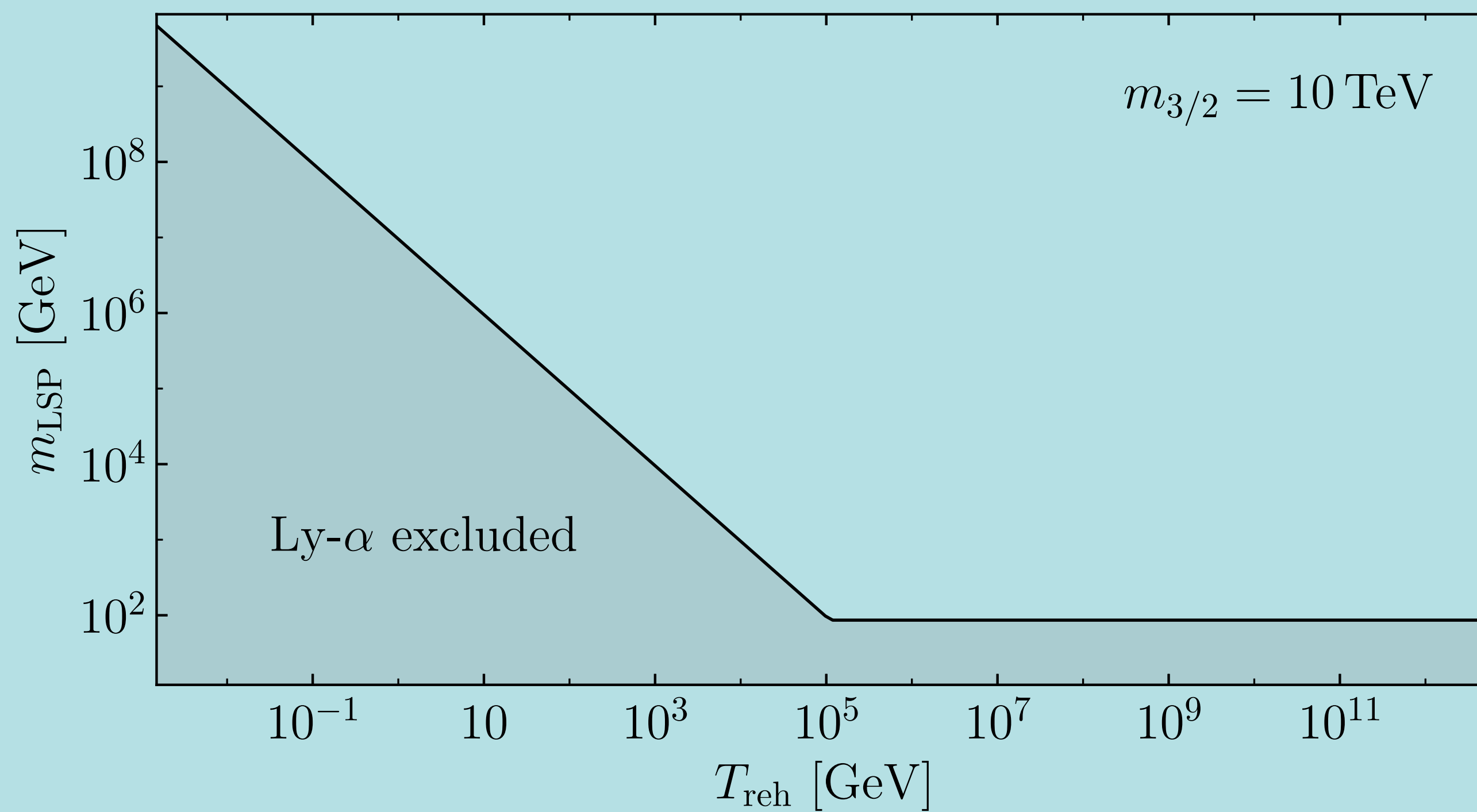


4. The end?

Constraints on dark matter from chain decay

$$\Phi \rightarrow \Psi_{3/2} \rightarrow \text{LSP}$$

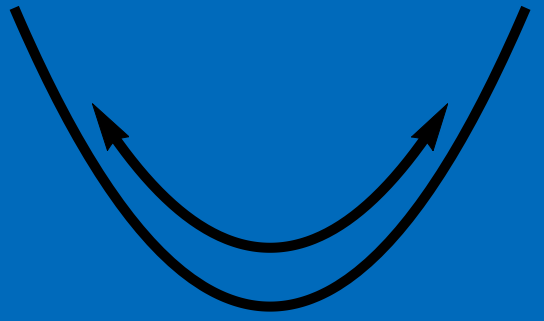
$$m_{\text{LSP}} > \begin{cases} 86 \text{ GeV} \left(\frac{m_{\text{WDM}}}{3 \text{ keV}} \right)^{4/3} \left(\frac{10 \text{ TeV}}{m_{3/2}} \right)^{1/2}, & T_{\text{reh}} \gg 10^5 \text{ GeV} \left(\frac{m_{3/2}}{10 \text{ TeV}} \right)^{1/2} \\ 95 \text{ GeV} \left(\frac{m_{\text{WDM}}}{3 \text{ keV}} \right)^{4/3} \left(\frac{10^5 \text{ GeV}}{T_{\text{reh}}} \right), & T_{\text{reh}} \ll 10^5 \text{ GeV} \left(\frac{m_{3/2}}{10 \text{ TeV}} \right)^{1/2} \end{cases}$$



