26/04/2021

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SHANGHAI JIAO TONG UNIVERSITY

How warm are non-thermal relics?

Constraining out-of-equilibrium dark matter

Marcos A. G. García
IFT-UAM

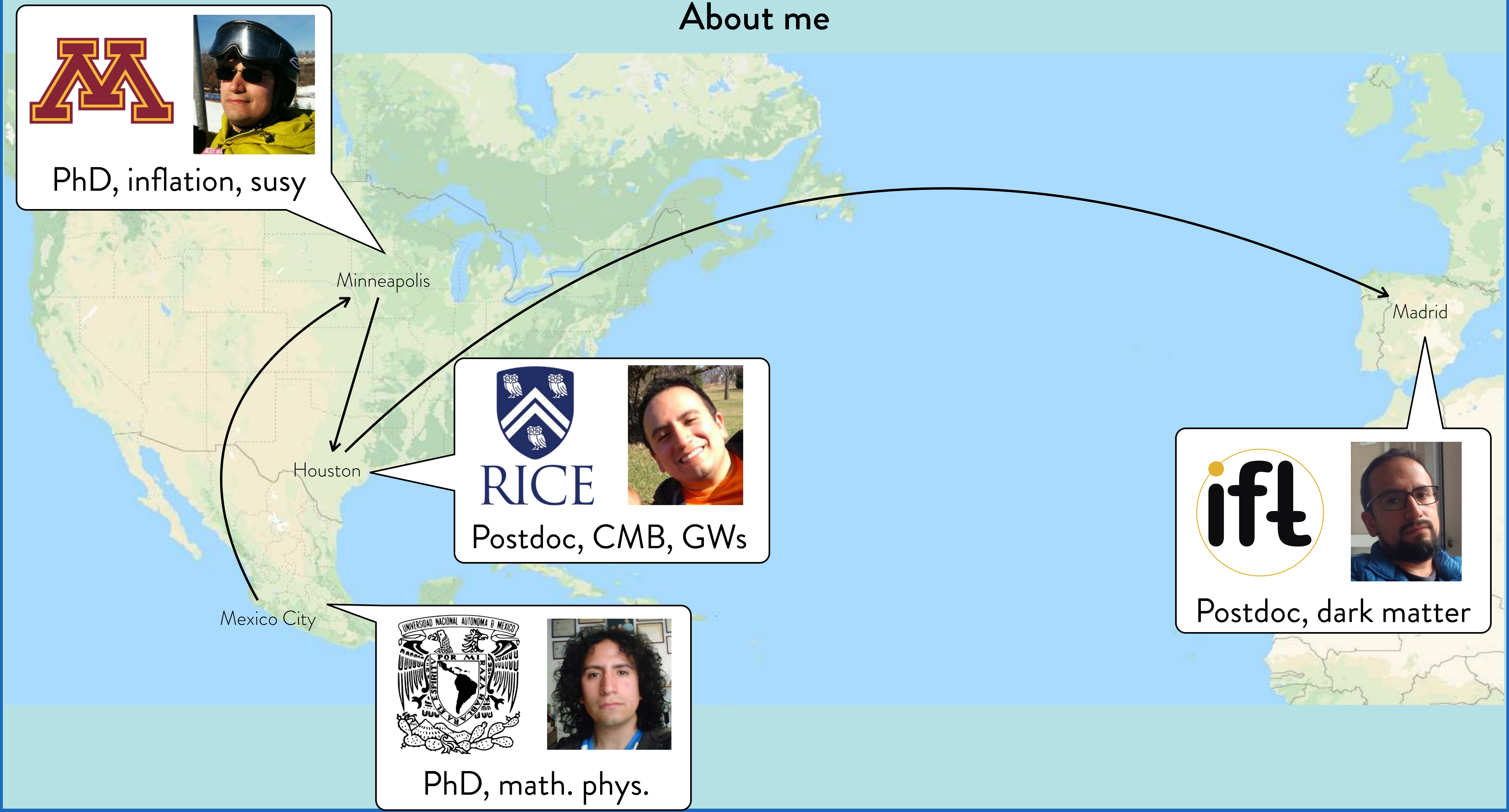
2011.13458	with G. Ballesteros and M. Pierre
2012.10756	with Y. Mambrini, K. Olive and K. Kaneta
2006.03325	with Y. Mambrini, K. Olive and S. Verner
2004.08404	with Y. Mambrini, K. Olive and K. Kaneta
1806.01865	with M. Amin
1709.01549	with Y. Mambrini, K. Olive and M. Peloso

Instituto de Física Teórica UAM-CSIC

Universidad Autónoma de Madrid

CSIC
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

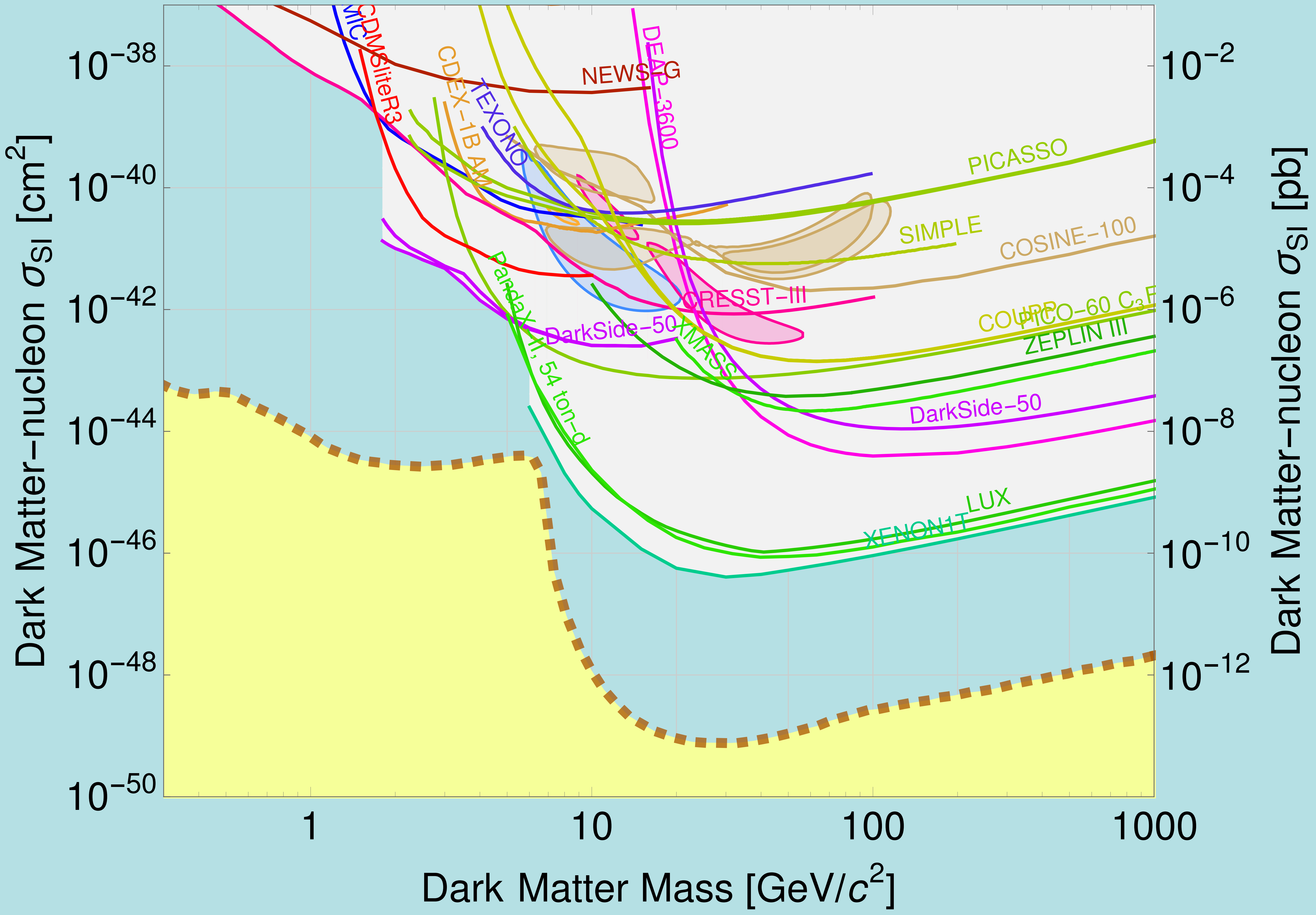
About me



PhD, inflation, susy
Minneapolis
Houston
RICE
Postdoc, CMB, GWs
Mexico City
UNIVERSIDAD NACIONAL AUTONOMA DE MEXICO
POR MI RAZA HABLARA EL ESPIRITU
PhD, math. phys.
Madrid
ift
Postdoc, dark matter

Case for Feebly Interacting Massive Particles (FIMPs) as dark matter

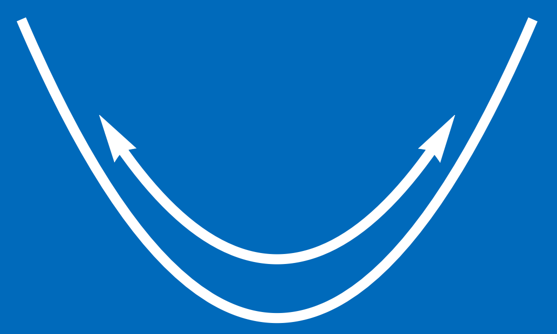
No detection of WIMPs yet!



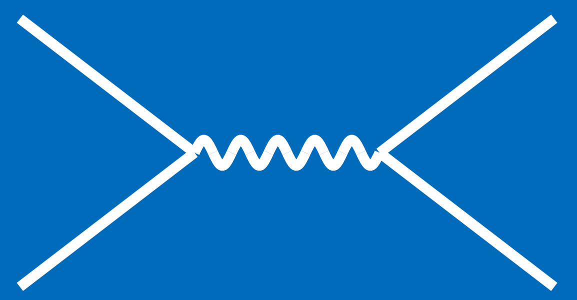
Consider FIMPs:

- Not in thermal equilibrium
- Produced via freeze-in
- Elusive (in)direct detection
- Dependence on initial conditions (inflation, reheating)

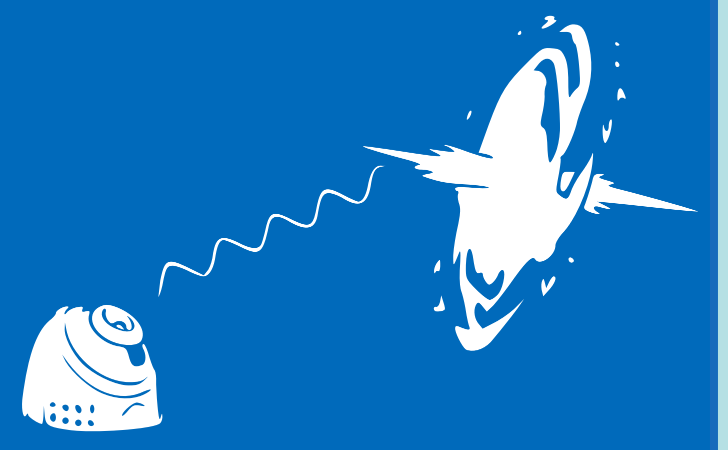
1. Reheating



2. Freeze-in



3. Lyman-α



4. The end?

The Goals

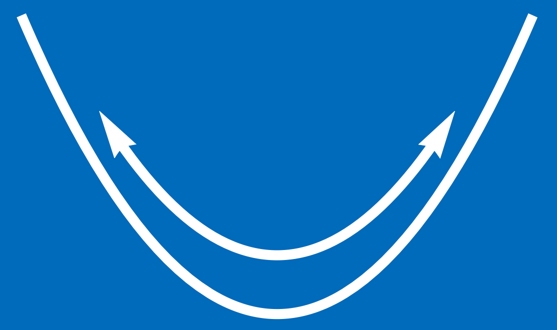
Understand the (perturbative) repopulation of the Universe after the end of inflation (rates, energy densities, temperatures, distributions,..)

Characterize the out-of-equilibrium production of dark matter (from particle/condensate decays, scatterings, misalignment...)

In the absence of a direct signal, can we say more beyond Ω_{DM}?
Use the Ly-α measurement of the power spectrum to impose constraints

A short summary of what we (don't) understand

1. Reheating



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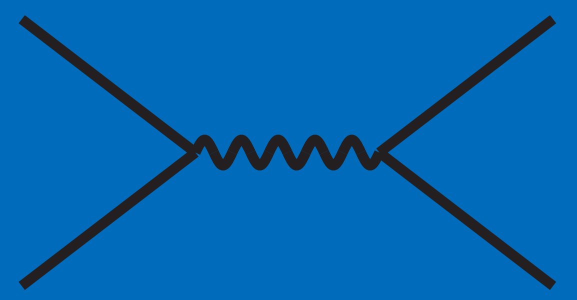

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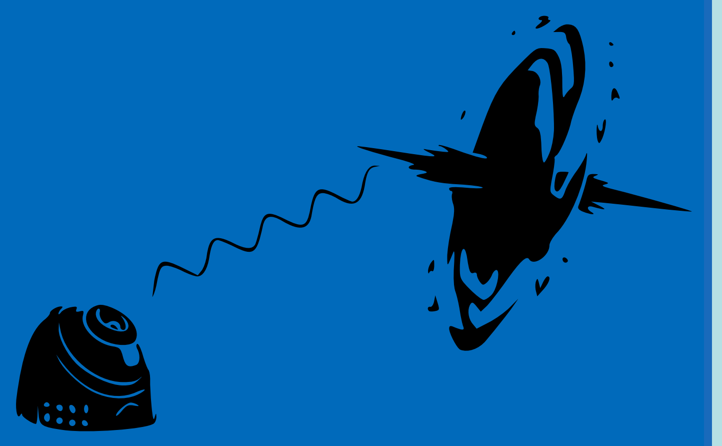


4. The end?

Accelerated expansion can be driven by a slowly rolling scalar field

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - V(\Phi) \right]$$

$$ds^2 = dt^2 - a(t)^2 d\boldsymbol{x}^2$$

$$G_{\mu\nu} = M_P^{-2} T_{\mu\nu}$$

$$\rho_\Phi = \frac{1}{2}\dot{\Phi}^2 + V(\Phi)$$

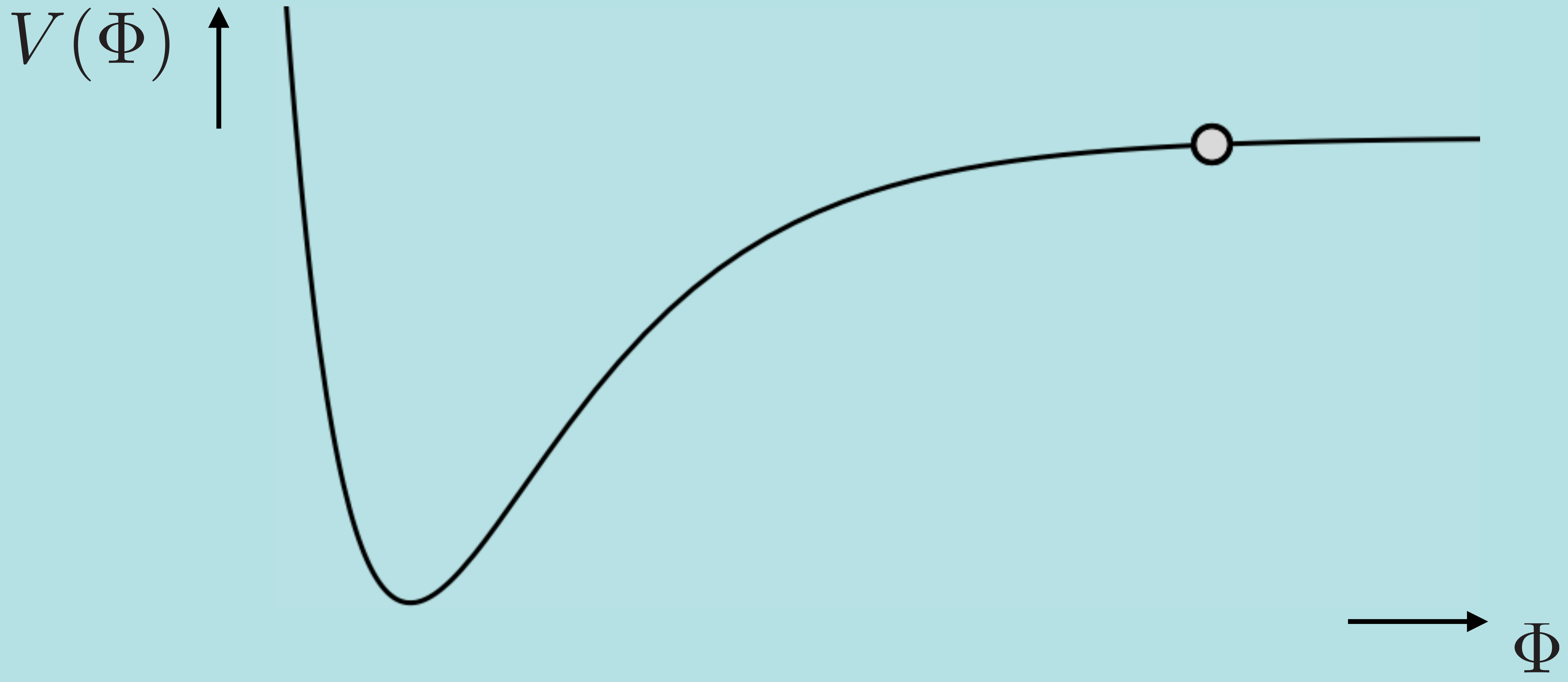
$$P_\Phi = \frac{1}{2}\dot{\Phi}^2 - V(\Phi)$$

$$H \equiv \frac{\dot{a}}{a} = \left(\frac{\rho_\Phi}{3M_P^2} \right)^{1/2}$$

$$\dot{\rho}_\Phi + 3H(\rho_\Phi + P_\Phi) = 0$$

or

$$\ddot{\Phi} + 3H\dot{\Phi} + V'(\Phi) = 0$$



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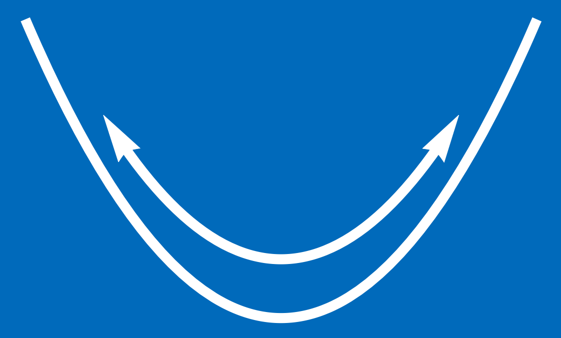

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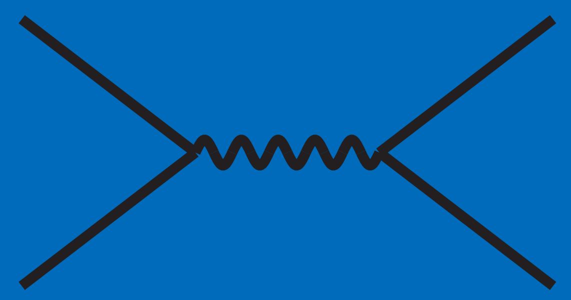
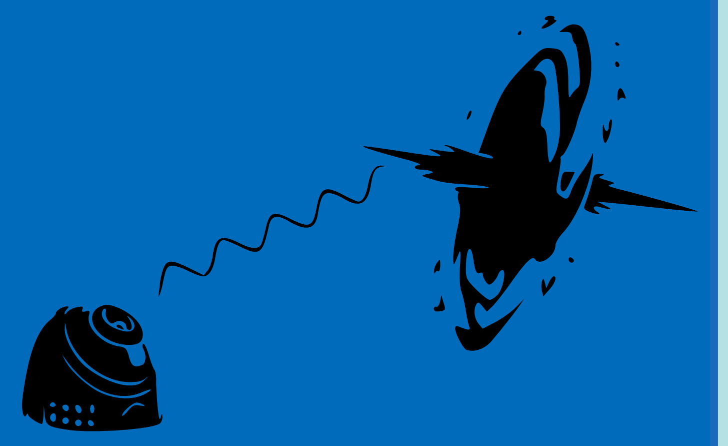