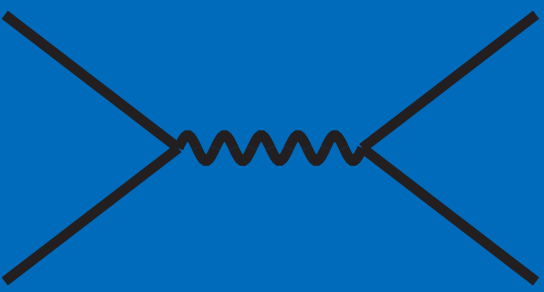
$$\Gamma_{3/2} = \frac{193}{384\pi} \frac{m_{3/2}^3}{M_P^2}$$

Constraints on dark matter from thermal freeze-in

1. Reheating



2. Freeze-in



3. Lyman- α



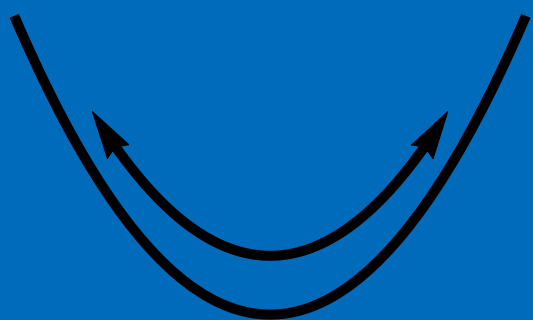
4. The end?

$$m_{\text{DM}} > \left(\frac{m_{\text{WDM}}}{3 \text{ keV}}\right)^{4/3} \left(\frac{106.75}{g_{*s}^{\text{reh}}}\right)^{1/3} \begin{cases} 7.3 \text{ keV}, & (n = 0, \text{ FF}) \\ 8.5 \text{ keV}, & (n = 2, \text{ FF}) \\ 8.5 \text{ keV}, & (n = 4, \text{ FF}) \end{cases}$$

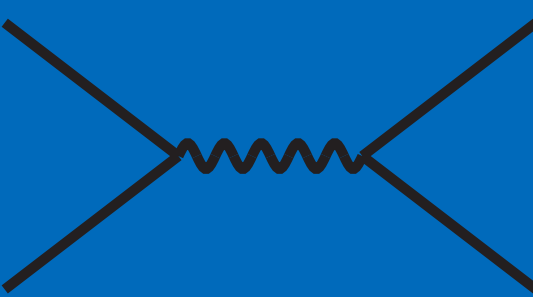
$$m_{\text{DM}} > \left(\frac{m_{\text{WDM}}}{3 \text{ keV}}\right)^{4/3} \left(\frac{106.75}{g_{*s}^{\text{reh}}}\right)^{1/3} \begin{cases} 9 \text{ keV} \ln^{-1/2} \left(\frac{T_{\text{max}}}{T_{\text{reh}}}\right), & n = 6, \\ 23 \text{ keV} \left(\frac{T_{\text{reh}}}{T_{\text{max}}}\right), & n = 8, \\ 52 \text{ keV} \left(\frac{T_{\text{reh}}}{T_{\text{max}}}\right)^{5/3}, & n \geq 10. \end{cases}$$

Constraints on dark matter from thermal freeze-in

1. Reheating



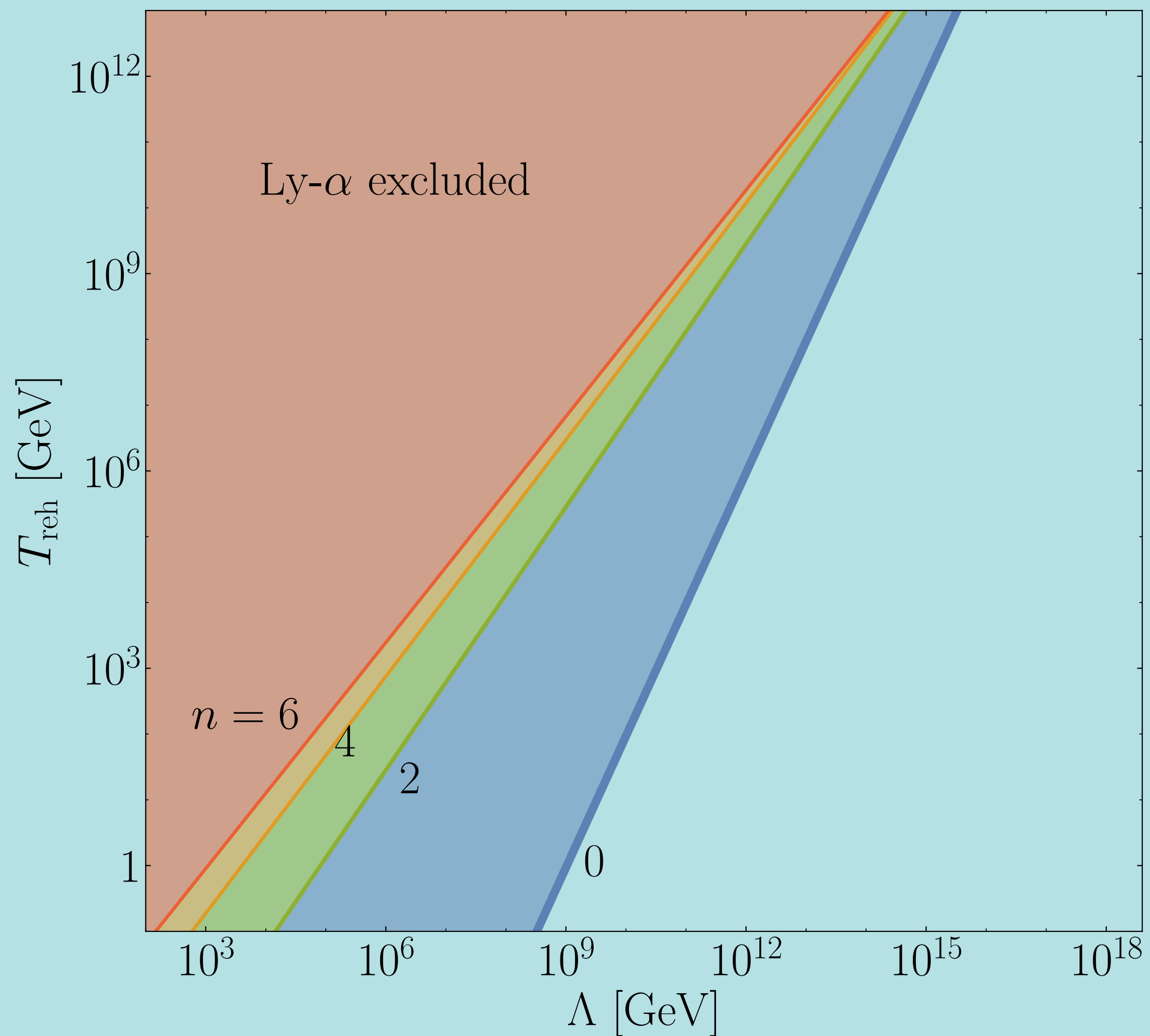
2. Freeze-in



3. Lyman- α

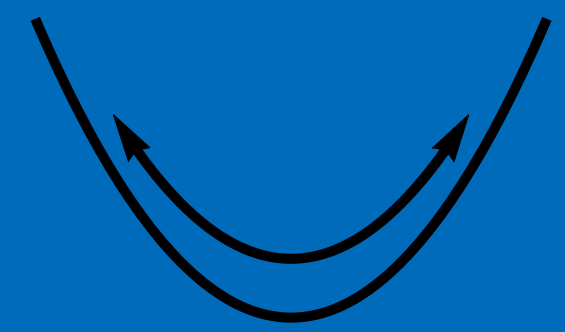


4. The end?

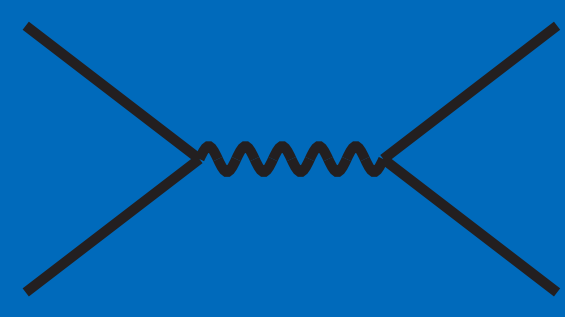


$$\sigma(s) = \frac{s^{n/2}}{\Lambda^{n+2}}$$

1. Reheating



2. Freeze-in