3. Lyman-α



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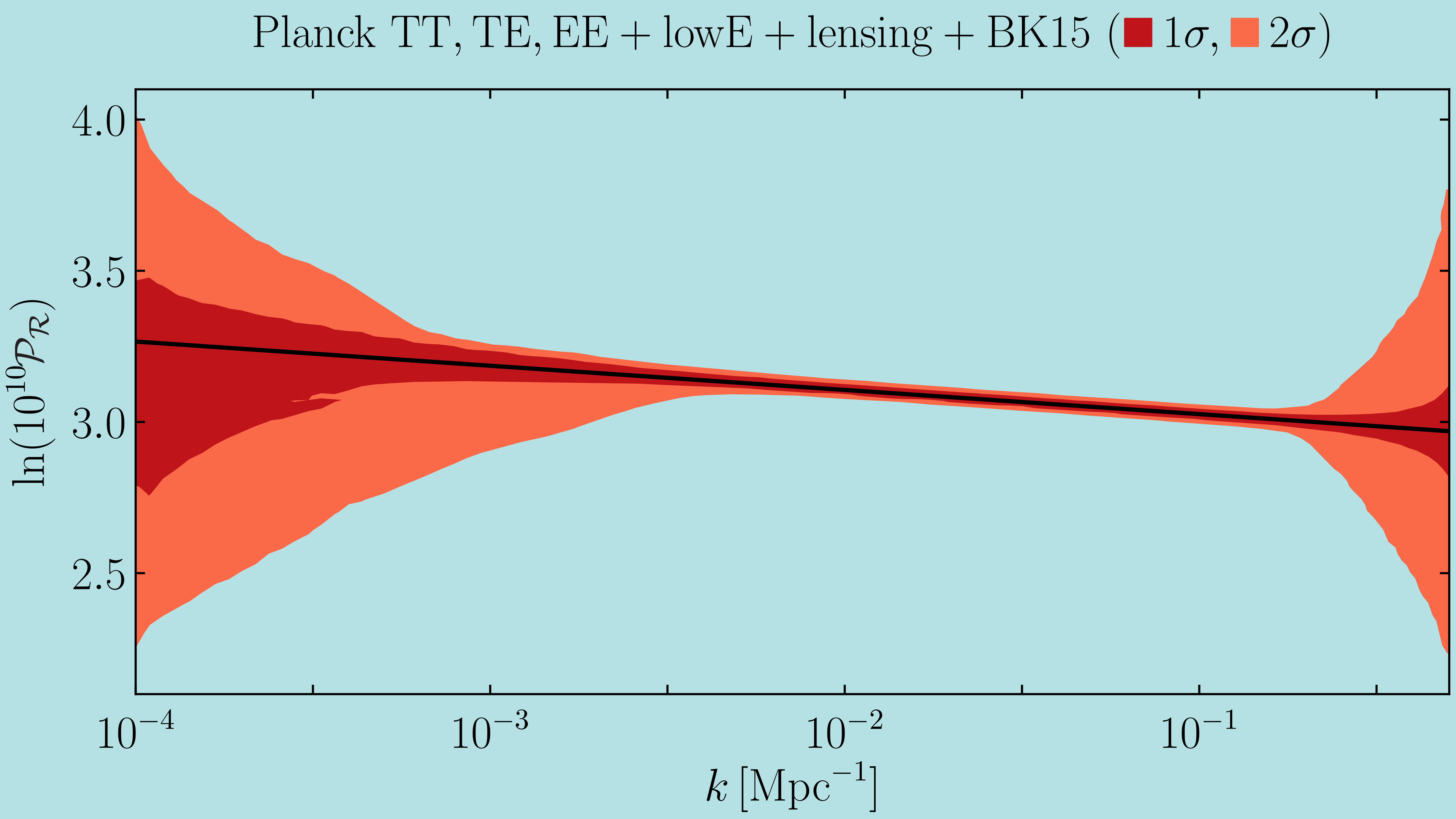
$$\Phi \rightarrow \Phi + \delta\Phi$$

$$g \rightarrow g + \delta g$$

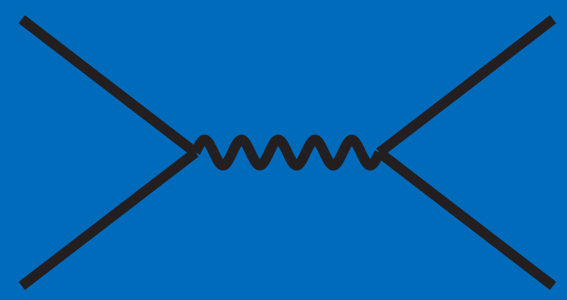
Y. Akrami et al. [Planck], Astron. Astrophys. 641 (2020) A10

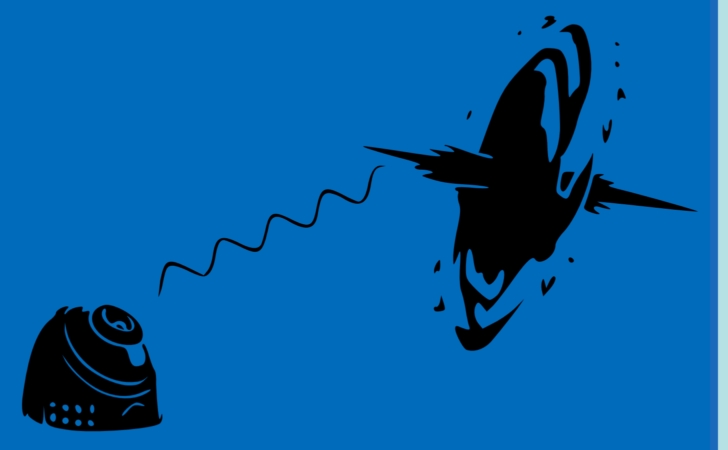


$$\mathcal{P}_\mathcal{R} = \frac{H_*^4}{4\pi^2 \dot{\phi}_*^2} \left(\frac{k}{aH} \right)^{n_s - 1}$$

$$\mathcal{P}_\mathcal{T} = \frac{2}{\pi^2} H_*^2 \left(\frac{k}{aH} \right)^{n_T}$$

$$(r < 0.06)$$





After inflation ends, reheating begins

$$\mathcal{S} = \int d^4x\,\sqrt{-g}\left[\frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - V(\Phi)\right]$$

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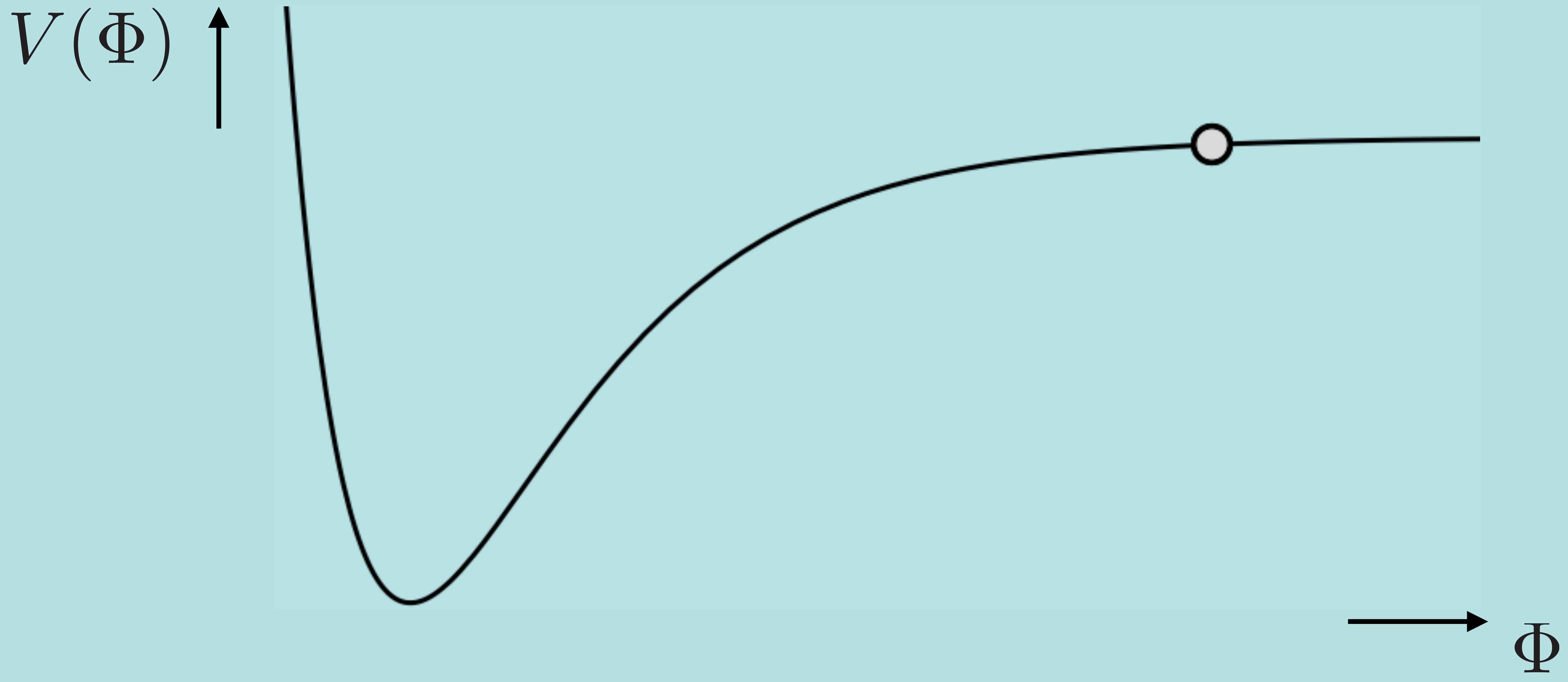

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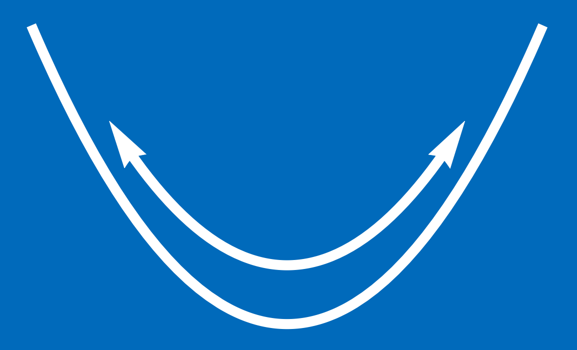
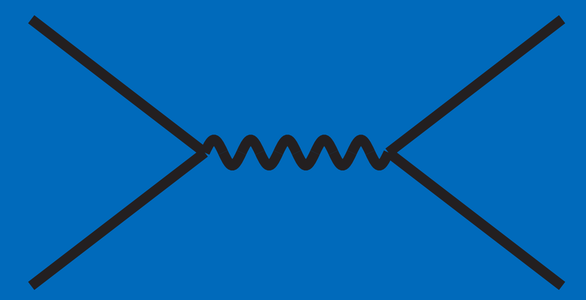


2. Freeze-in



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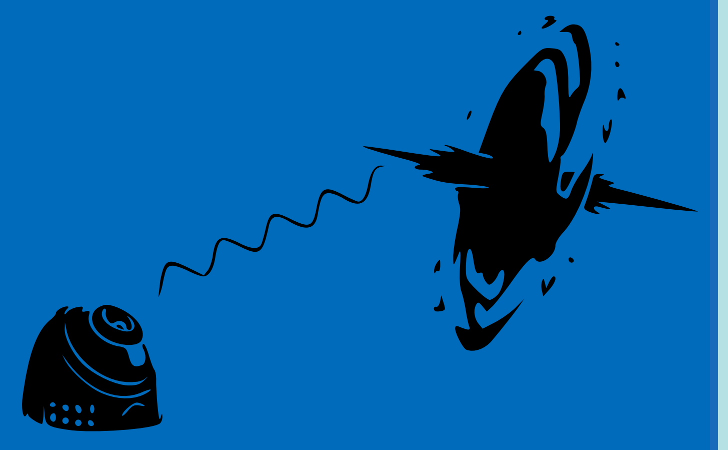
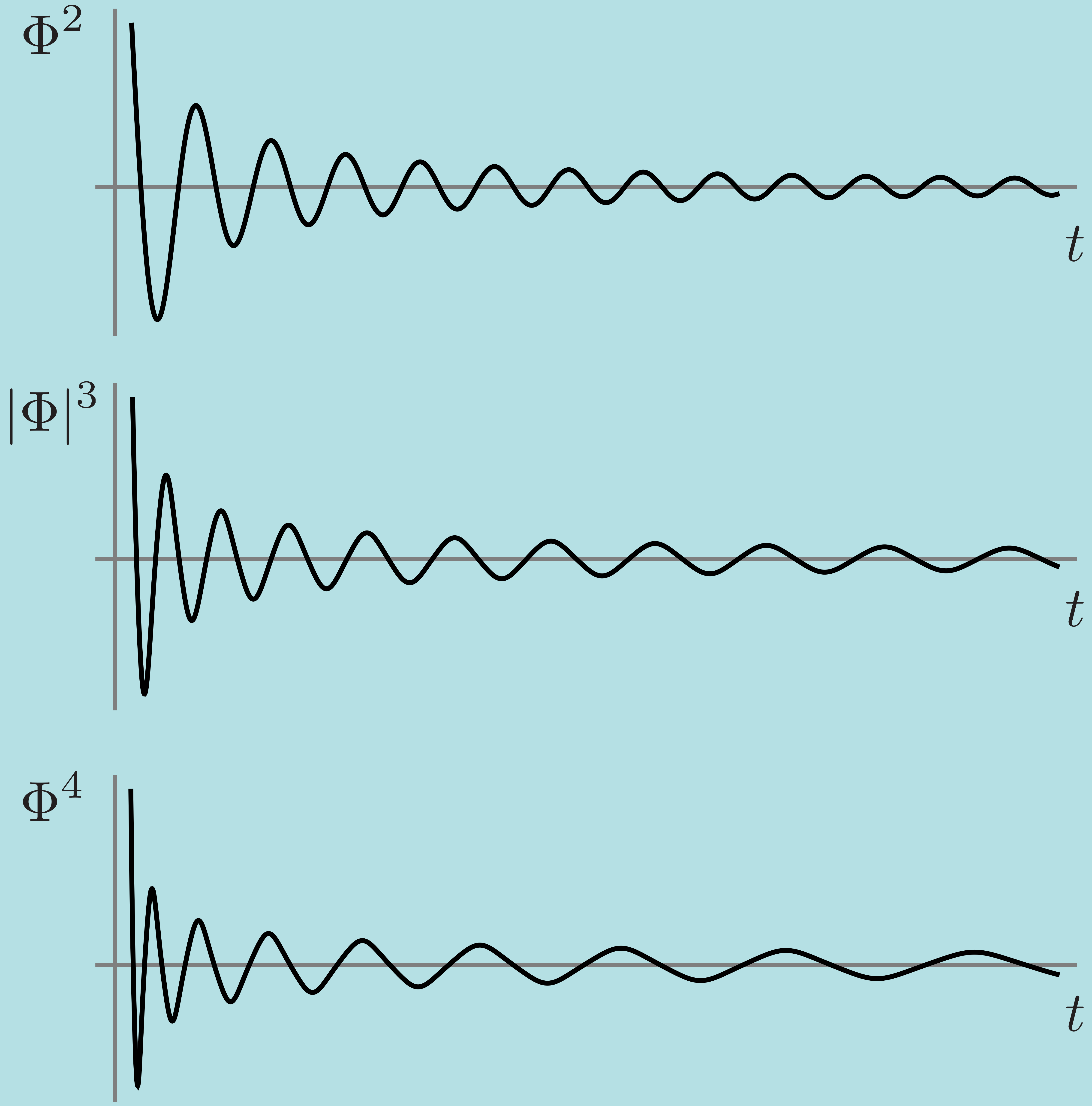


4. The end?

Reheating depends on the shape of the potential



$$V(\Phi) \propto |\Phi|^k$$

$$\downarrow$$

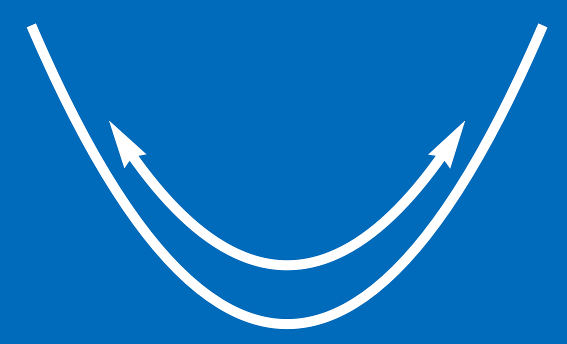
over one oscillation,

$$\langle \dot{\Phi}^2 \rangle \simeq \langle \Phi V'(\Phi) \rangle$$

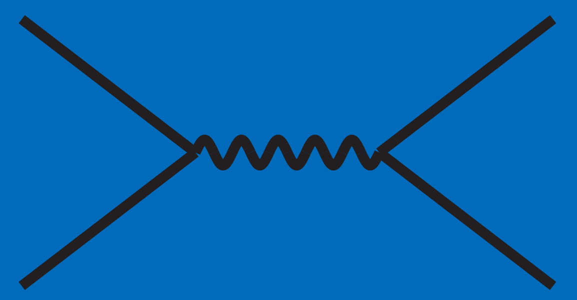
$$\downarrow$$

$$\langle P_\Phi \rangle = \frac{k-2}{k+2} \langle \rho_\Phi \rangle$$

1. Reheating



2. Freeze-in



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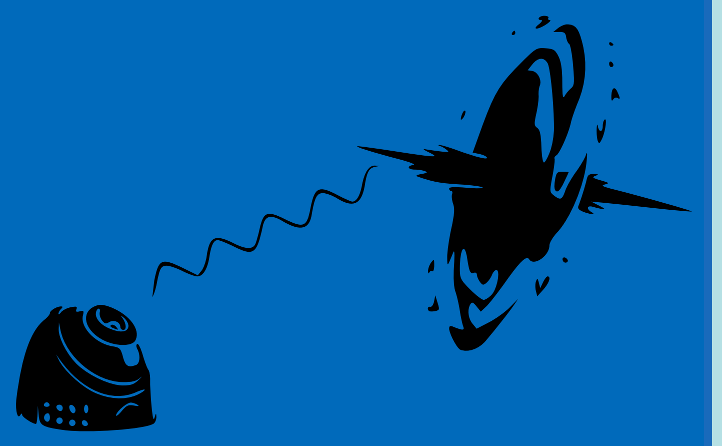

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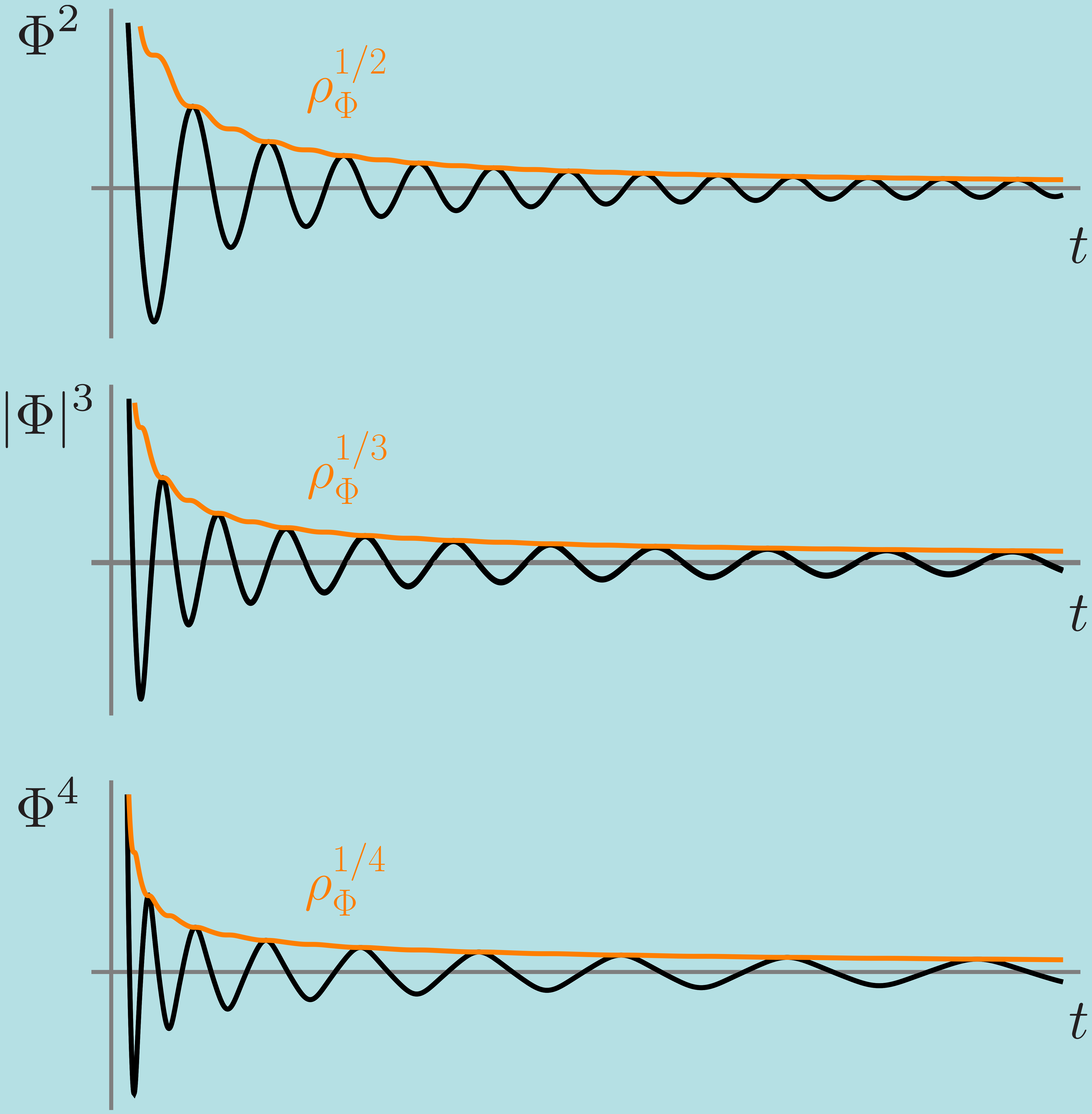


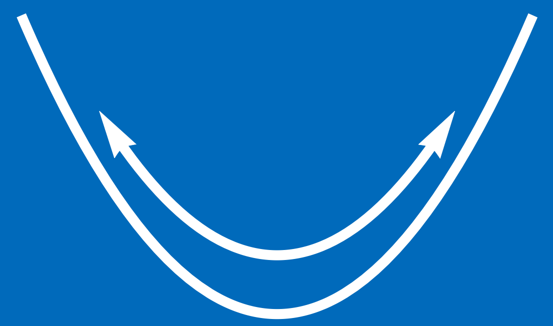
~ matter

$$\rho_\Phi = \rho_{\rm end}\left(\frac{a}{a_{\rm end}}\right)^{-\frac{6k}{k+2}}$$

$$a \propto t^{\frac{k+2}{3k}}$$

~ radiation

1. Reheating



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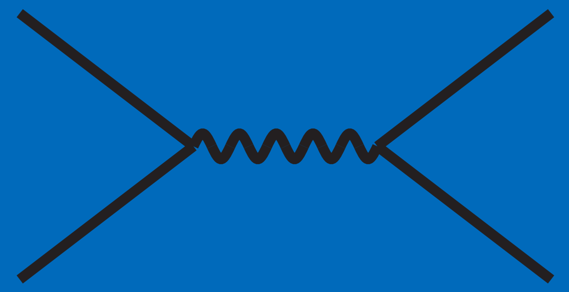

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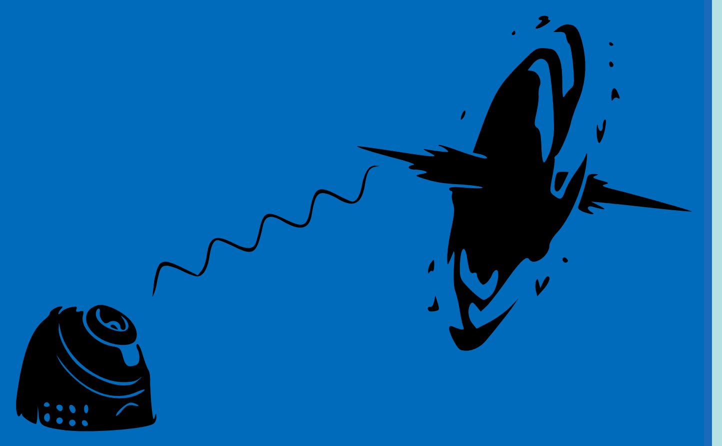