3. Lyman- α

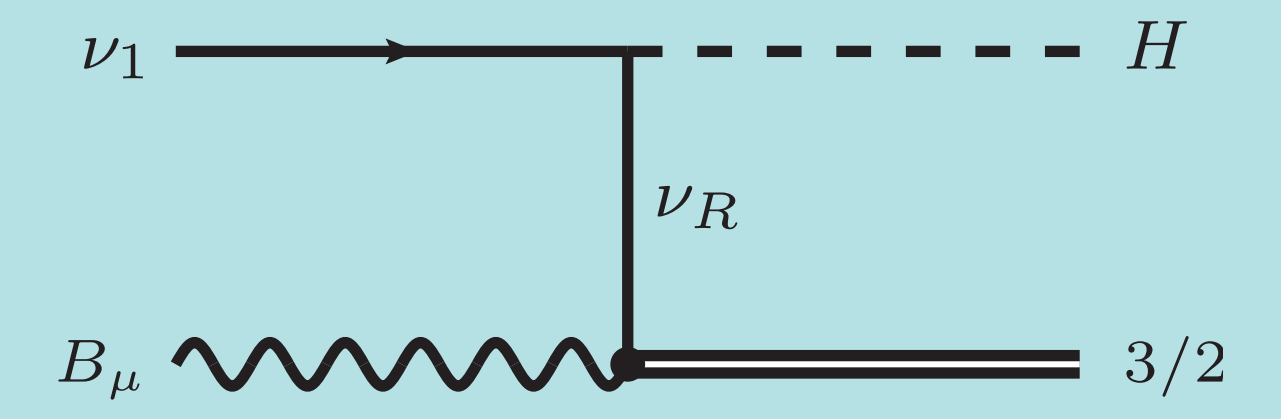
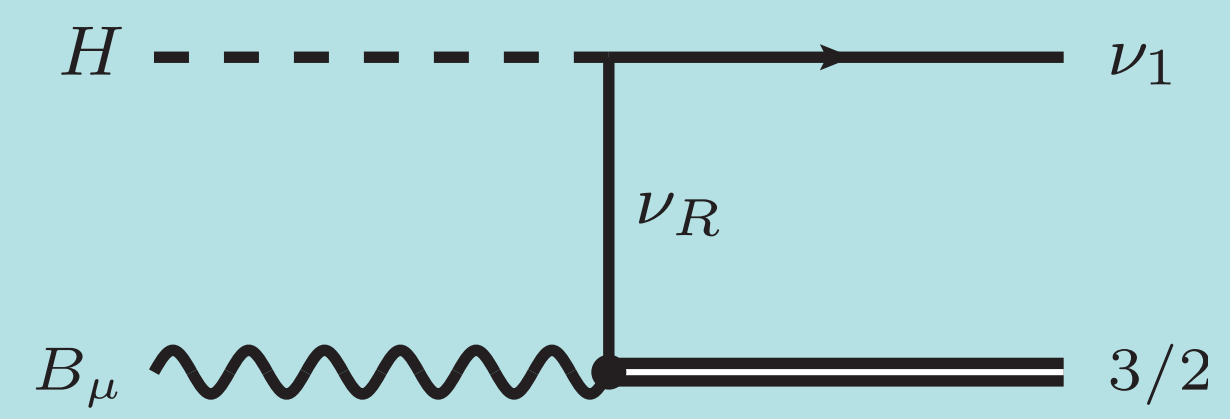
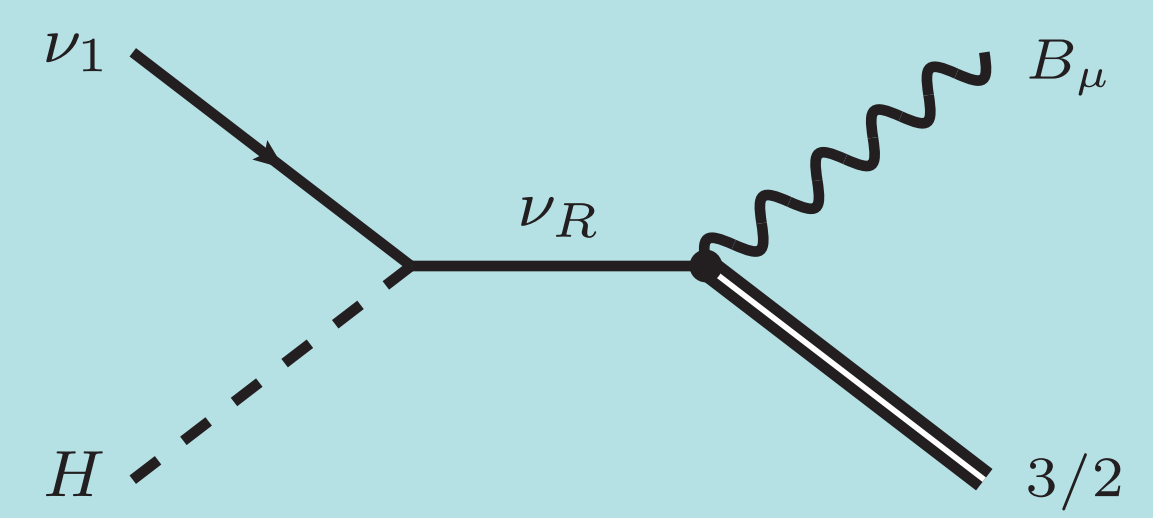


4. The end?

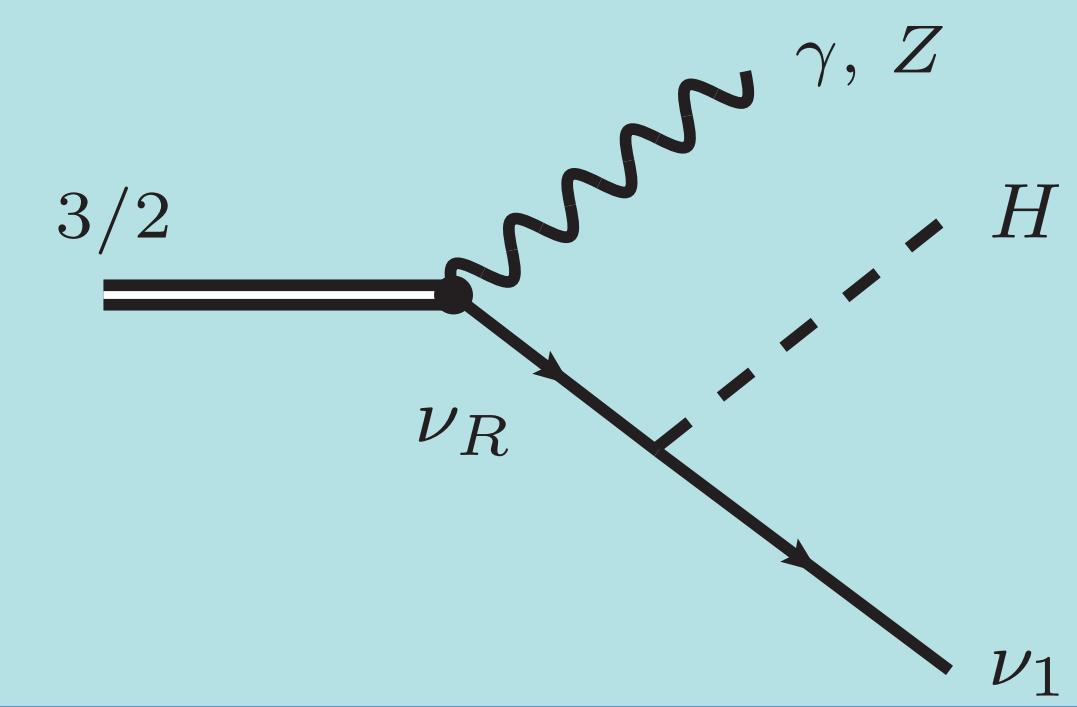
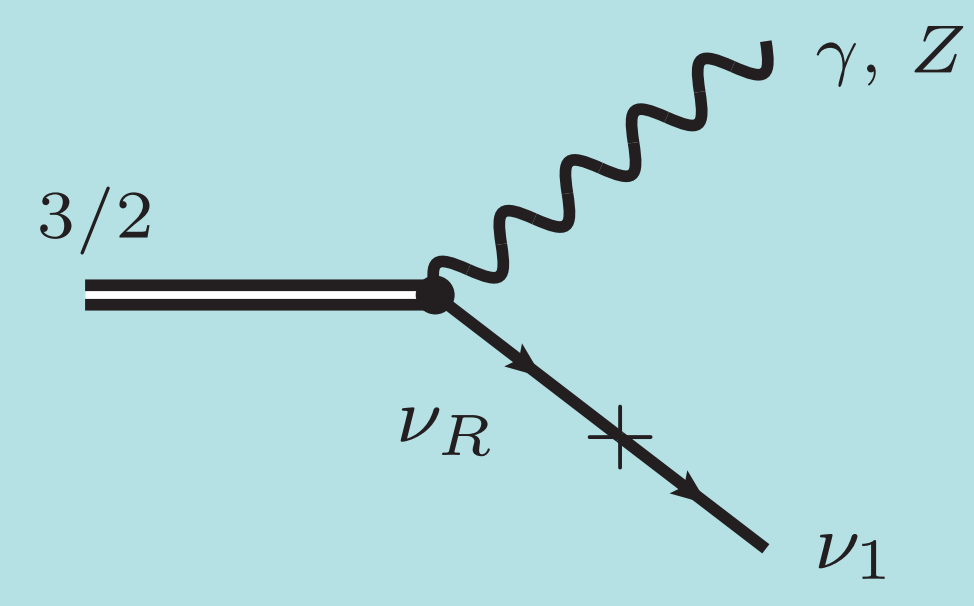
Non-thermal freeze-in: a non-supersymmetric decaying spin-3/2 particle