4. The end?

Particle production in the expanding Universe

$$\chi + a + b + \cdots \longleftrightarrow i + j + \cdots$$

$$\frac{\partial f_\chi}{\partial t} - H|\boldsymbol{p}|\frac{\partial f_\chi}{\partial |\boldsymbol{p}|} = \mathcal{C}[f_\chi(|\boldsymbol{p}|, t)]$$

$$\begin{aligned}\mathcal{C}[f_\chi] = -\frac{1}{2p_0}\int \frac{g_a d^3\boldsymbol{p}_a}{(2\pi)^3 2p_{a0}} \frac{g_b d^3\boldsymbol{p}_b}{(2\pi)^3 2p_{b0}} \cdots \frac{g_i d^3\boldsymbol{p}_i}{(2\pi)^3 2p_{i0}} \frac{g_j d^3\boldsymbol{p}_j}{(2\pi)^3 2p_{j0}} \cdots \\ \times (2\pi)^4\, \delta^{(4)}(p_\chi + p_a + p_b + \cdots - p_i - p_j - \cdots) \\ \times \Big[|\mathcal{M}|^2_{\chi+a+b+\cdots \longrightarrow i+j+\cdots}\, f_a f_b \cdots f_\chi (1 \pm f_i)(1 \pm f_j) \cdots \\ - |\mathcal{M}|^2_{i+j+\cdots \longrightarrow \chi+a+b+\cdots}\, f_i f_j \cdots (1 \pm f_a)(1 \pm f_b) \cdots (1 \pm f_\chi) \Big]\end{aligned}$$

$$n_\chi(t) = \frac{g_\chi}{(2\pi)^3}\int d^3\boldsymbol{p}\, f_\chi(p_0, t) \qquad \rho_\chi(t) = \frac{g_\chi}{(2\pi)^3}\int d^3\boldsymbol{p}\; p_0\, f_\chi(p_0, t)$$

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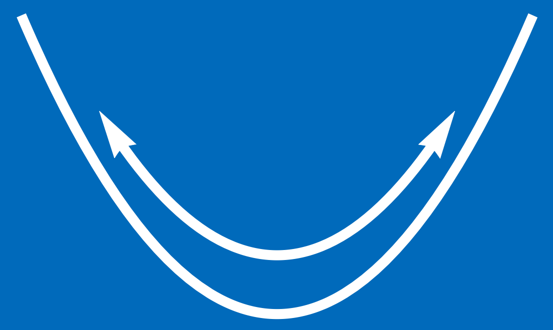

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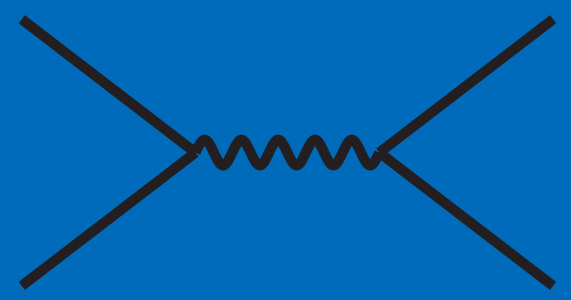

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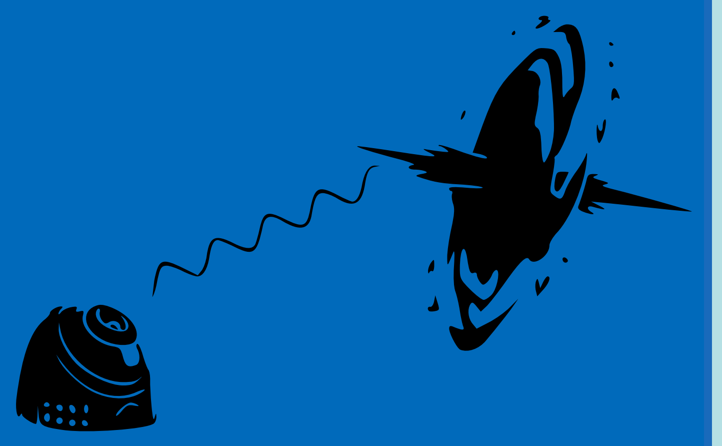


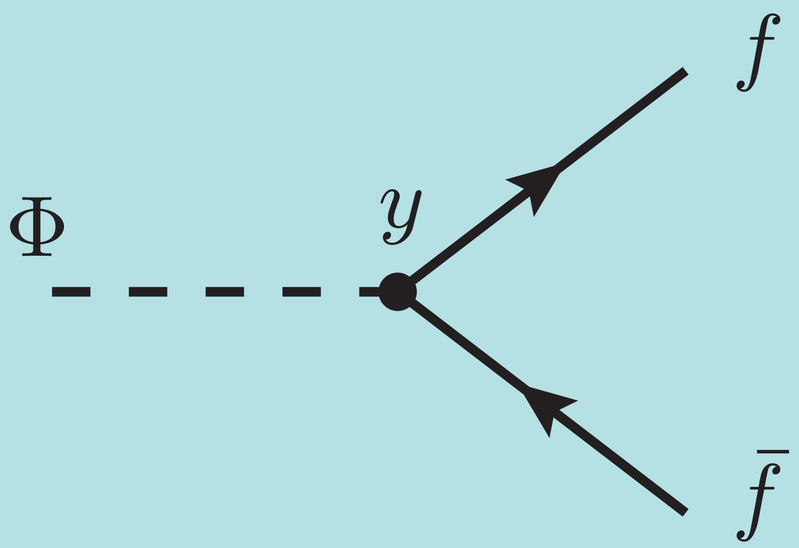
4. The end?

Integrated Boltzmann equations

$$\dot{\rho}_\Phi + 3\left(\frac{2k}{k+2}\right) H\rho_\Phi = -\Gamma_\Phi(t)\rho_\Phi$$

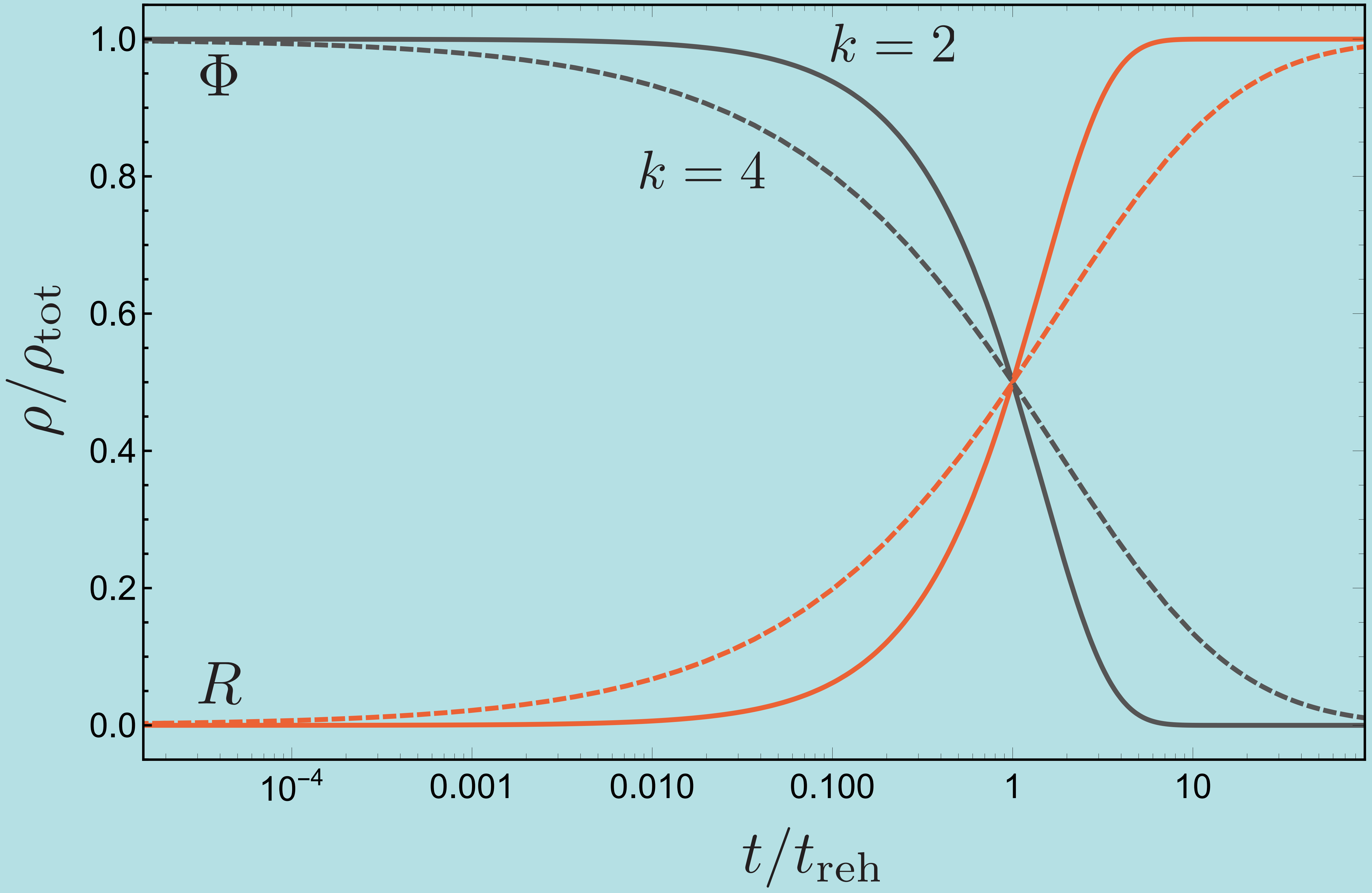
$$\dot{\rho}_R + 4H\rho_R = \Gamma_\Phi(t)\rho_\Phi$$

$$3M_P^2 H^2 = \rho_\Phi + \rho_R$$



$$\Gamma_\Phi = \frac{y^2}{8\pi} m_\Phi(t)\,,$$

$$m_\Phi^2 \equiv \partial_\phi^2 V(\Phi) \propto \rho_\Phi^{\frac{k-2}{k}}$$



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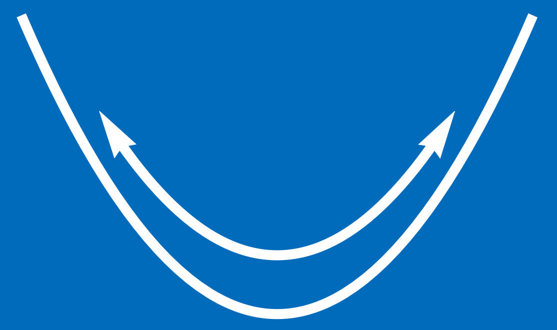

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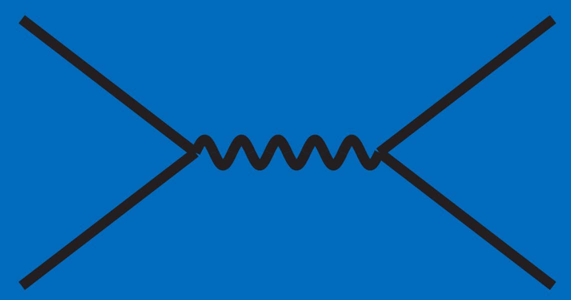

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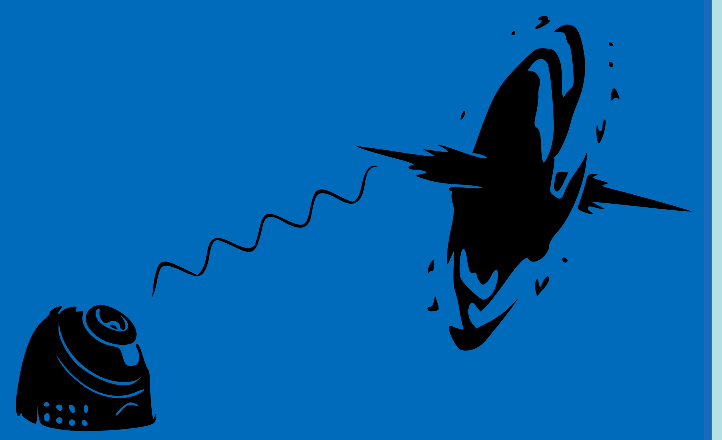

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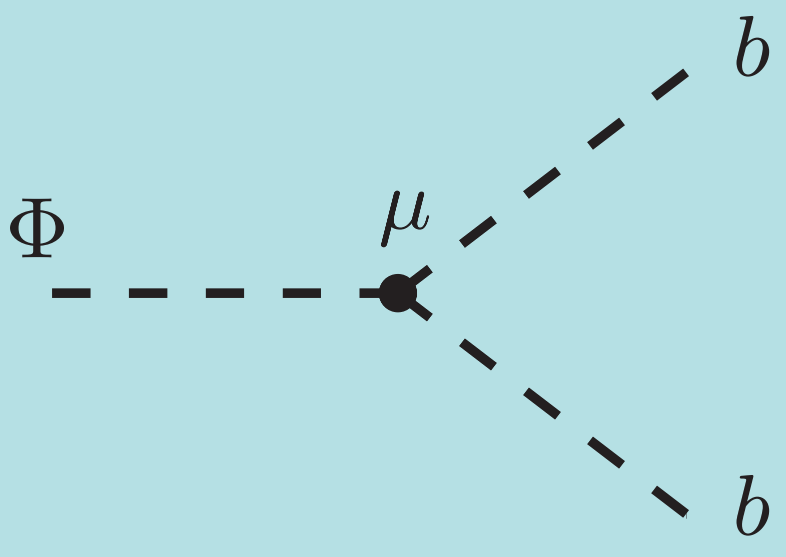

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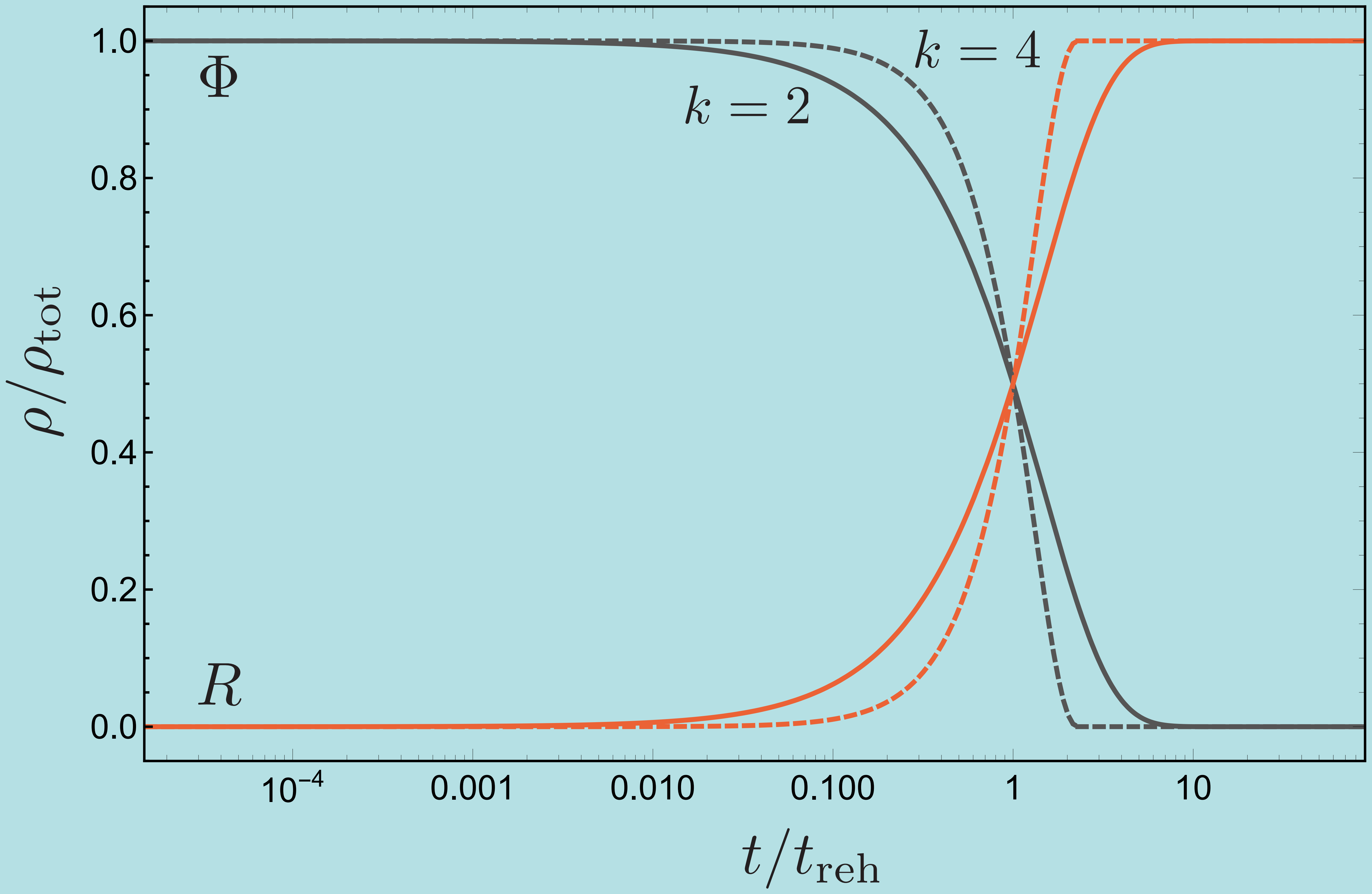
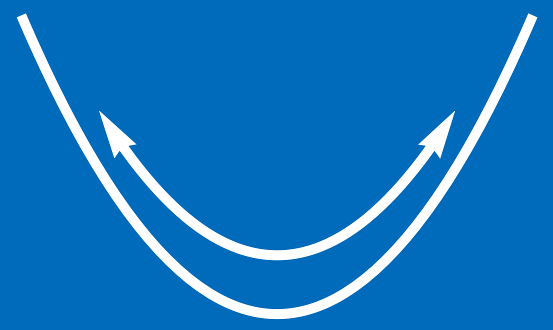
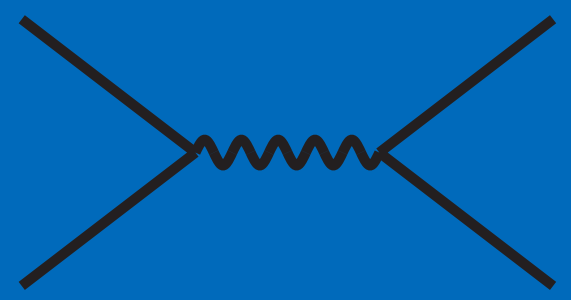
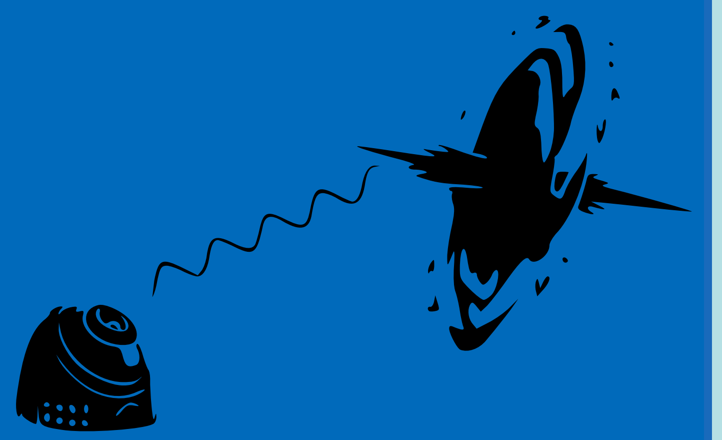
$$m_\Phi^2 \equiv \partial_\phi^2 V(\Phi) \propto \rho_\Phi^{\frac{k-2}{k}}$$