$$\mathcal{L} = \frac{i\alpha}{2M_P} \bar{\nu}_R \gamma^\mu [\gamma^\rho, \gamma^\sigma] \Psi_\mu F_{\rho\sigma} + \text{h.c.}$$

$n = 4$ freeze-in

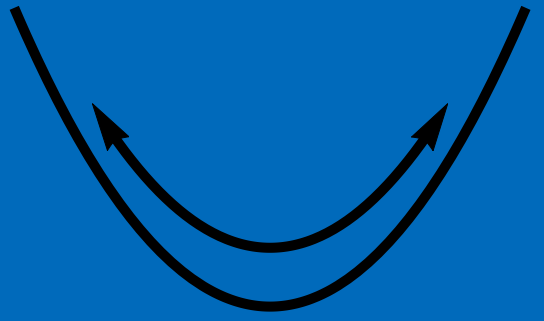


2 & 3-body decays



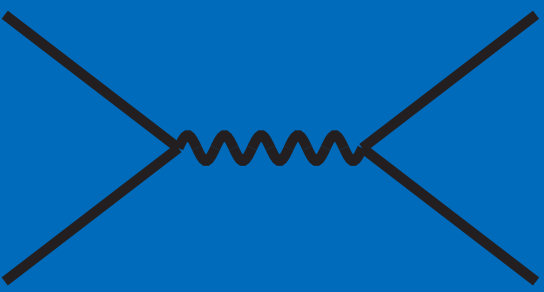
Non-thermal freeze-in: a non-supersymmetric decaying spin-3/2 particle

1. Reheating



$$\Omega_{3/2}^{\text{thermal}} h^2 \simeq 0.1 \left(\frac{\alpha_1}{1.1 \times 10^{-3}} \right)^2 \left(\frac{106.9}{g_{*s}^{\text{reh}}} \right)^{3/2} \left(\frac{T_{\text{reh}}}{10^{10} \text{ GeV}} \right)^5 \left(\frac{m_\nu}{0.15 \text{ eV}} \right) \left(\frac{10^{14} \text{ GeV}}{m_R} \right) \left(\frac{10^4 \text{ GeV}}{m_{3/2}} \right)$$

2. Freeze-in



$$\Omega_{3/2}^{\text{non-thermal}} h^2 \simeq 0.1 \left(\frac{\alpha_1}{1.1 \times 10^{-3}} \right)^2 \left(\frac{g_{*s}^{\text{reh}}}{106.5} \right)^{7/10} \left(\frac{0.030}{\alpha_{\text{SM}}} \right)^{16/5} \left(\frac{m_\Phi}{3 \times 10^{13} \text{ GeV}} \right)^{14/5} \\ \times \left(\frac{m_\nu}{0.15 \text{ eV}} \right) \left(\frac{10^4 \text{ GeV}}{m_{3/2}} \right) \left(\frac{10^{14} \text{ GeV}}{m_R} \right) \left(\frac{T_{\text{reh}}}{10^{10} \text{ GeV}} \right)^{19/5} \left(\frac{\text{Br}_\nu}{7 \times 10^{-4}} \right)$$

3. Lyman- α

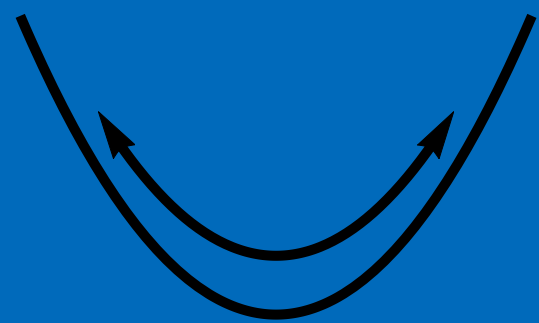


$$m_{3/2} > 0.44 \text{ keV} \left(\frac{m_{\text{WDM}}}{3 \text{ keV}} \right)^{4/3} \left(\frac{\alpha_{\text{SM}}}{0.03} \right)^{-32/15} \left(\frac{c}{3/5} \right)^{2/15} \\ \times \left(\frac{106.75}{g_{*s}^{\text{reh}}} \right)^{1/5} \left(\frac{10^{10} \text{ GeV}}{T_{\text{reh}}} \right)^{7/15} \left(\frac{m_\Phi}{3 \times 10^{13} \text{ GeV}} \right)^{23/15}$$

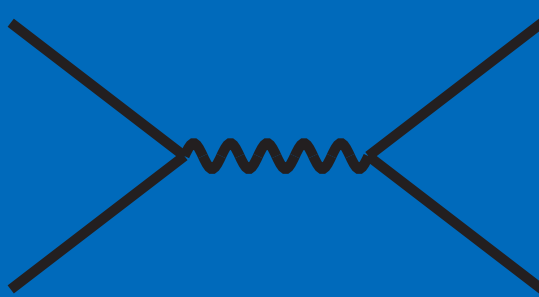
4. The end?

Non-thermal freeze-in: a non-supersymmetric decaying spin-3/2 particle

1. Reheating



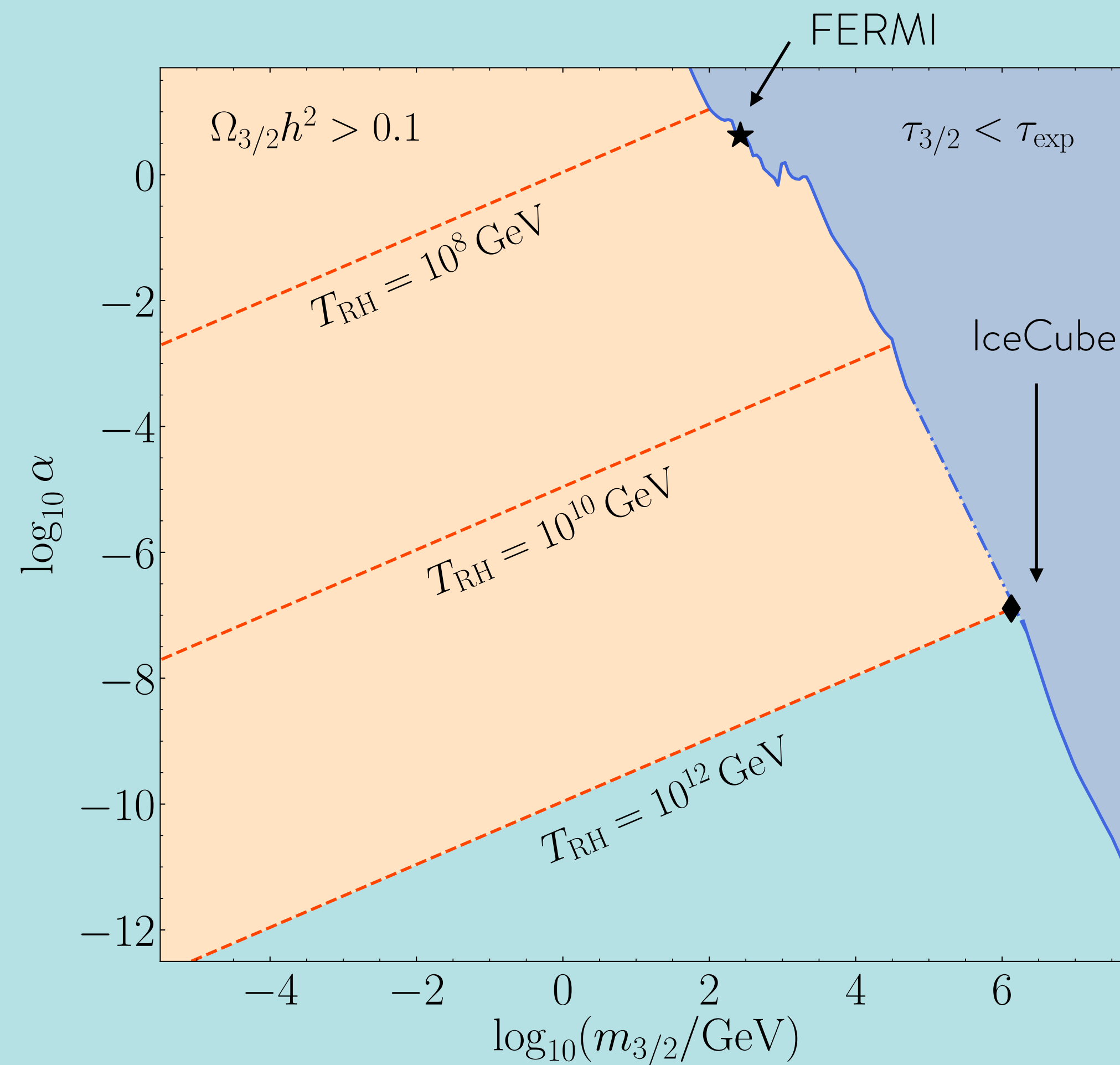
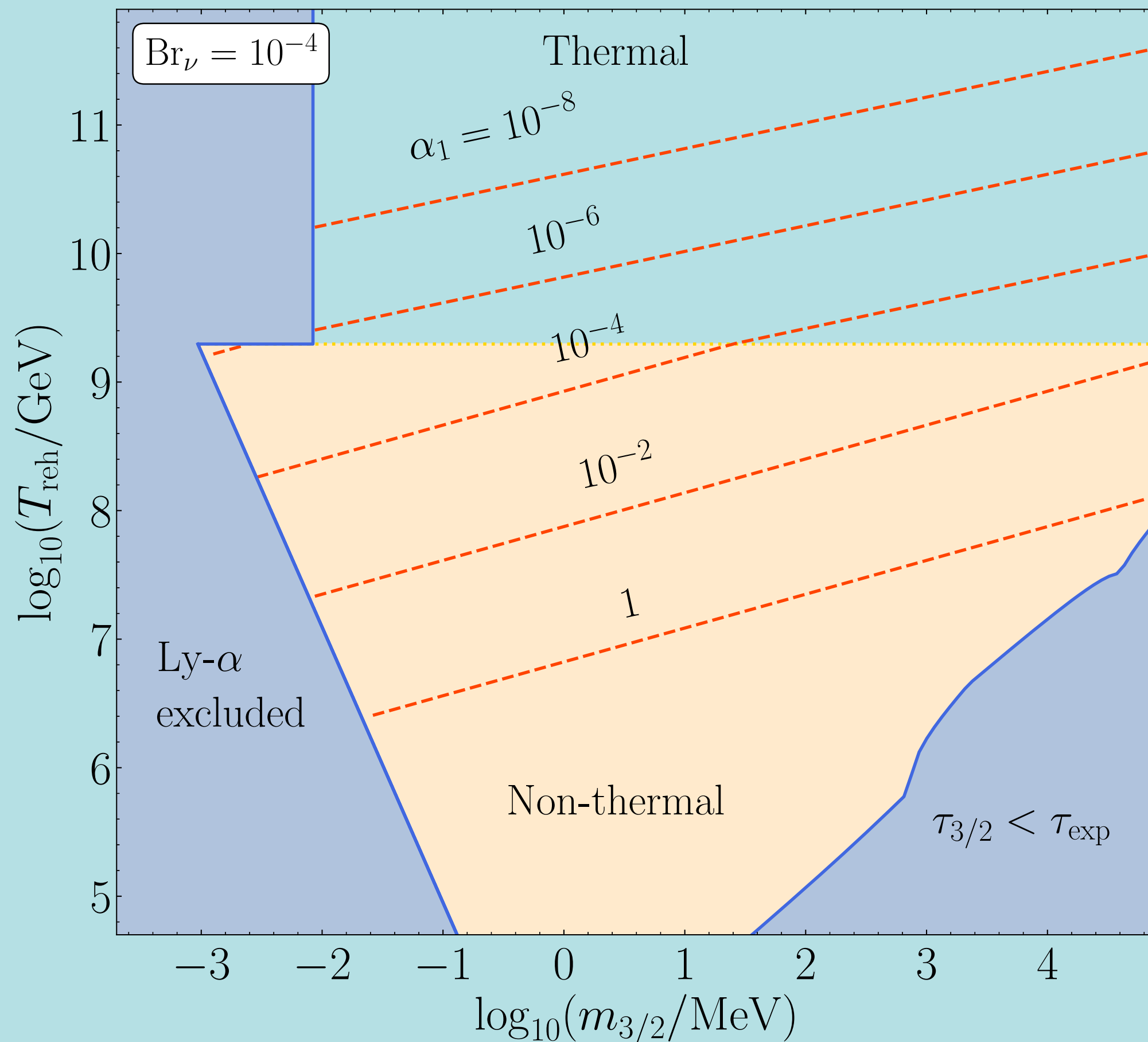
2. Freeze-in