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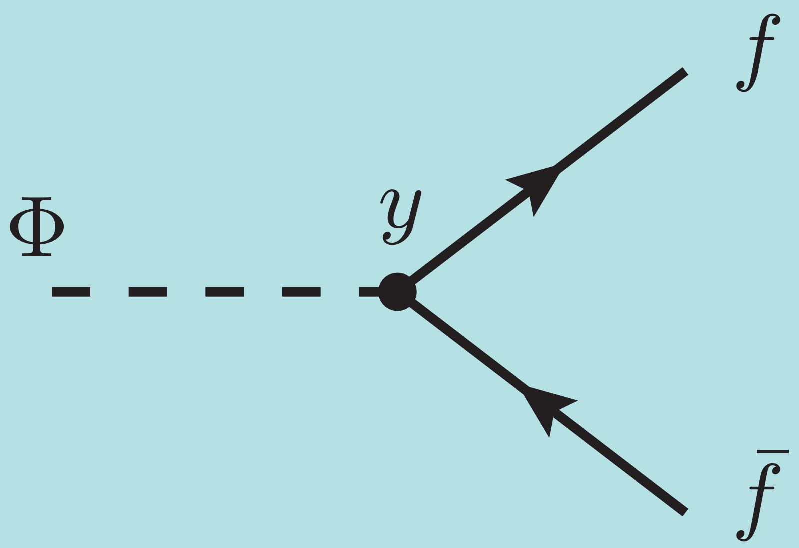
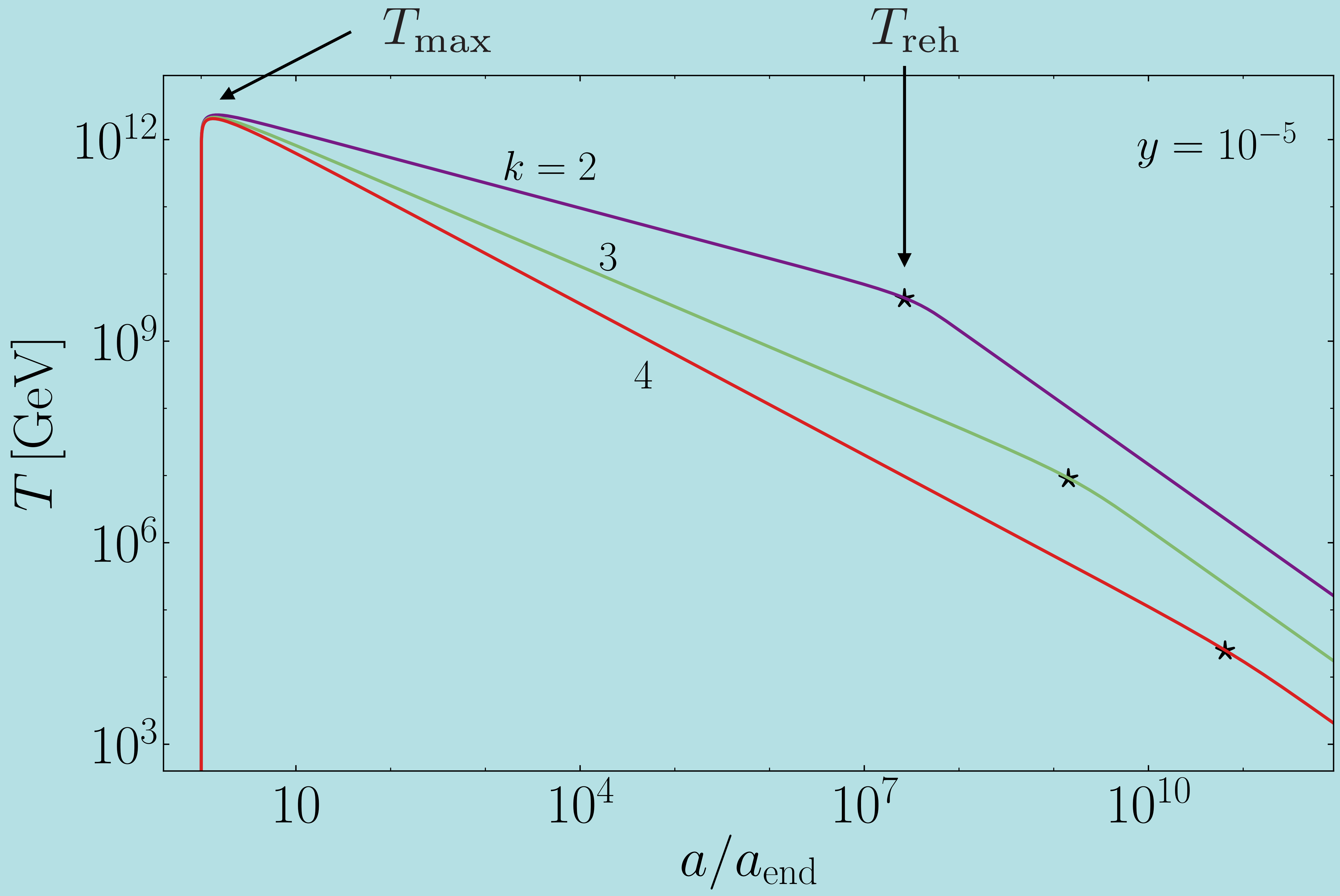


$$\Gamma_\Phi = \frac{y^2}{8\pi} m_\Phi(t)\,,$$

$$m_\Phi^2 \equiv \partial_\phi^2 V(\Phi) \propto \rho_\Phi^{\frac{k-2}{k}}$$

$$T = \left(\frac{30\rho_R}{\pi^2 g_*}\right)^{1/4}$$

$$\propto a^{-\frac{3k-3}{2k+4}}$$



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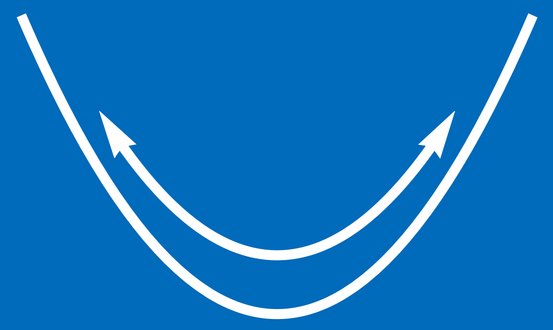

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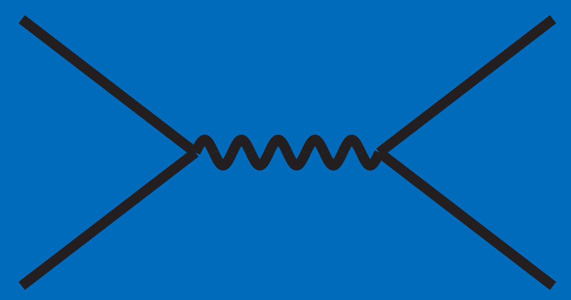

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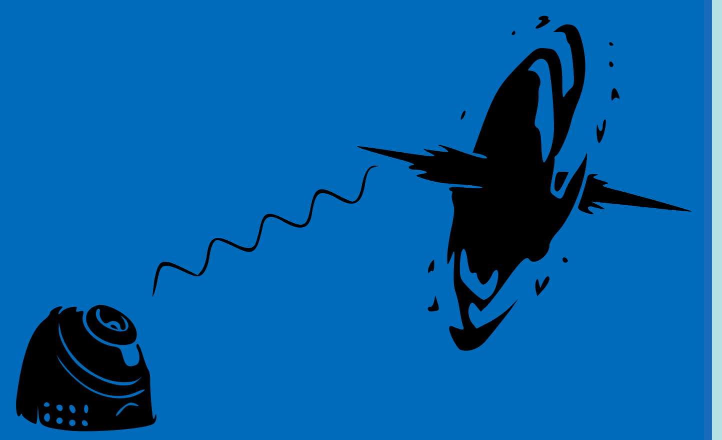
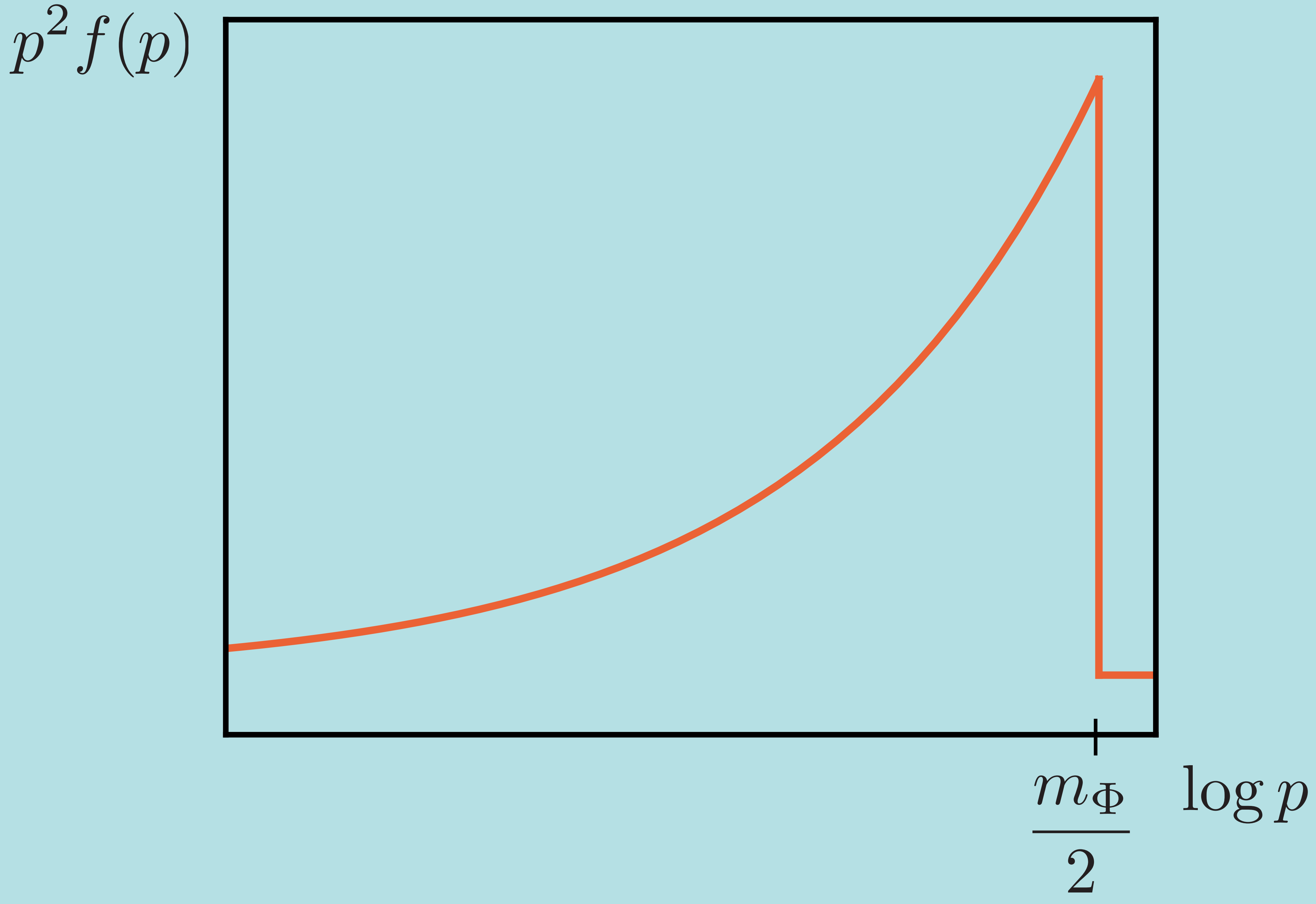


4. The end?

A closer look: thermalization is not instantaneous! (quadratic only)

$$\mathcal{C}[f_\chi(p,t)] \;=\; \frac{8\pi^2}{g_\chi m_\Phi^2} n_\Phi \Gamma_{\Phi\to\chi\psi}\delta(p - m_\Phi/2)$$

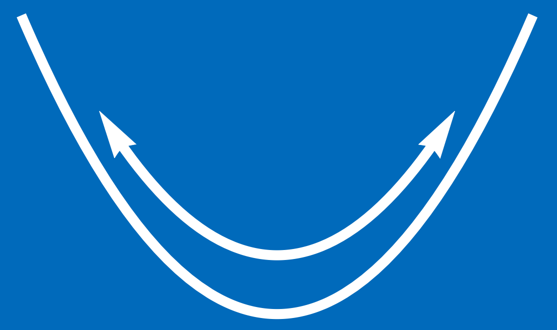
$$f_\chi(p,t) \;=\; \frac{16\pi^2\Gamma_{\Phi\to\chi\psi} n_\Phi(\hat{t})}{g_\chi m_\Phi^3 H(\hat{t})}\theta(t-\hat{t})\,, \qquad \frac{a(t)}{a(\hat{t})} = \frac{m_\Phi}{2p}$$



$$f_\chi(p,t) \;\simeq\; \frac{24\pi^2 n_\chi(t)}{g_\chi m_\Phi^3}\left(\frac{m_\Phi}{2p}\right)^{3/2} \theta(m_\Phi/2 - p)$$

$$(t \ll t_{\rm reh})$$

1. Reheating



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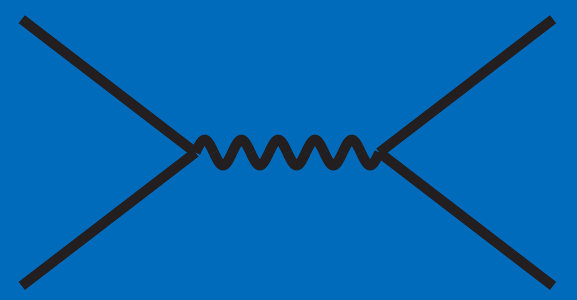

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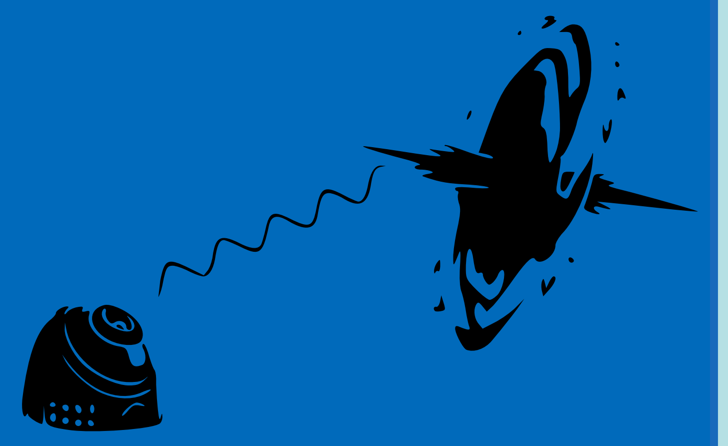

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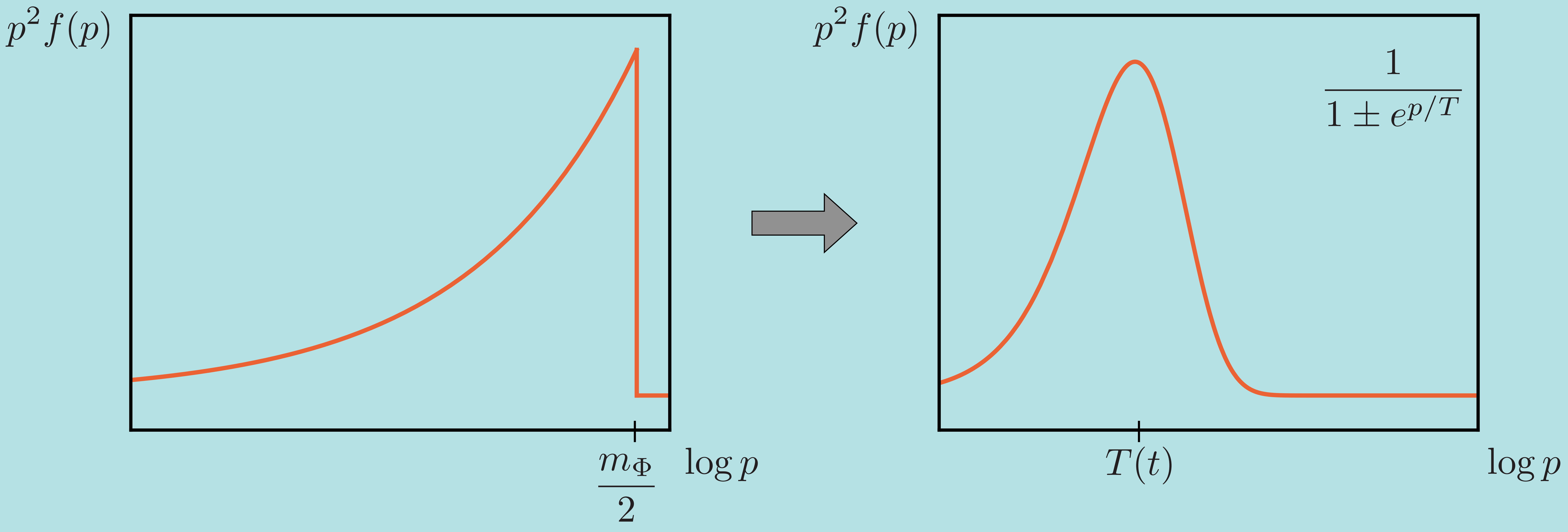
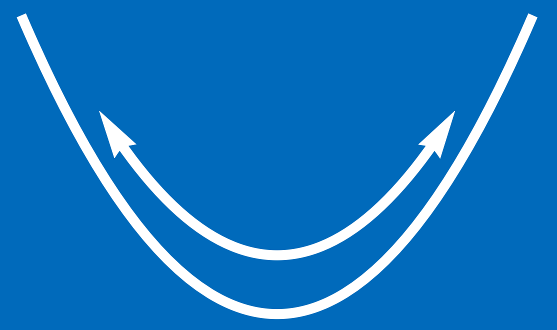
A closer look: thermalization is not instantaneous! (quadratic only)

$$\mathcal{C}[f_\chi(p,t)] = \frac{8\pi^2}{g_\chi m_\Phi^2} n_\Phi \Gamma_{\Phi\to\chi\psi}\delta(p - m_\Phi/2)$$

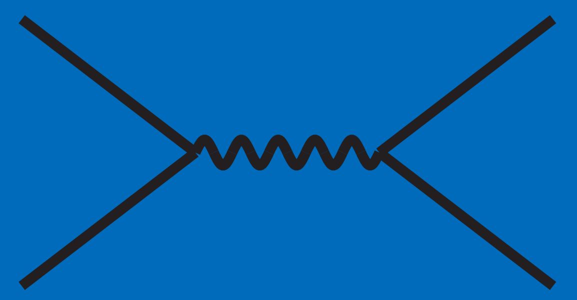
$$f_\chi(p,t) = \frac{16\pi^2 \Gamma_{\Phi\to\chi\psi} n_\Phi(\hat{t})}{g_\chi m_\Phi^3 H(\hat{t})}\theta(t-\hat{t})\,, \qquad \frac{a(t)}{a(\hat{t})} = \frac{m_\Phi}{2p}$$



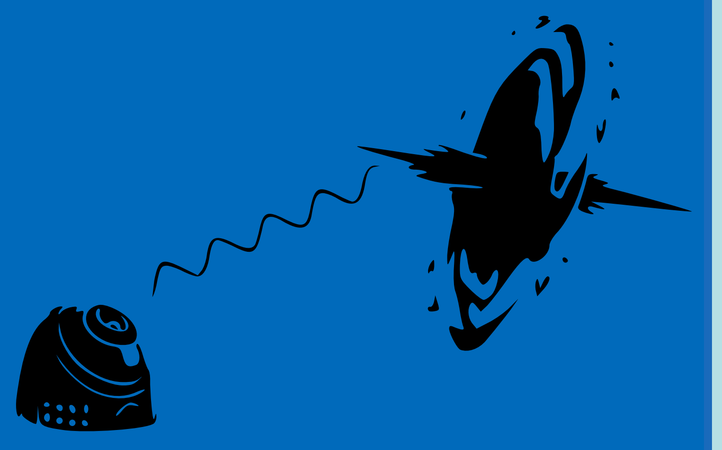
1. Reheating



2. Freeze-in

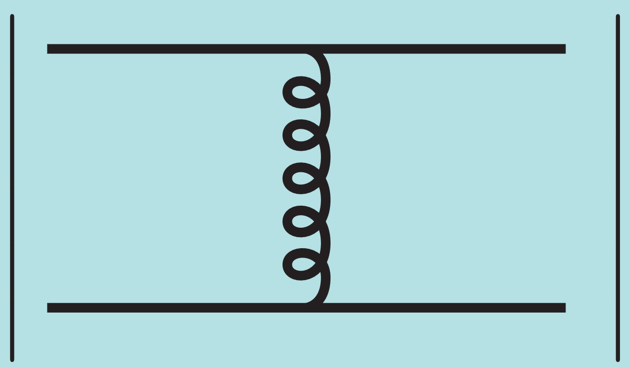


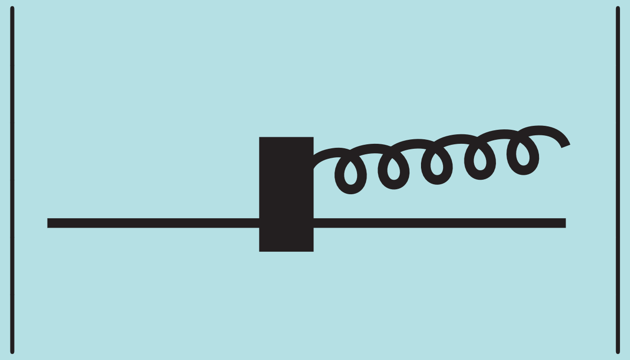
3. Lyman-α



4. The end?

$$\frac{\partial f_\chi}{\partial t} - Hp\frac{\partial f_\chi}{\partial p} = \left| \;\cdot\; \right|^2 + \left| \;\cdot\; \right|^2 + \cdots$$



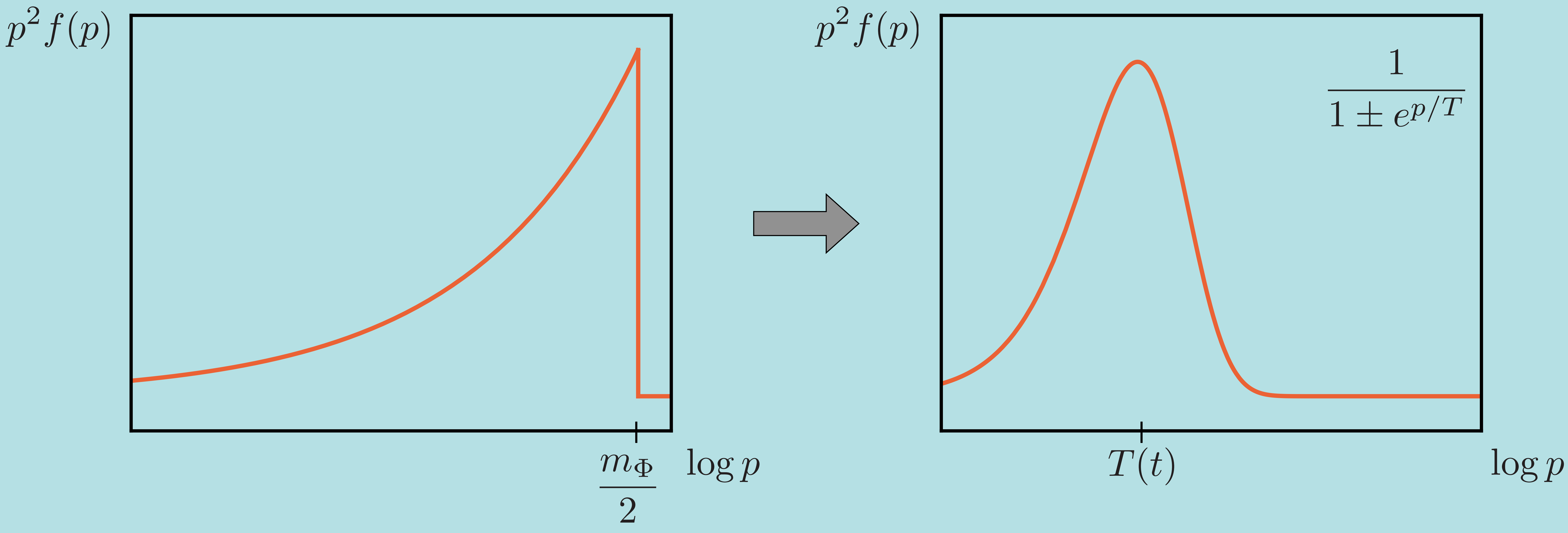


$$\equiv -\mathcal{C}^{2\leftrightarrow 2}[f_\chi] - \mathcal{C}^{\text{“}1\leftrightarrow 2\text{”}}[f_\chi] + \cdots ,$$

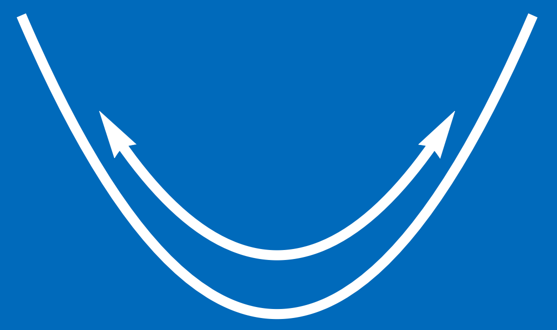
K. Harigaya, K. Mukaida, JHEP 05 (2014) 006

K. Mukaida, M. Yamada, JCAP 02 (2016) 003

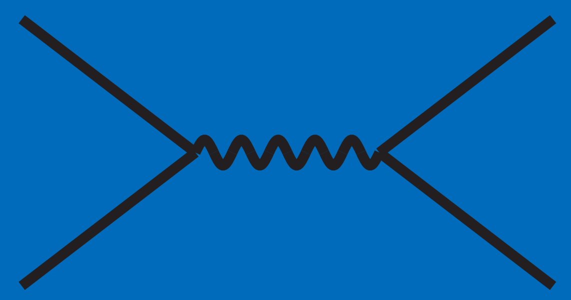
$$\Gamma_\Phi t_{\rm th} \simeq \alpha_{\rm SM}^{-16/5} \left(\frac{\Gamma_\Phi m_\Phi^2}{M_P^3} \right)^{2/5}$$



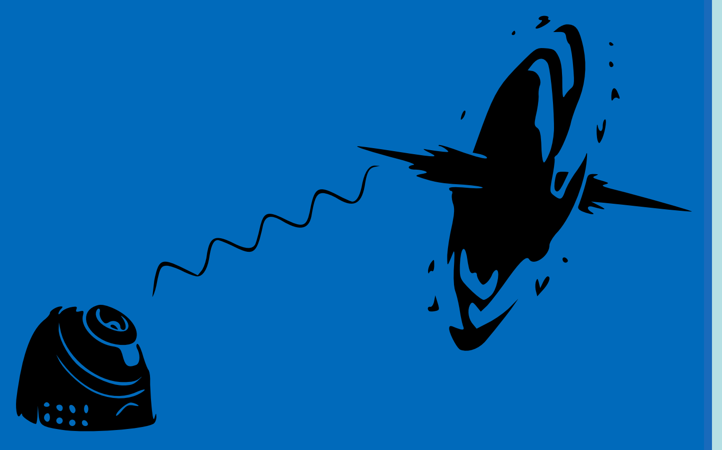
1. Reheating



2. Freeze-in

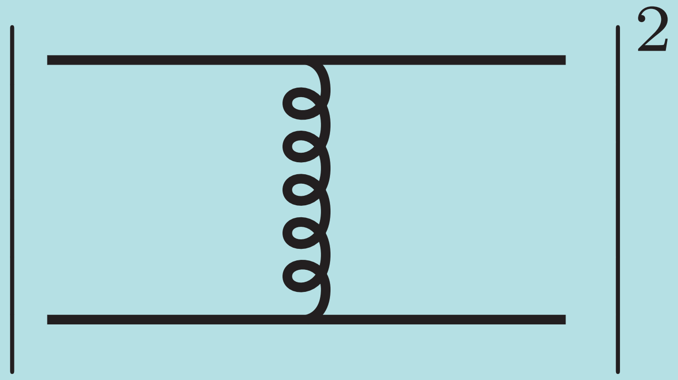


3. Lyman-α

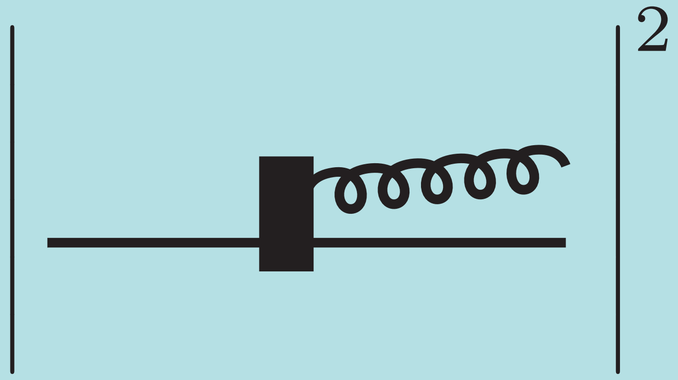


4. The end?

$$\frac{\partial f_\chi}{\partial t} - Hp\frac{\partial f_\chi}{\partial p} =$$



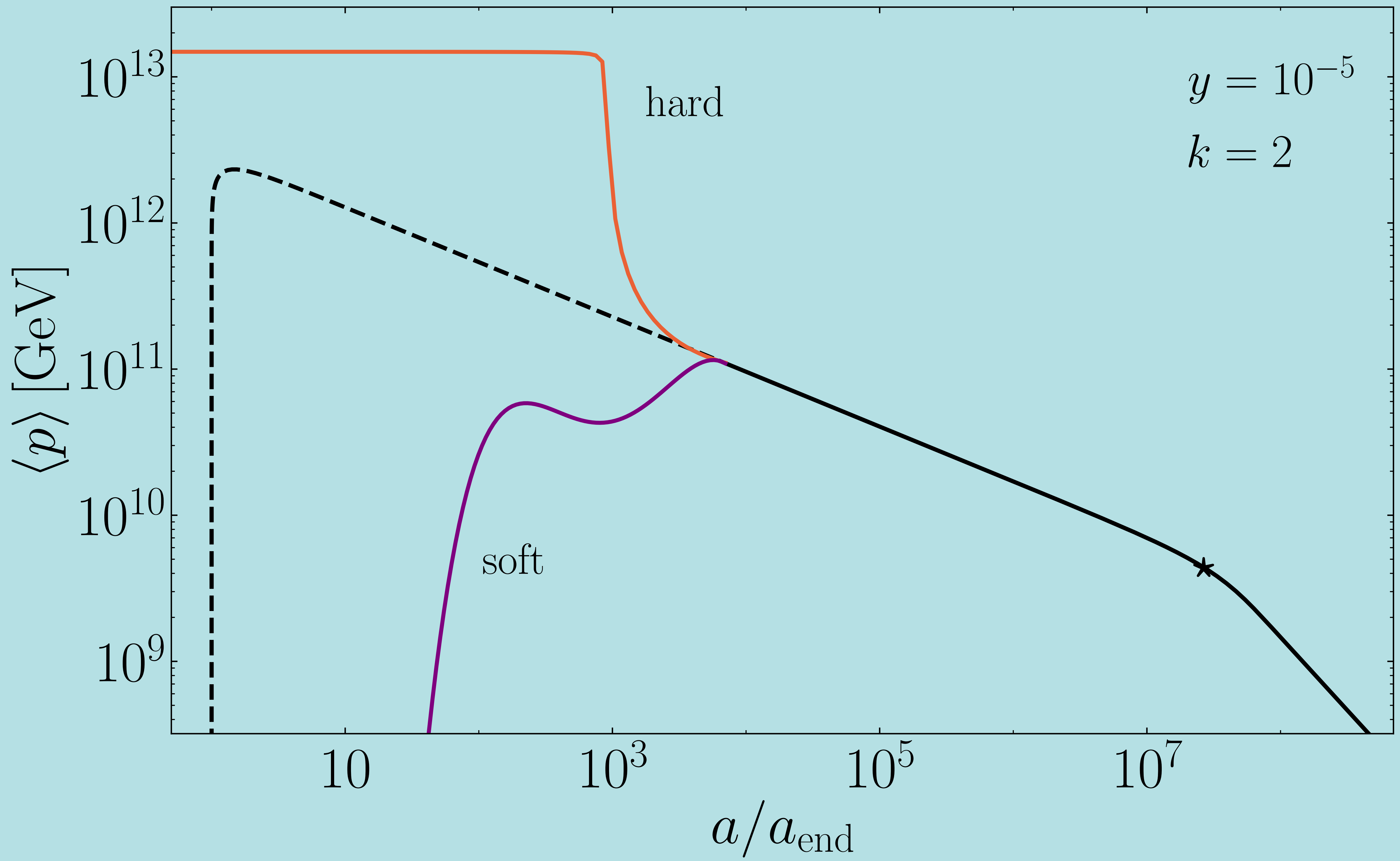
$$+$$



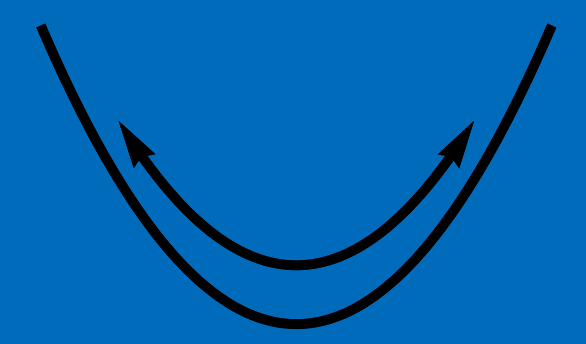
$$+ \cdots$$

$$\equiv -\mathcal{C}^{2\leftrightarrow 2}[f_\chi] - \mathcal{C}^{\text{“}1\leftrightarrow 2\text{”}}[f_\chi] + \cdots ,$$

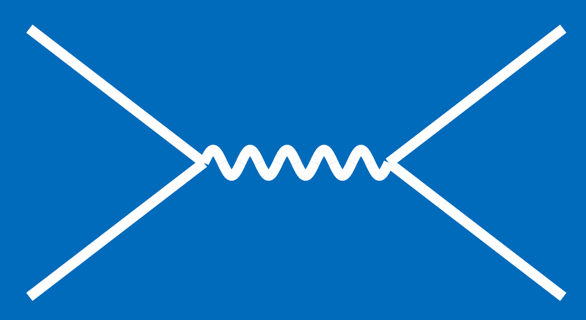
$$\Gamma_\Phi t_{\mathrm{th}} \simeq \alpha_{\mathrm{SM}}^{-16/5}\left(\frac{\Gamma_\Phi m_\Phi^2}{M_P^3}\right)^{2/5}$$



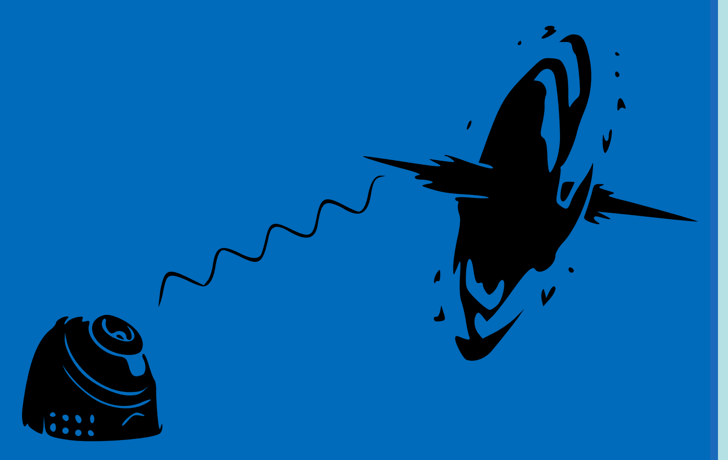
1. Reheating



2. Freeze-in

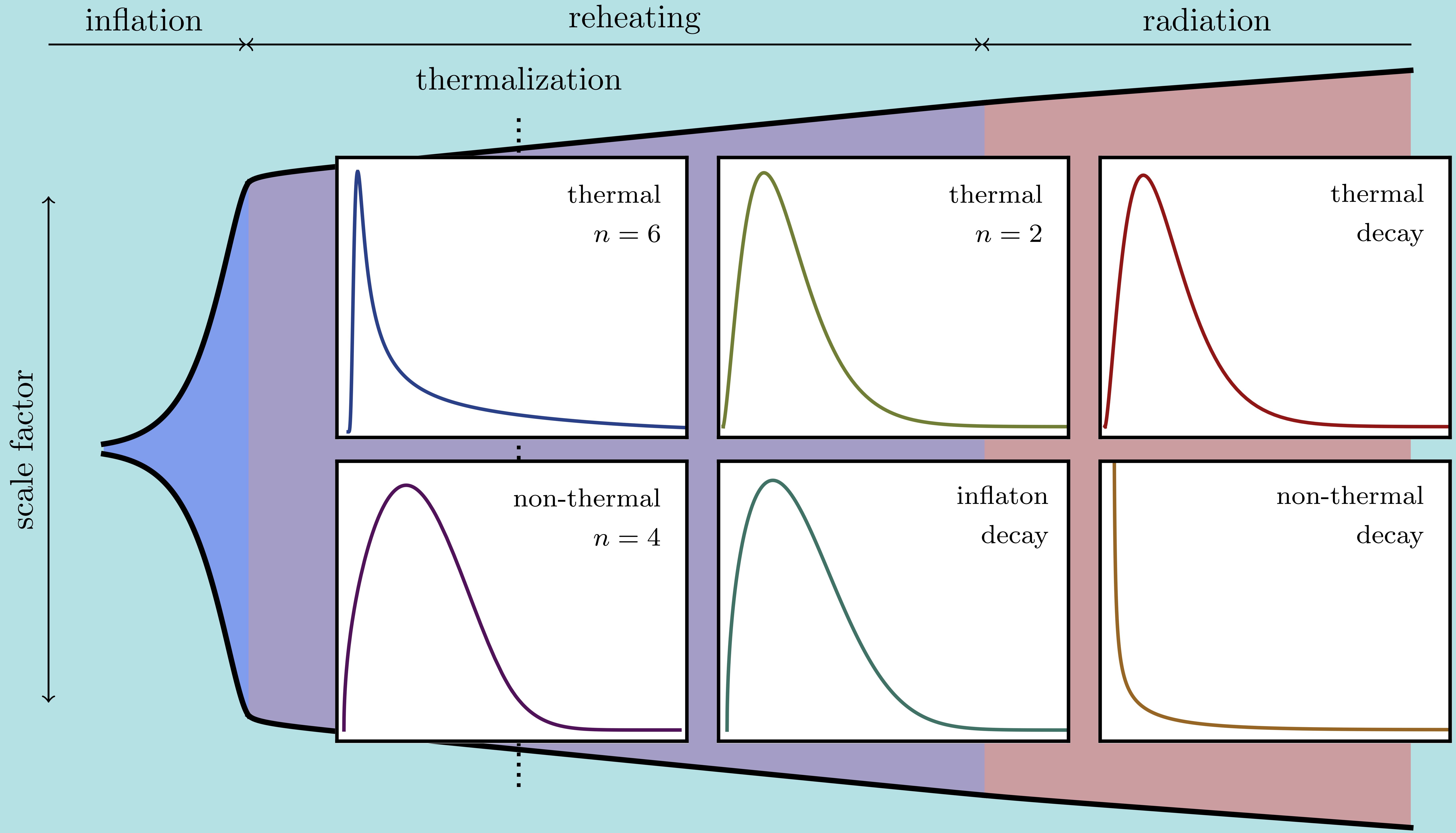


3. Lyman-α

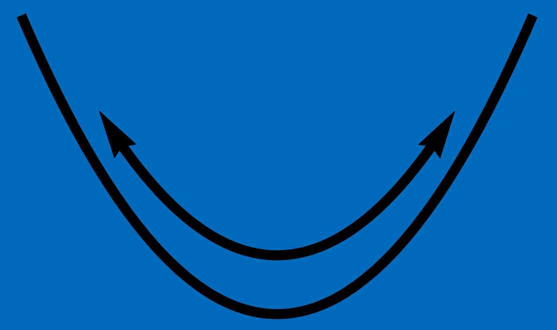


4. The end?

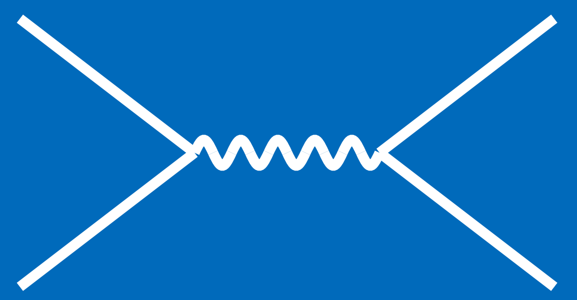
Freeze-in: producing dark matter out-of-equilibrium



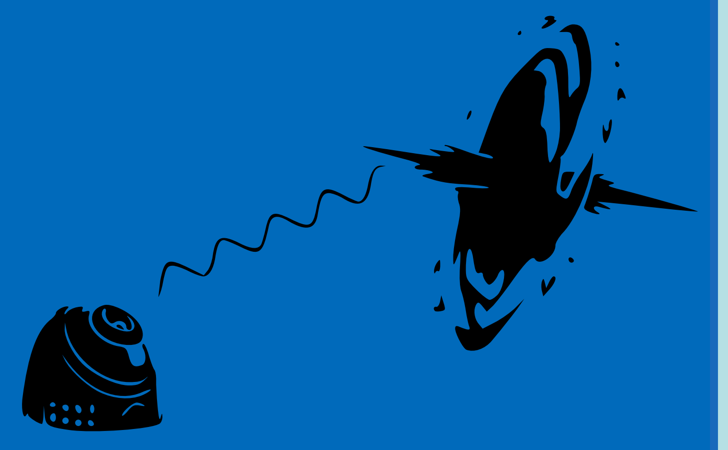
1. Reheating



2. Freeze-in



3. Lyman-α



4. The end?

Decay of an oscillating condensate (inflaton, non-stabilized moduli, ...)

Notation. If $f_\chi(p) \equiv f_\chi(p/p_0)$ at decoupling $(t = t_*)$, then for $t > t_*$

$$f_\chi\left(\frac{p}{p_0}\frac{a(t)}{a_*}\right) = f_\chi\left(\frac{p\,a(t)/a_0}{p_0\,a_*/a_0}\right) = f_\chi\left(\frac{p_{\rm co}}{\underbrace{p_0\,a_*/a_0}_{T_*}}\right) \equiv f_\chi(q)$$

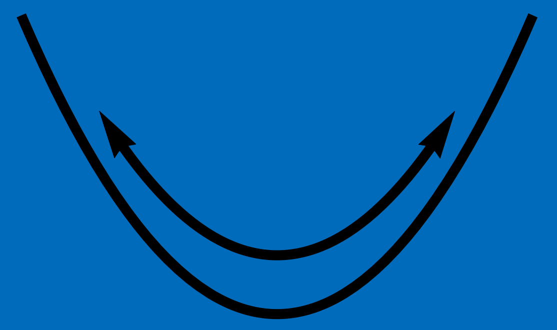
T_* (or $T_{\rm NCDM}$)

Standard in literature and cosmology codes (CLASS)

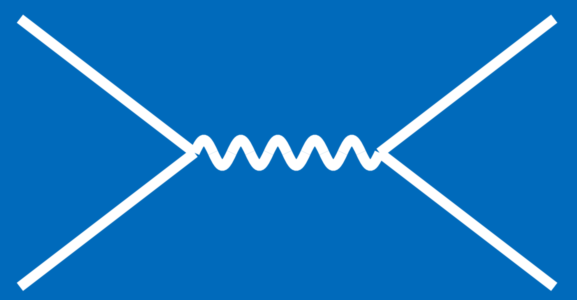
C. Ma, E. Bertschinger, Astrophys. J. 455 (1995) 7

D. Blas, J. Lesgourgues, T. Tram, JCAP 07 (2011) 034

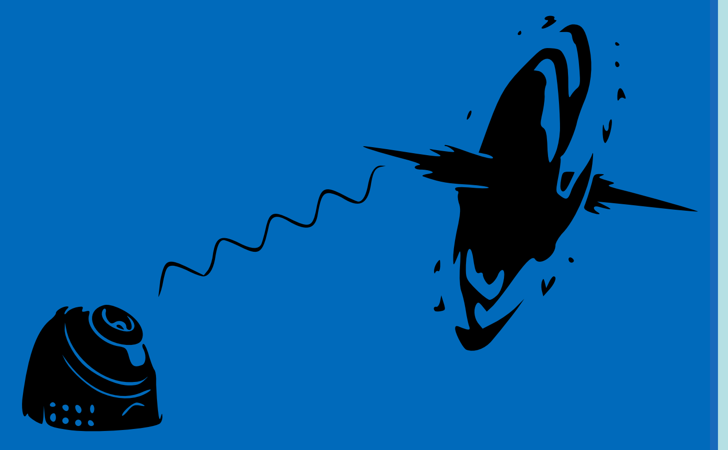
1. Reheating



2. Freeze-in



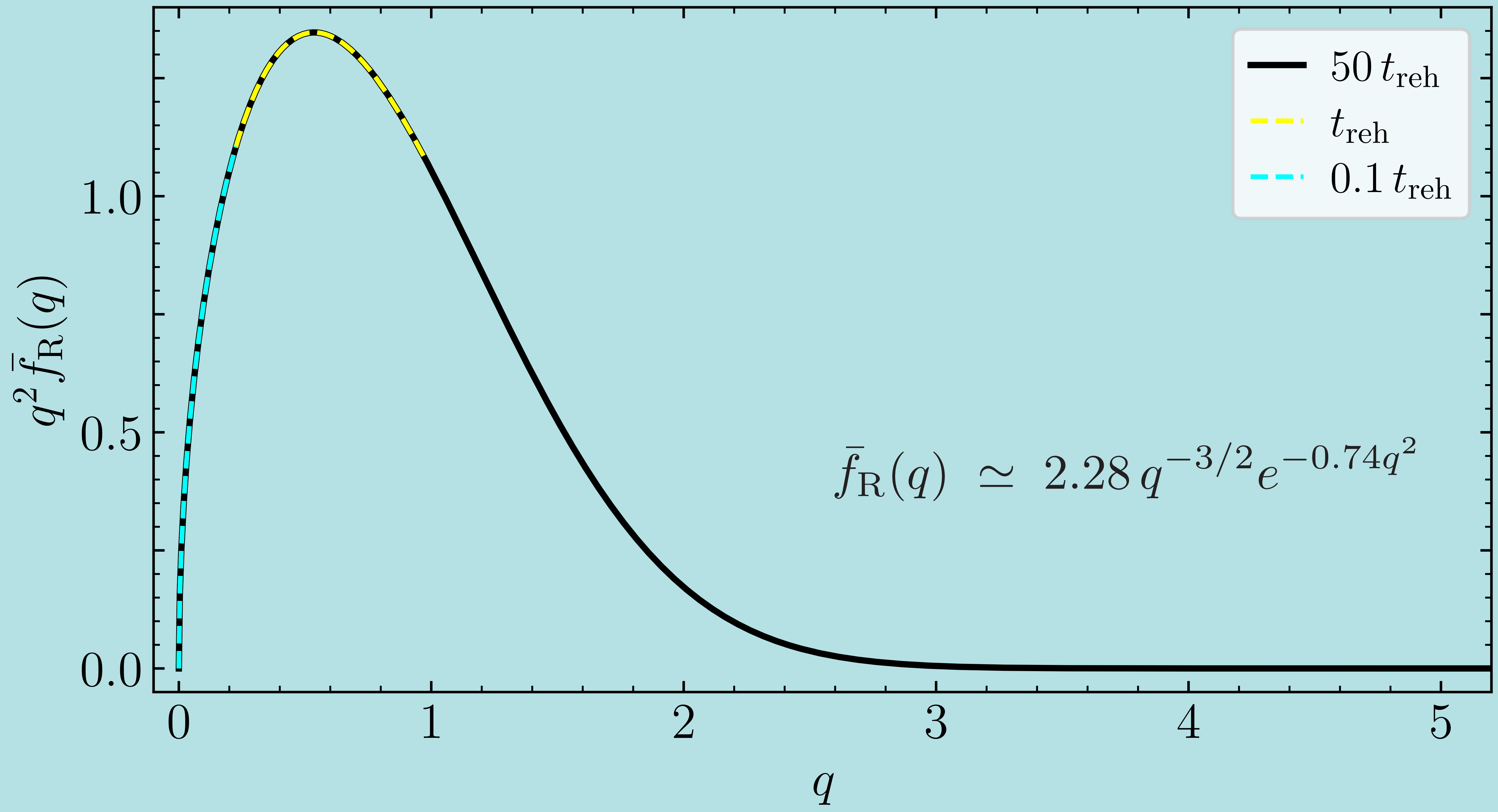
3. Lyman-α



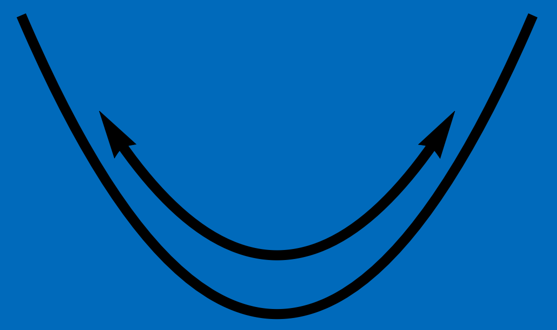
4. The end?