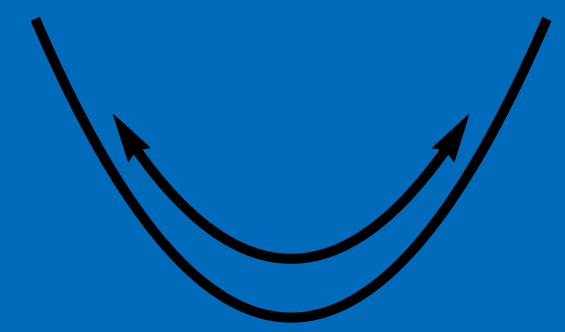
3. Lyman- α



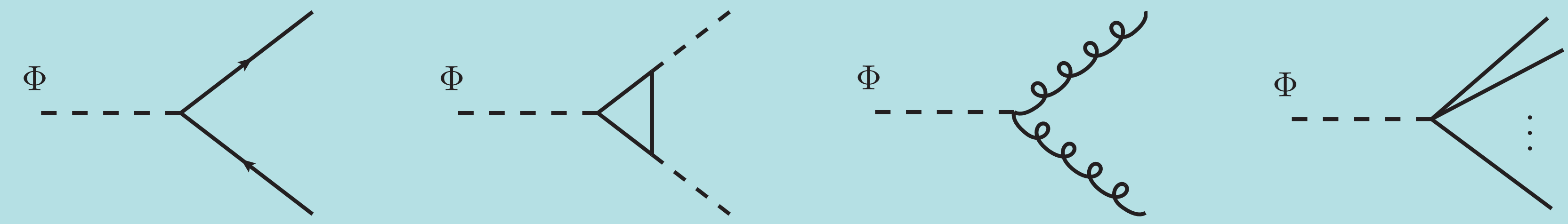
4. The end?