Decay of an oscillating condensate (inflaton, non-stabilized moduli, ...)

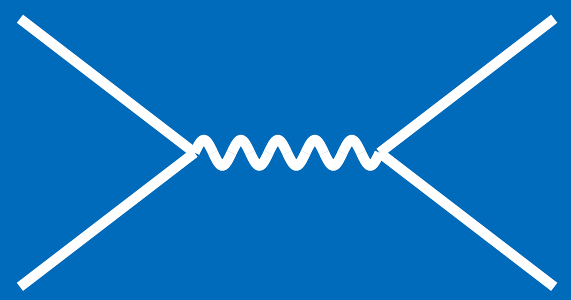
$$f_\chi(p,t)\,d^3\boldsymbol{p} \;=\; \frac{4\pi^4 \mathrm{Br}_\chi g_{*s}^{\mathrm{reh}}}{5 g_\chi}\left(\frac{T_{\mathrm{reh}}}{m_\Phi}\right)^4 \left(\frac{a_0}{a(t)}\right)^3 T_*^3\, \bar{f}_{\mathrm{R}}(q)\, d^3\boldsymbol{q}\,, \qquad T_* \;=\; \left(\frac{g_{*s}^0}{g_{*s}^{\mathrm{reh}}}\right)^{1/3} \frac{m_\Phi}{2T_{\mathrm{reh}}}\, T_0$$



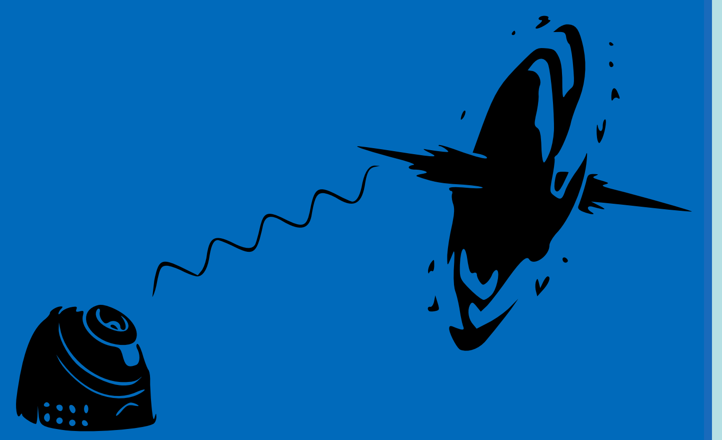
1. Reheating



2. Freeze-in



3. Lyman-α



4. The end?

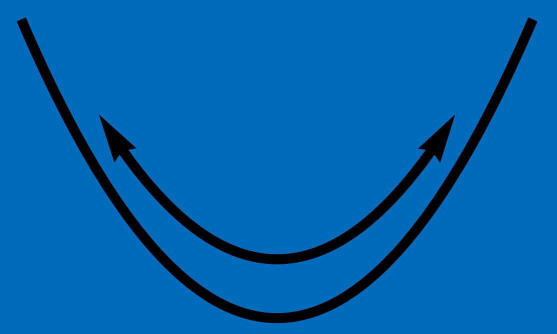
Decay of an oscillating condensate (inflaton, non-stabilized moduli, ...)

$$f_\chi(p,t)\,d^3\boldsymbol{p} \;=\; \frac{4\pi^4 \mathrm{Br}_\chi g_{*s}^{\mathrm{reh}}}{5 g_\chi}\left(\frac{T_{\mathrm{reh}}}{m_\Phi}\right)^4 \left(\frac{a_0}{a(t)}\right)^3 T_*^3\, \bar{f}_{\mathrm{R}}(q)\, d^3\boldsymbol{q}\,, \qquad T_* \;=\; \left(\frac{g_{*s}^0}{g_{*s}^{\mathrm{reh}}}\right)^{1/3} \frac{m_\Phi}{2T_{\mathrm{reh}}}\, T_0$$

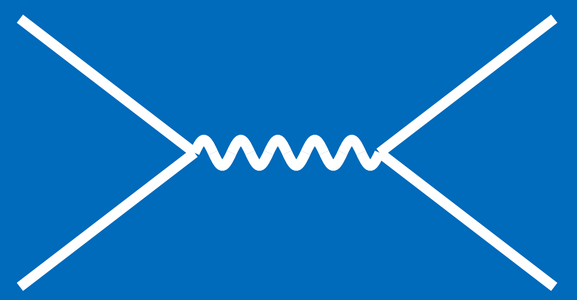
$$n_\chi(t) \;\simeq\; 0.70\pi^2 \mathrm{Br}_\chi g_{*s}^{\mathrm{reh}} \left(\frac{T_{\mathrm{reh}}}{m_\Phi}\right)^4 \left(\frac{a_0}{a(t)}\right)^3 T_*^3$$

$$\Omega_\chi h^2 \;\simeq\; 0.1 \left(\frac{\mathrm{Br}_\chi}{5.5\times 10^{-4}}\right)\left(\frac{m_{\mathrm{DM}}}{1\,\mathrm{MeV}}\right)\left(\frac{T_{\mathrm{reh}}}{10^{10}\,\mathrm{GeV}}\right)\left(\frac{3\times 10^{13}\,\mathrm{GeV}}{m_\Phi}\right)$$

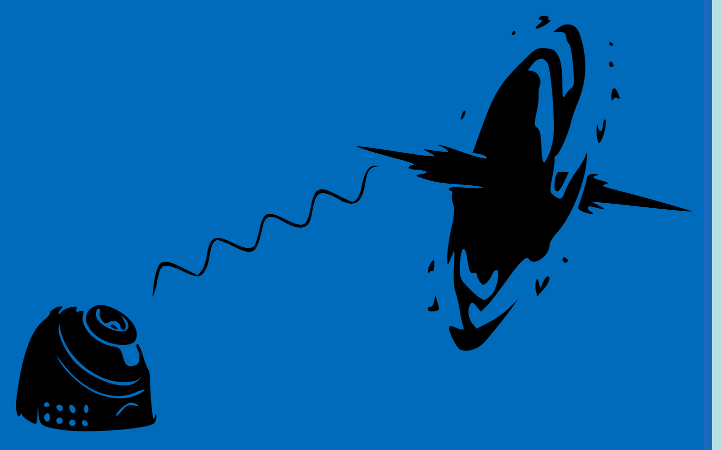
1. Reheating



2. Freeze-in



3. Lyman-α



4. The end?

Out-of-equilibrium decay chain $\Phi \to A \to \chi$

$$\Omega_\chi h^2 \simeq 0.1 \left(\frac{\mathrm{Br}_\chi}{5.5\times 10^{-4}}\right)\left(\frac{m_{\mathrm{DM}}}{1\,\mathrm{MeV}}\right)\left(\frac{T_{\mathrm{reh}}}{10^{10}\,\mathrm{GeV}}\right)\left(\frac{3\times 10^{13}\,\mathrm{GeV}}{m_\Phi}\right)$$

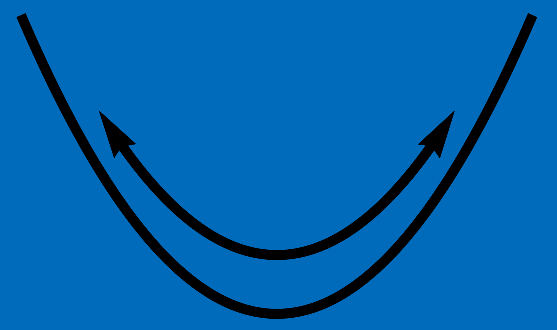
$$f_\chi(p,t)\, d^3\boldsymbol{p} = \frac{24\pi^3\sqrt{10 g_{*s}^{\mathrm{reh}}}\,\mathrm{Br}_\chi \mathrm{Br}_A \Gamma_A M_P}{5 g_A m_A^2}\left(\frac{T_{\mathrm{reh}}}{m_\phi}\right)^2 \mathcal{F}(q,\mathcal{R})\left(\frac{a_0}{a(t)}\right)^3 T_*^3 d^3\boldsymbol{q}$$

$$\mathcal{F}(q,\mathcal{R}) = q^{-2}\int_0^{\mathcal{R}} dy\, y^2 \int_{\left|q-\frac{y^2}{q}\right|}^{\infty} \frac{z\,dz}{\sqrt{q^2+4y^2}}\, \bar{f}_R(z) \simeq \begin{cases} \bar{f}_{\mathrm{D,NR}}(q)\,, & \mathcal{R}\gg 1\,,\\ \frac{\mathcal{R}^3}{3}\,\bar{f}_{\mathrm{D,R}}(q)\,, & \mathcal{R}\ll 1\,. \end{cases}$$

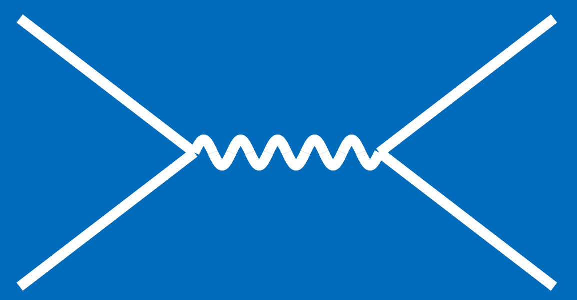
$$\mathcal{R} \equiv \left(\frac{g_{*s}^{\mathrm{reh}}}{g_{*s}^{\mathrm{dec}}}\right)^{1/3} \frac{m_A T_{\mathrm{reh}}}{m_\phi T_{\mathrm{dec}}}$$

(for thermal decays, see also K. Petraki and A. Kusenko, Phys. Rev. D 77 (2008) 065014; K. J. Bae, A. Kamada, S. P. Liew and K. Yanagi, JCAP 01 (2018) 054)

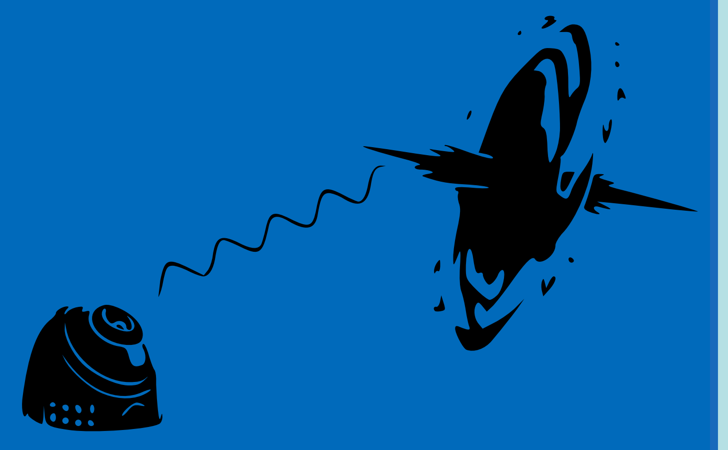
1. Reheating



2. Freeze-in

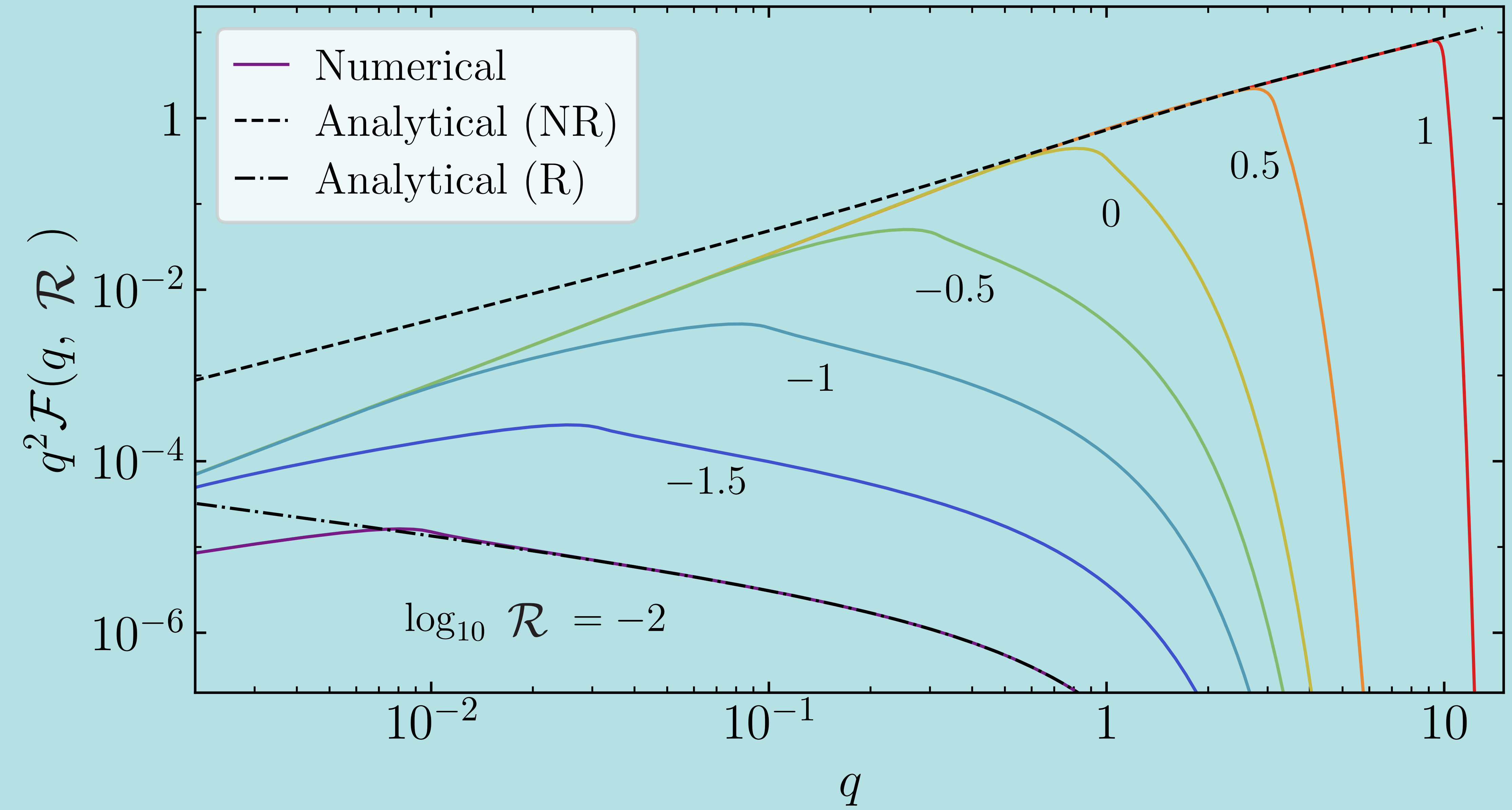


3. Lyman-α

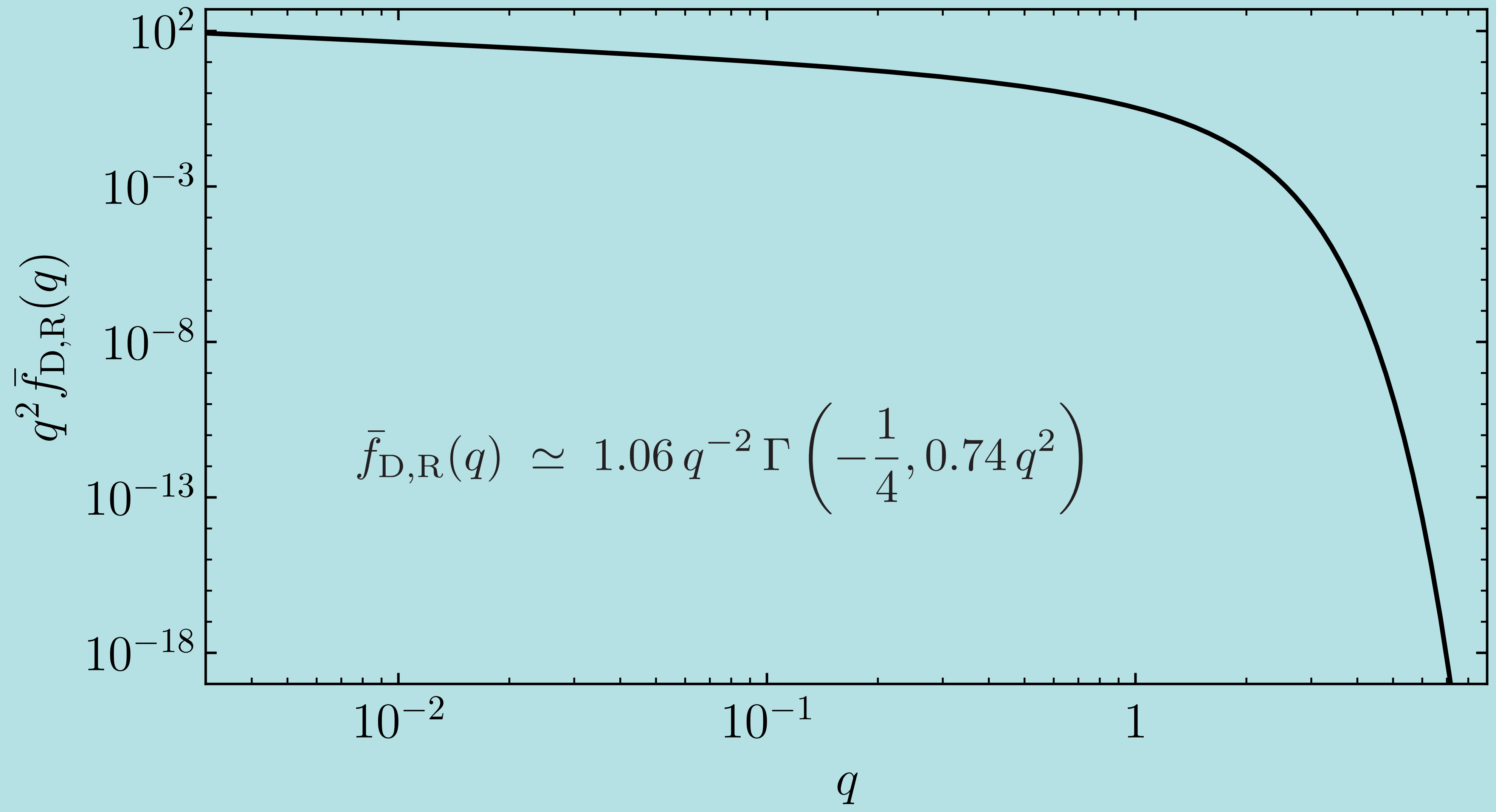


4. The end?

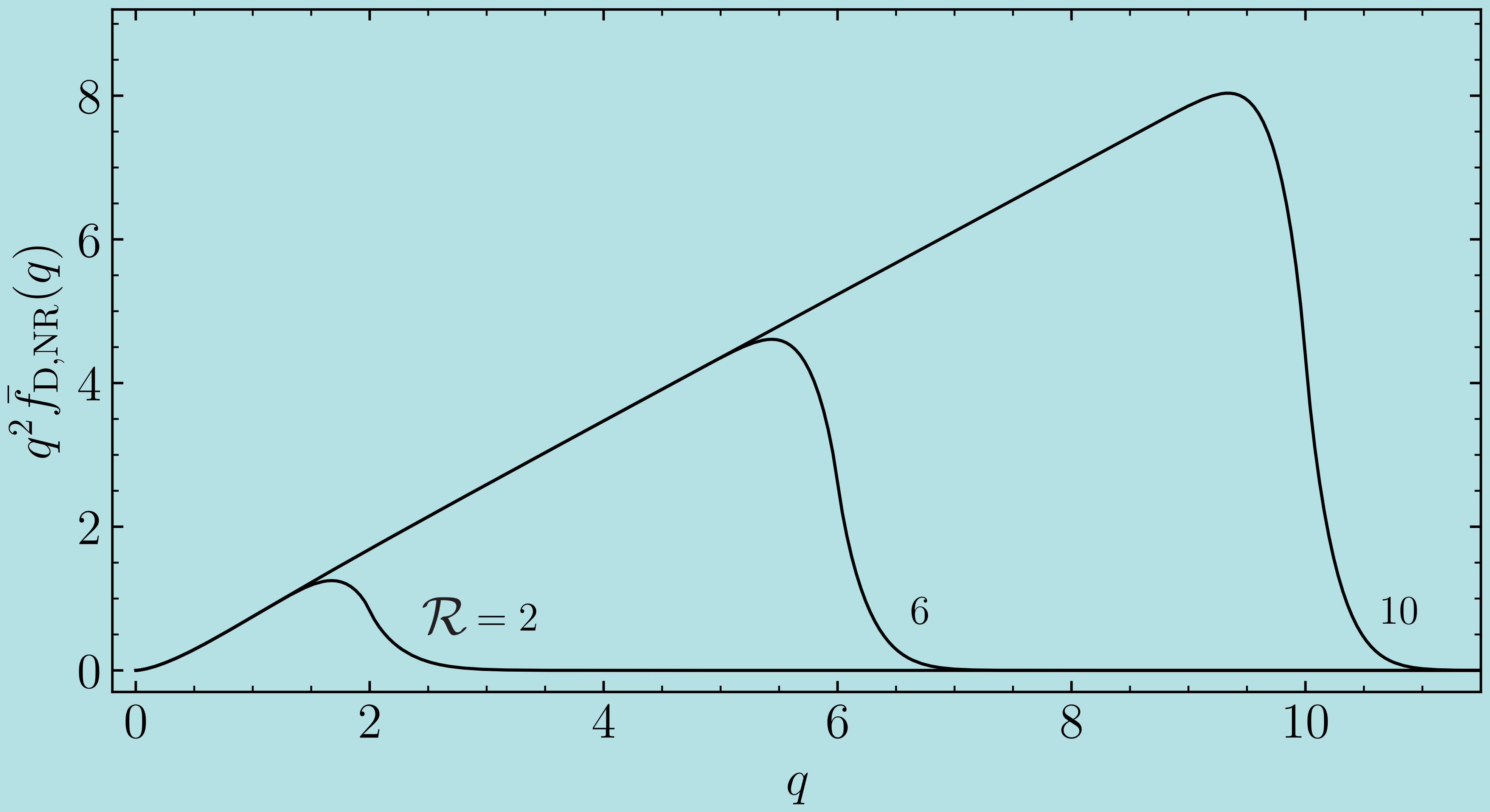
Out-of-equilibrium decay chain $\Phi \to A \to \chi$



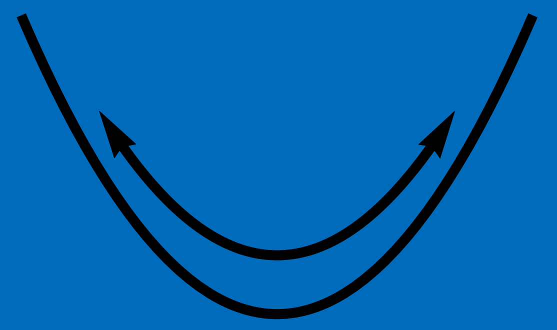
R



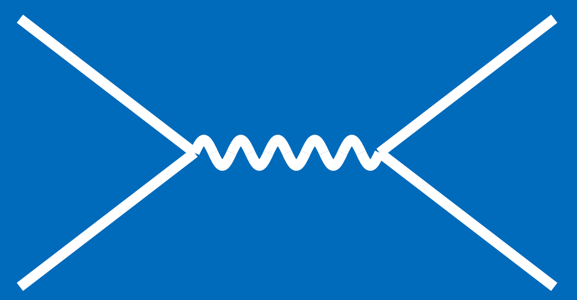
NR



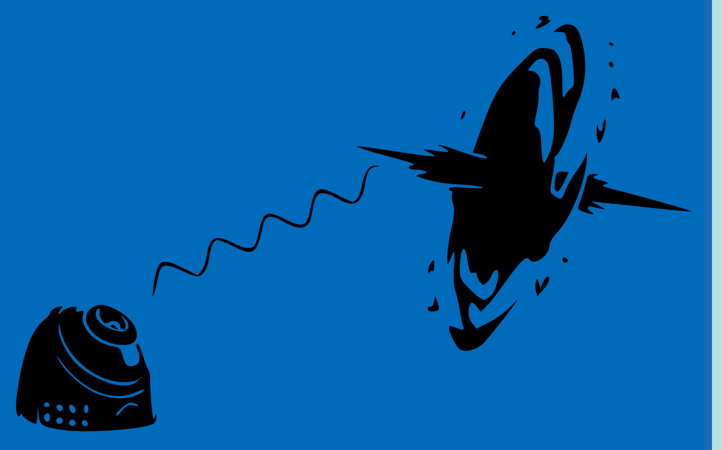
1. Reheating



2. Freeze-in

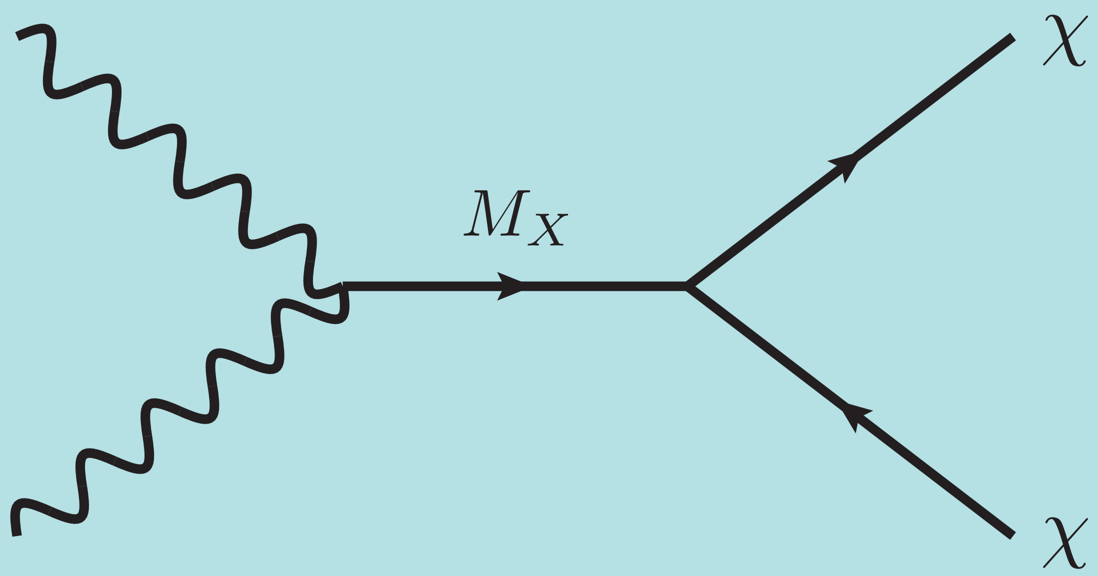


3. Lyman-α

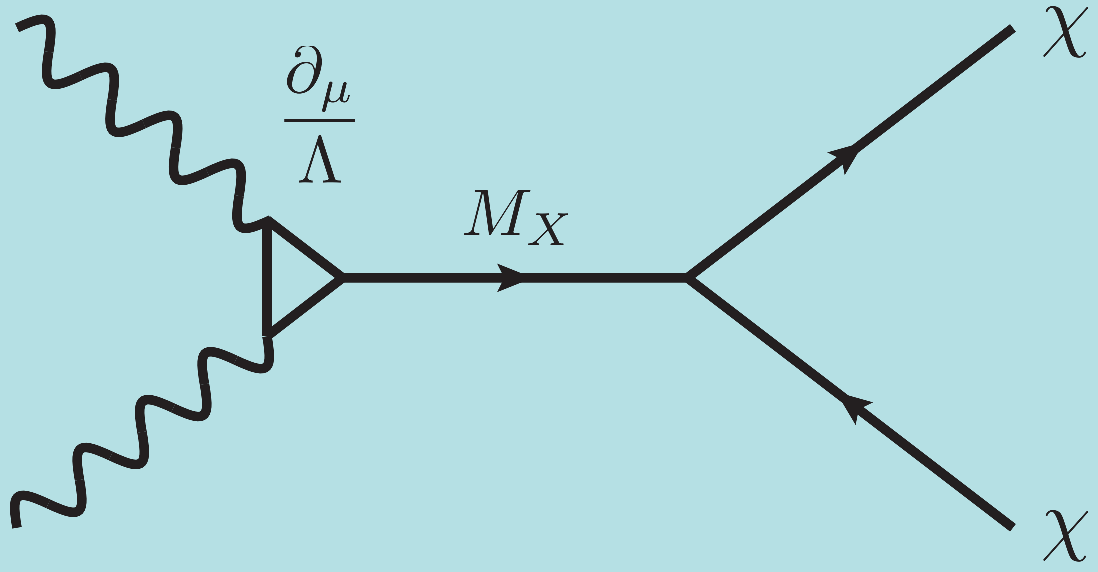


4. The end?

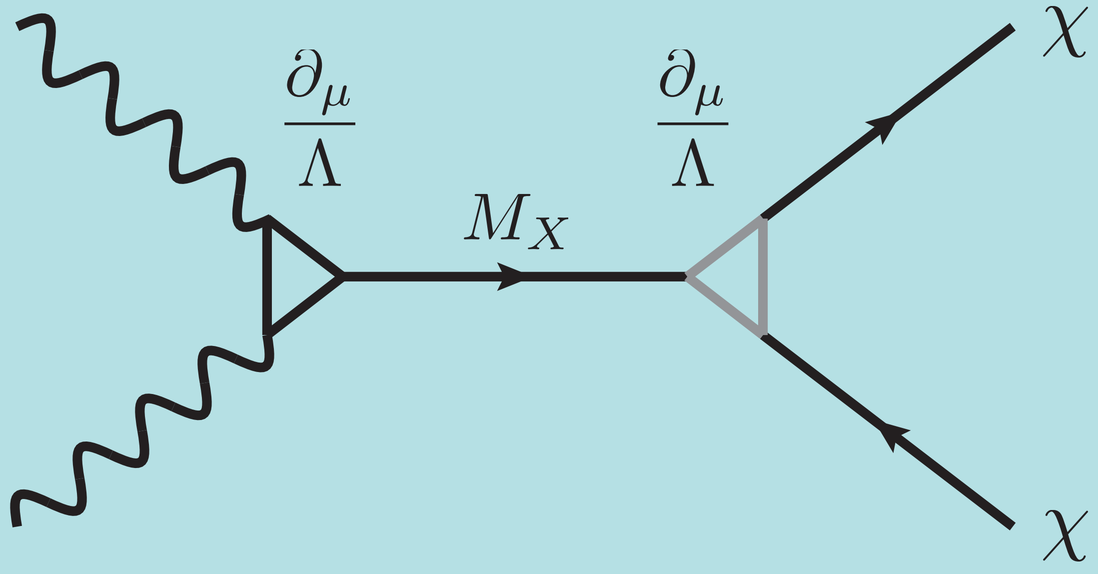
Freezing-in through scatterings



$$\sigma(s) = \frac{s}{M_X^4}$$



$$\sigma(s) = \frac{s^2}{\Lambda^2 M_X^4}$$



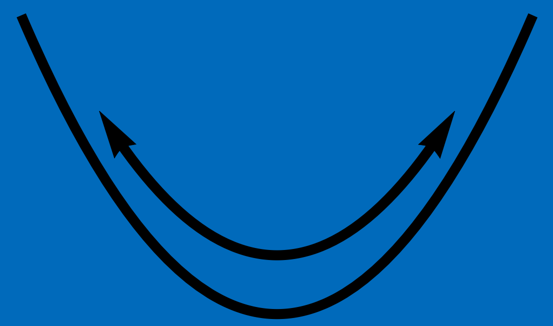
$$\sigma(s) = \frac{s^3}{\Lambda^4 M_X^4}$$

Production at $t \leq t_{\rm reh}$ if

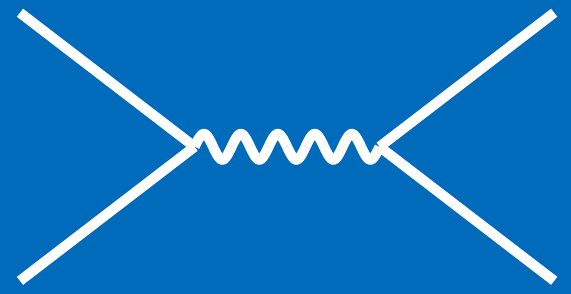
$$\sigma(s) = \frac{s^{n/2}}{\Lambda^{n+2}}$$

with $n > -1$

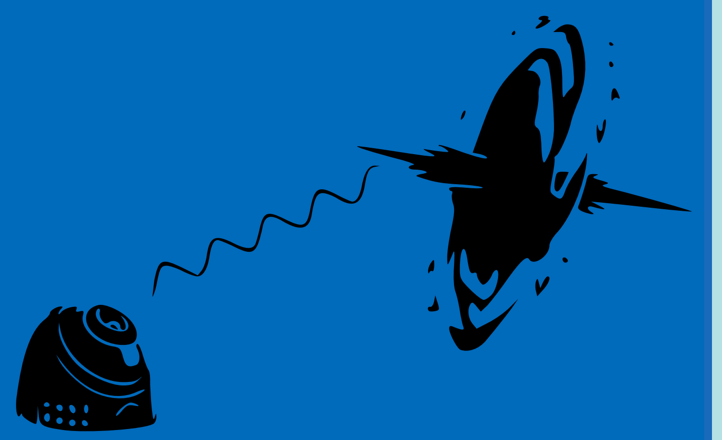
1. Reheating



2. Freeze-in



3. Lyman-α



4. The end?

Freezing-in through scatterings (examples)

- $n = 0:$ Low scale susy gravitino, $\sigma \propto M_P^{-2}$. Axino, $\sigma \propto f_a^{-2}$

  V. Rychkov, A. Strumia, Phys. Rev. D 75 (2007) 075011 ; A. Strumia, JHEP 06 (2010) 036

- $n = 2:$ $SO(10) \rightarrow SU(4) \times SU(2)_L \times U(1)_R \rightarrow \mathrm{SM}$ (heavy DM), $\sigma \propto s/M_{\mathrm{int}}^4$

  Light spin-2 mediator (light DM), $\sigma \propto s/M_P^4$

  Y. Mambrini et al., Phys. Rev. Lett. 110 (2013) 241306; N. Bernal et al., Phys. Rev. D 97 (2018) 115020

- $n = 4:$ Non-susy spin-3/2 DM + sterile neutrino, $\sigma(s) \propto (s/m_{3/2} m_R M_P)^2$

- $n = 6:$ High scale susy gravitino (heavy DM), $\sigma \propto s^3/(m_{3/2} M_P)^4$

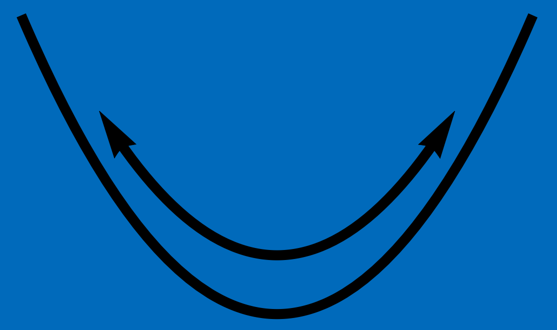
  Heavy spin-2 mediator (light DM), $\sigma \propto s^3/(m_{\tilde{h}} M_P)^4$

  K. Benakli et al., Phys. Rev. D 95 (2017) 095002; N. Bernal et al., Phys. Rev. D 97 (2018) 115020

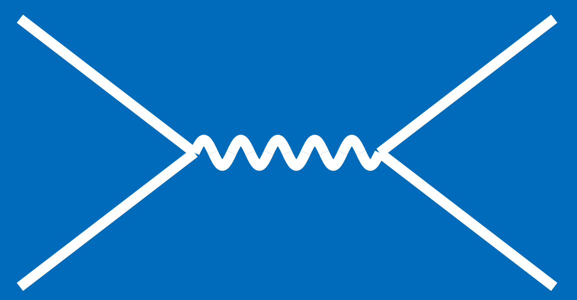
- $n > 6:$ Vector non-Abelian DM with heavy Z'

  G. Bhattacharyya, M. Dutra, Y. Mambrini, M. Pierre, Phys. Rev. D 98 (2018) 035038

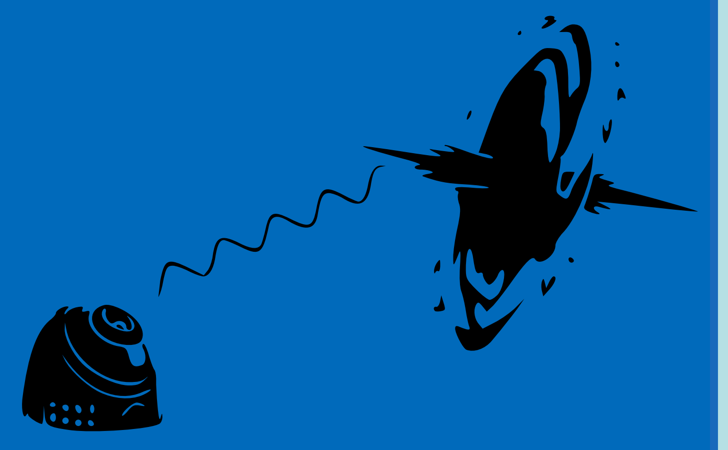
1. Reheating



2. Freeze-in



3. Lyman-α



4. The end?

Thermal freeze-in

$n < 6:$

$$\Omega_\chi^{(n)} h^2 \simeq \frac{g_A g_B g_\psi g_\chi \sqrt{c}\, 2^{n+3} \Gamma(\frac{n}{2}+3)^2 \zeta(\frac{n}{2}+3)^2 \mathcal{S}(n)}{(6-n)(n+4)} \left(\frac{106.75}{g_{*s}^{\rm reh}}\right)^{3/2} \left(\frac{T_{\rm reh}}{\Lambda}\right)^{n+1} \left(\frac{10^{16}\,{\rm GeV}}{\Lambda}\right) \left(\frac{m_{\rm DM}}{1\,{\rm keV}}\right)$$

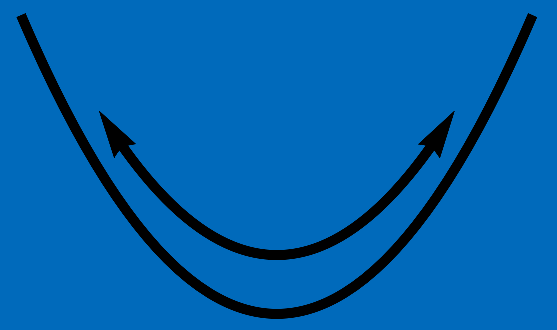
$n = 6:$

$$\Omega_\chi^{(6)} h^2 = g_A g_B g_\psi g_\chi \sqrt{c}\, \mathcal{S}(6) \left(\frac{106.75}{g_{*s}^{\rm reh}}\right)^{3/2} \left(\frac{m_{\rm DM}}{1.2\,{\rm keV}}\right) \left(\frac{T_{\rm reh}}{10^6\,{\rm GeV}}\right)^{7} \left(\frac{10^8\,{\rm GeV}}{\Lambda}\right)^{8} \ln\left(\frac{T_{\rm max}}{T_{\rm reh}}\right)$$

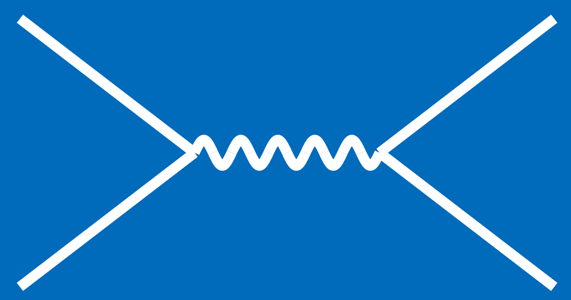
$n > 6:$

$$\Omega_\chi^{(n)} h^2 \simeq \frac{g_A g_B g_\psi g_\chi \sqrt{c}\, 2^{n+3} \Gamma(\frac{n+4}{2}) \Gamma(\frac{n+6}{2})}{n-6} \left(\frac{106.75}{g_{*s}^{\rm reh}}\right)^{3/2} \times \left(\frac{T_{\rm max}}{\Lambda}\right)^{n+1} \left(\frac{T_{\rm reh}}{T_{\rm max}}\right)^{7} \left(\frac{10^{16}\,{\rm GeV}}{\Lambda}\right) \left(\frac{m_{\rm DM}}{1.8\,{\rm keV}}\right)$$

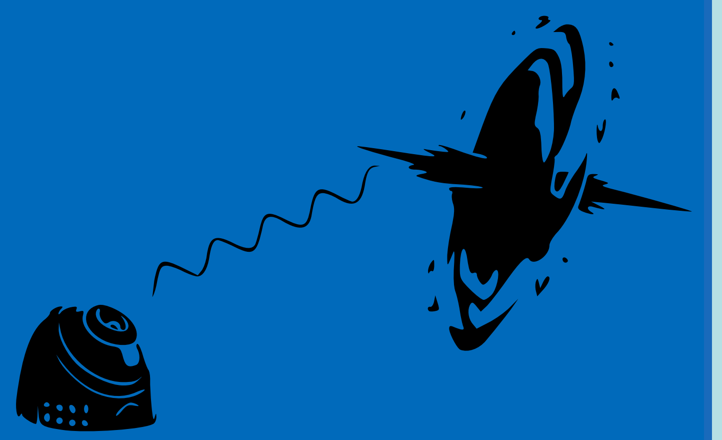
1. Reheating



2. Freeze-in



3. Lyman-α



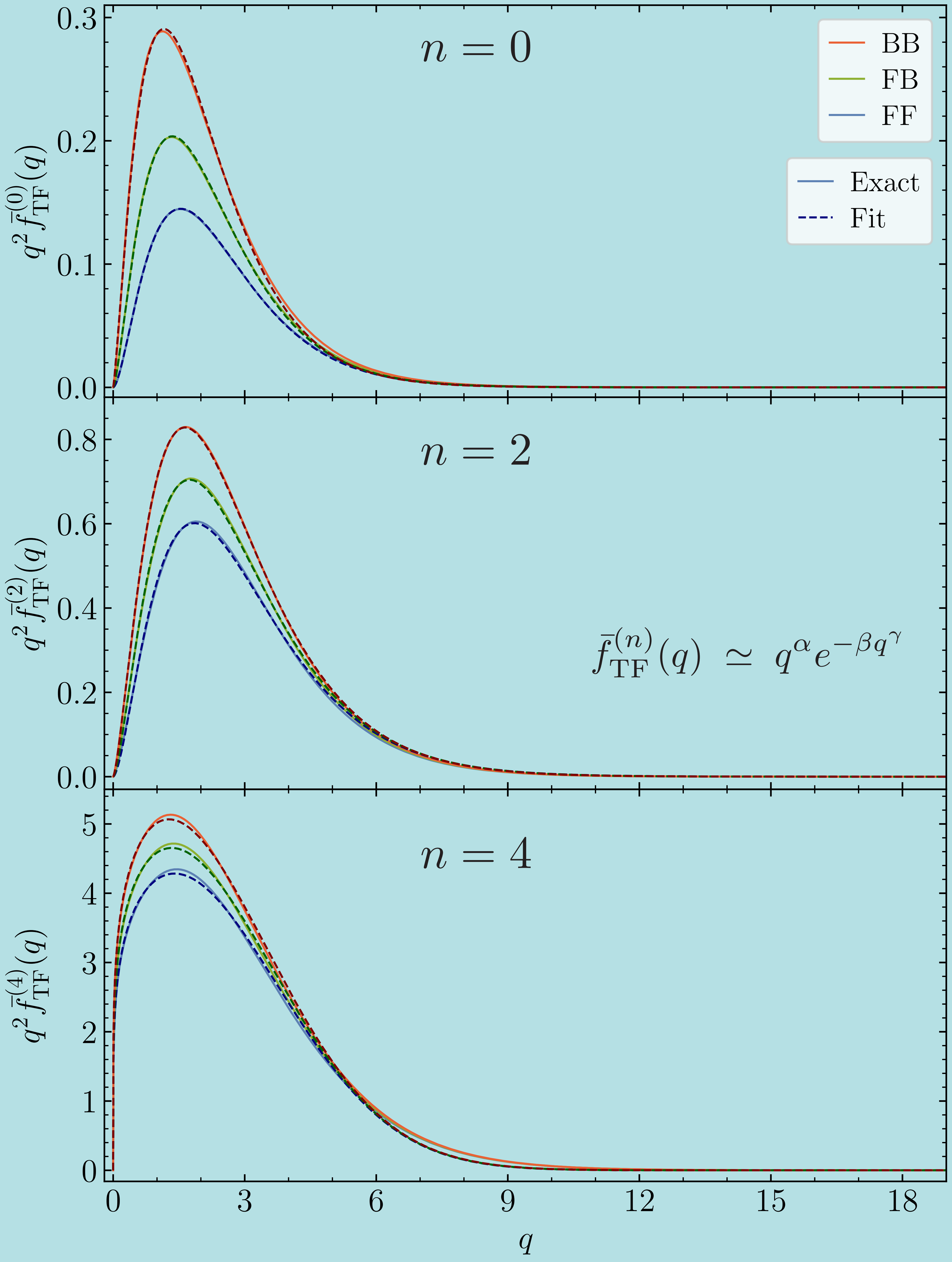
4. The end?