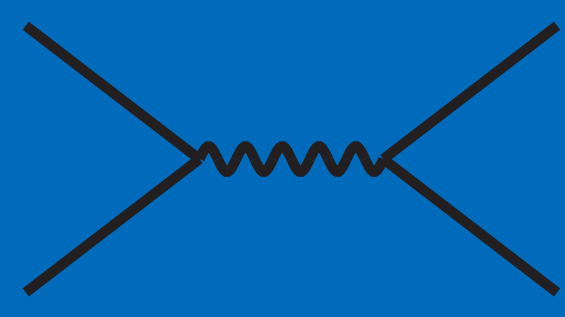
1. Reheating



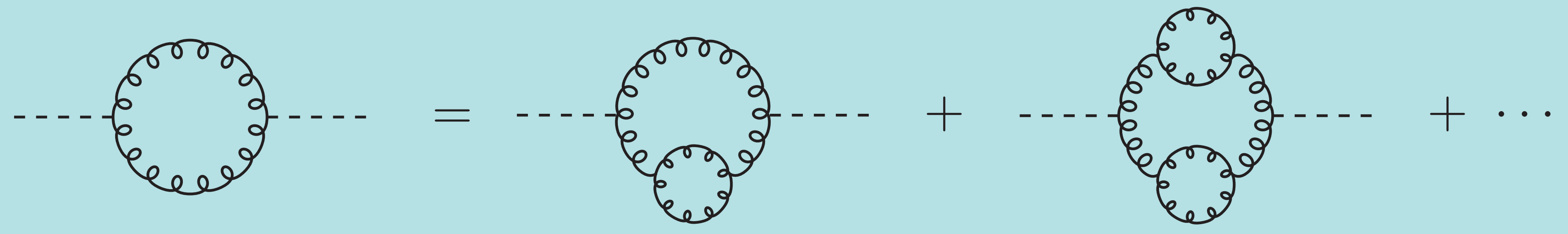
For $k \neq 2$ the nature of the final state matters



2. Freeze-in



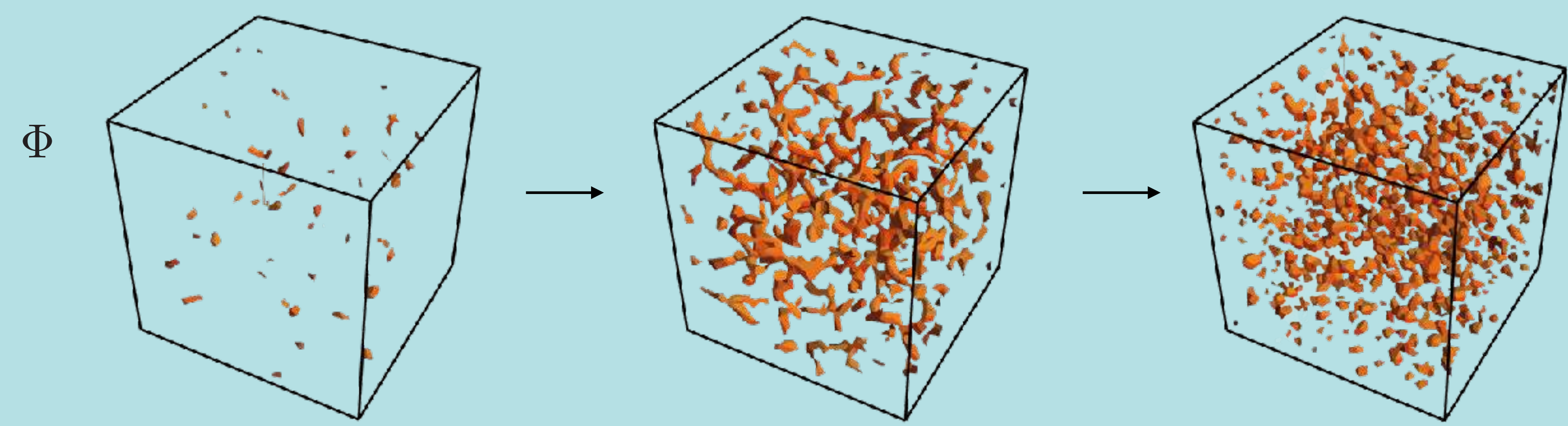
In-medium effects cannot be neglected



3. Lyman- α



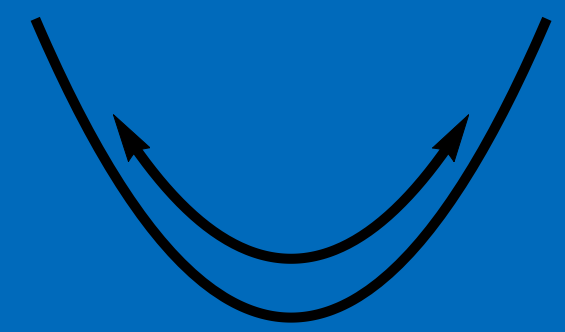
Interactions fragment homogeneous fields



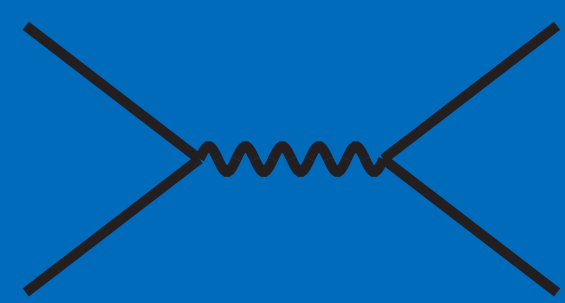
4. The end?

So, how warm are non-thermal relics?

1. Reheating



2. Freeze-in



3. Lyman- α



4. The end?