Thermal freeze-in

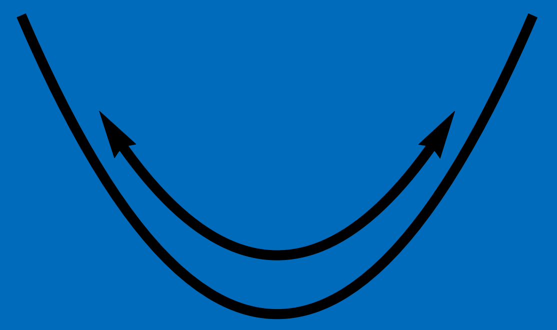
$n < 6:$

$$f_\chi(p,t)\,d^3\boldsymbol{p} \;\simeq\; \frac{3\cdot 2^{n+6}\Gamma(\frac{n+4}{2})g_A g_B g_\psi M_P T_{\rm reh}^{n+1}}{5(2\pi)^3\Lambda^{n+2}} \times \left(\frac{6c}{g_{*s}^{\rm reh}}\right)^{1/2}\left(\frac{a_0}{a(t)}\right)^3 T_*^3\,\bar{f}_{\rm TF}^{(n)}(q)\,d^3\boldsymbol{q}$$

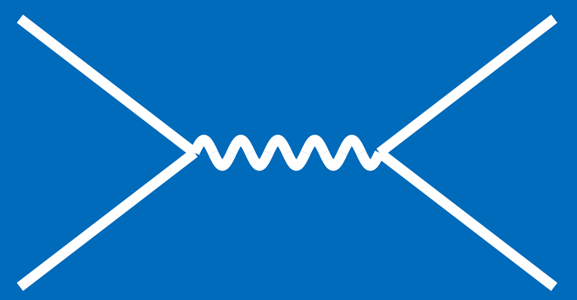
$$T_* = \left(\frac{g_{*s}^0}{g_{*s}^{\rm reh}}\right)^{1/3} T_0$$



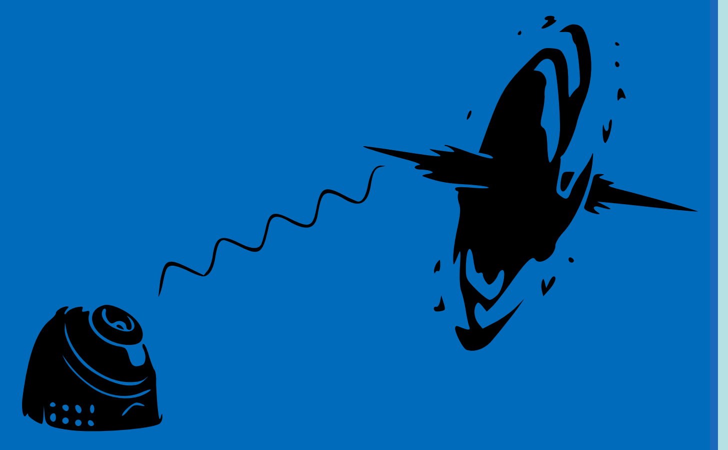
1. Reheating



2. Freeze-in



3. Lyman-α

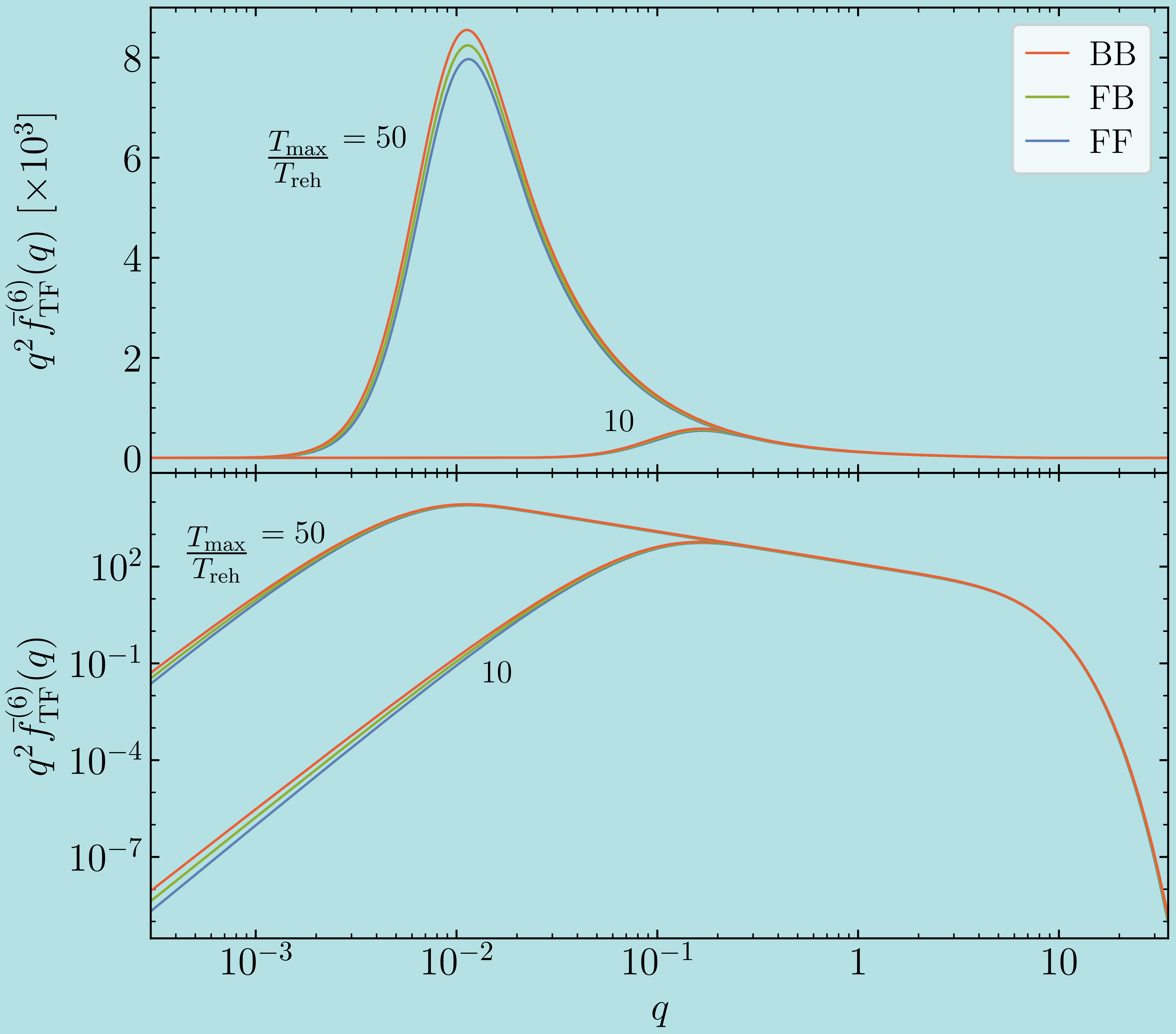


4. The end?

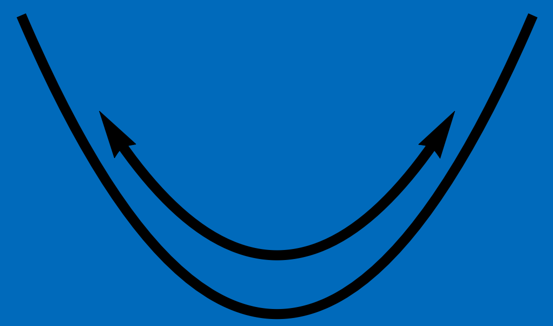
Thermal freeze-in

$n \geq 6$:

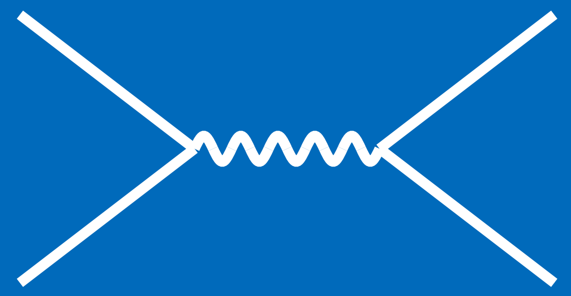
$$\bar{f}^{(n)}_{\rm TF}(q) \simeq q^{\frac{3}{5}(1-n)}\left[\Gamma\left(\frac{11}{10}n-\frac{3}{5},q\right) - \Gamma\left(\frac{11}{10}n-\frac{3}{5},q\left(\frac{T_{\rm max}}{T_{\rm reh}}\right)^{5/3}\right)\right]$$



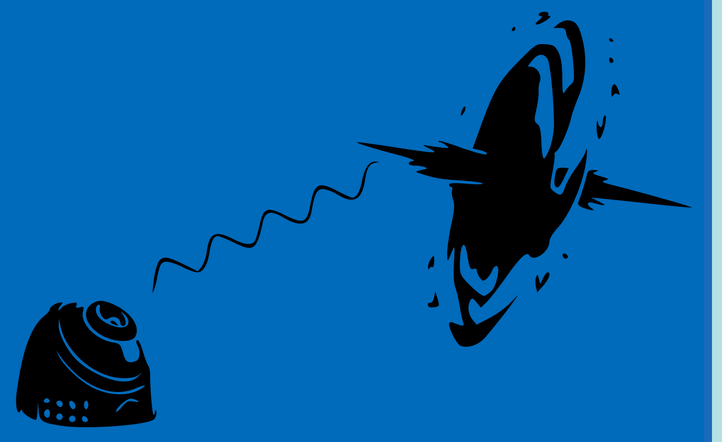
1. Reheating



2. Freeze-in



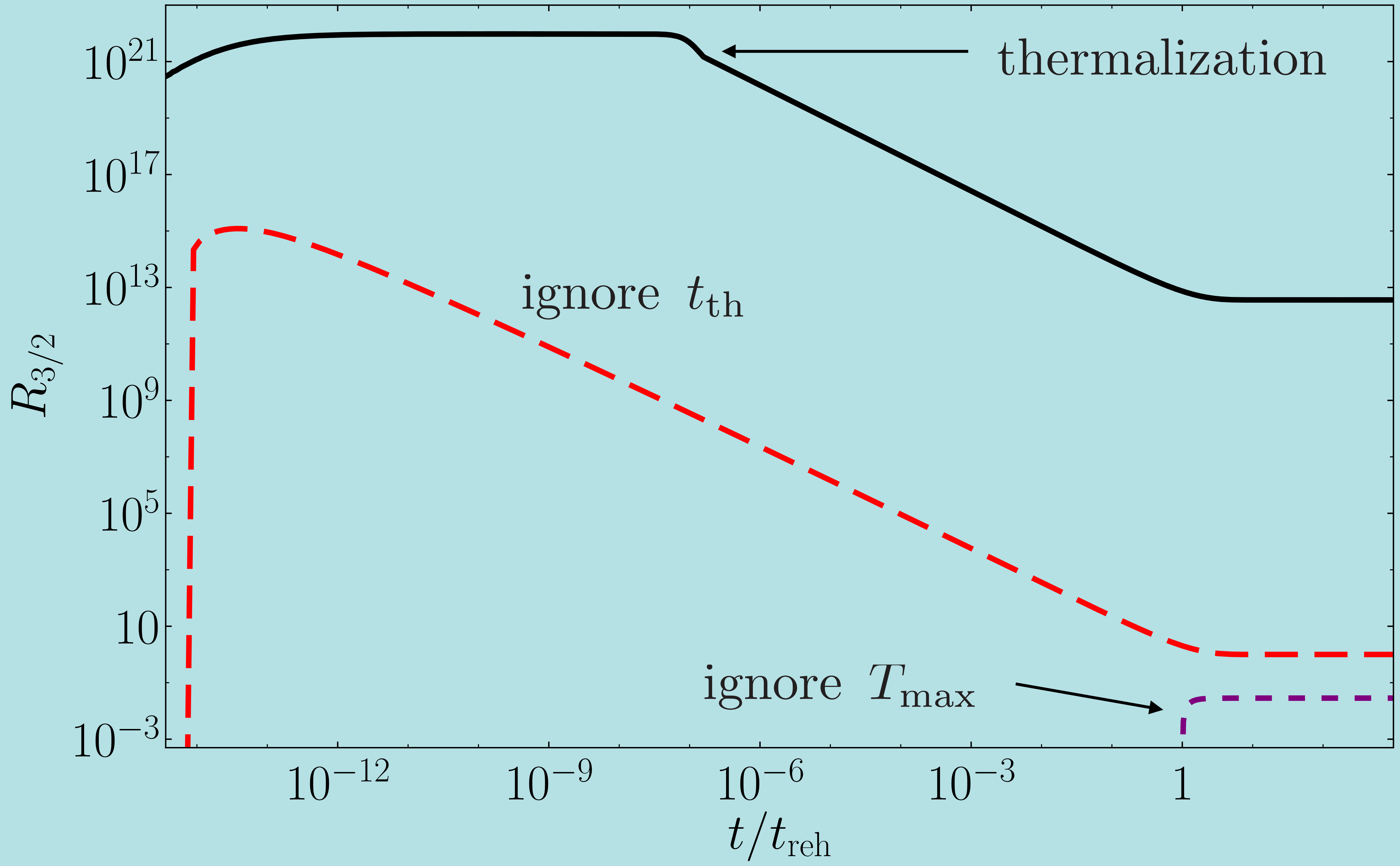
3. Lyman-α



4. The end?

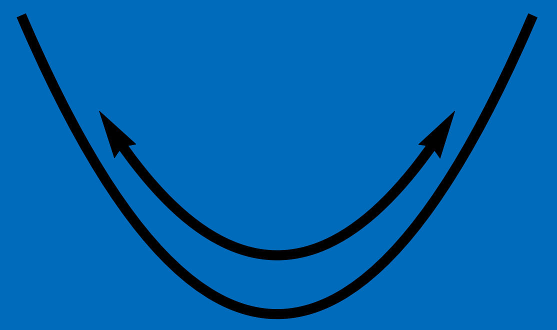
Non-thermal freeze-in

High scale supersymmetry breaking: $m_{\rm susy} \gg m_\Phi \gg m_{3/2} \qquad (n = 6)$

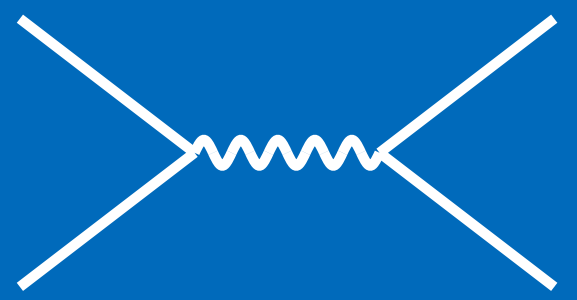


$$R_{3/2} \equiv \frac{n_{3/2}^{\rm NT}}{n_{3/2}^{\rm T}}$$

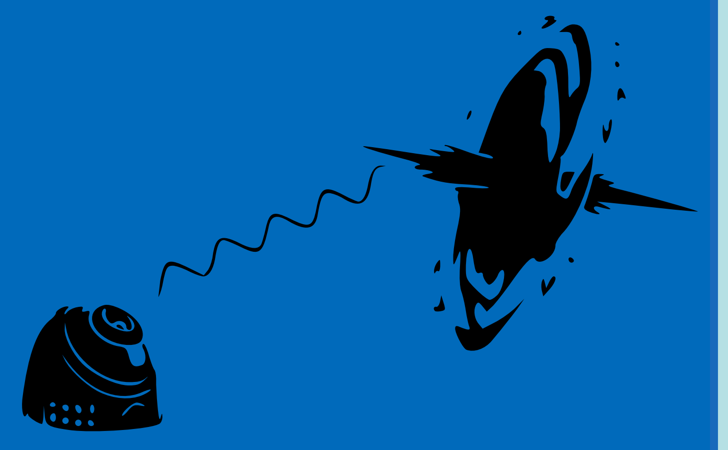
1. Reheating



2. Freeze-in

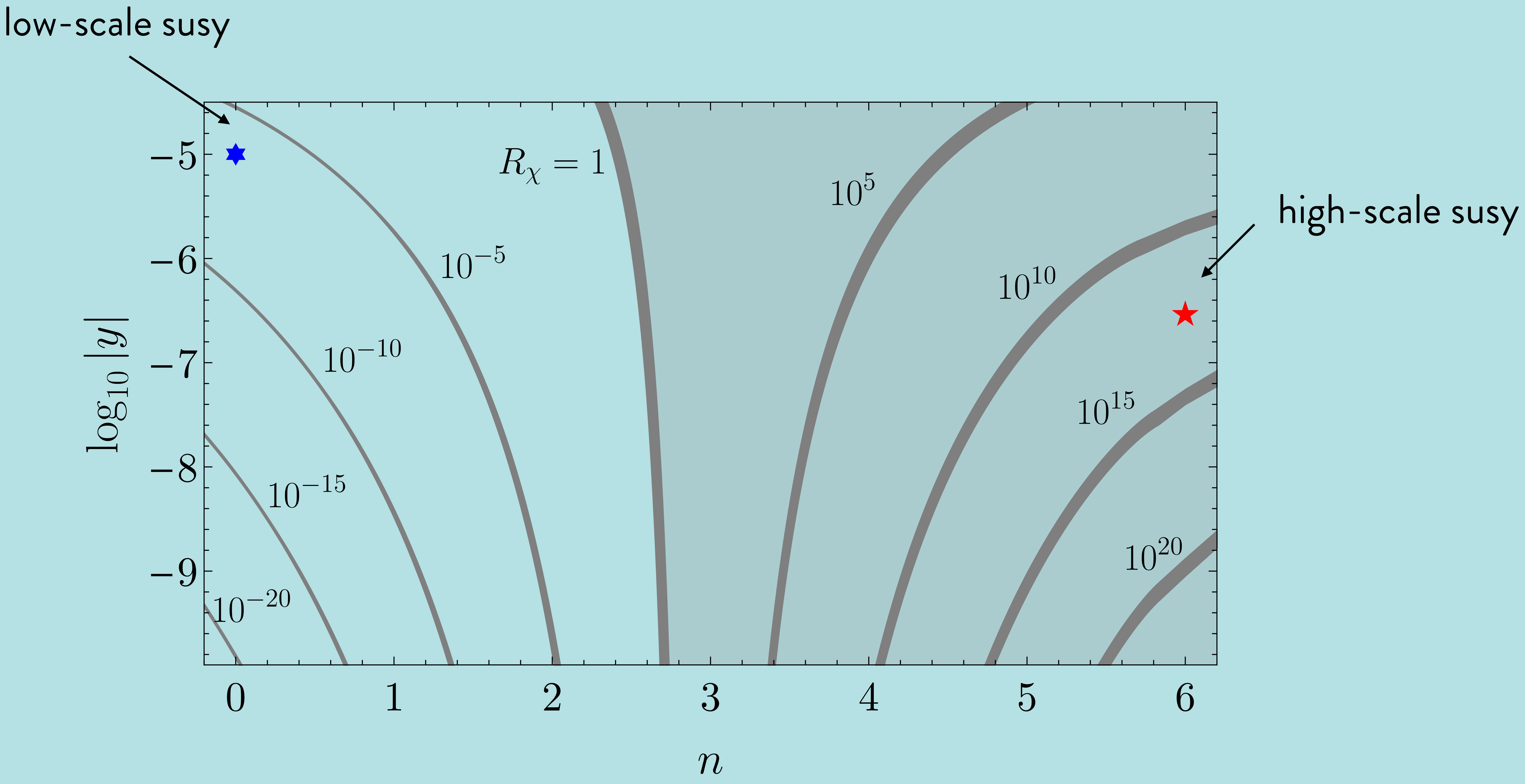


3. Lyman-α



4. The end?

Non-thermal freeze-in

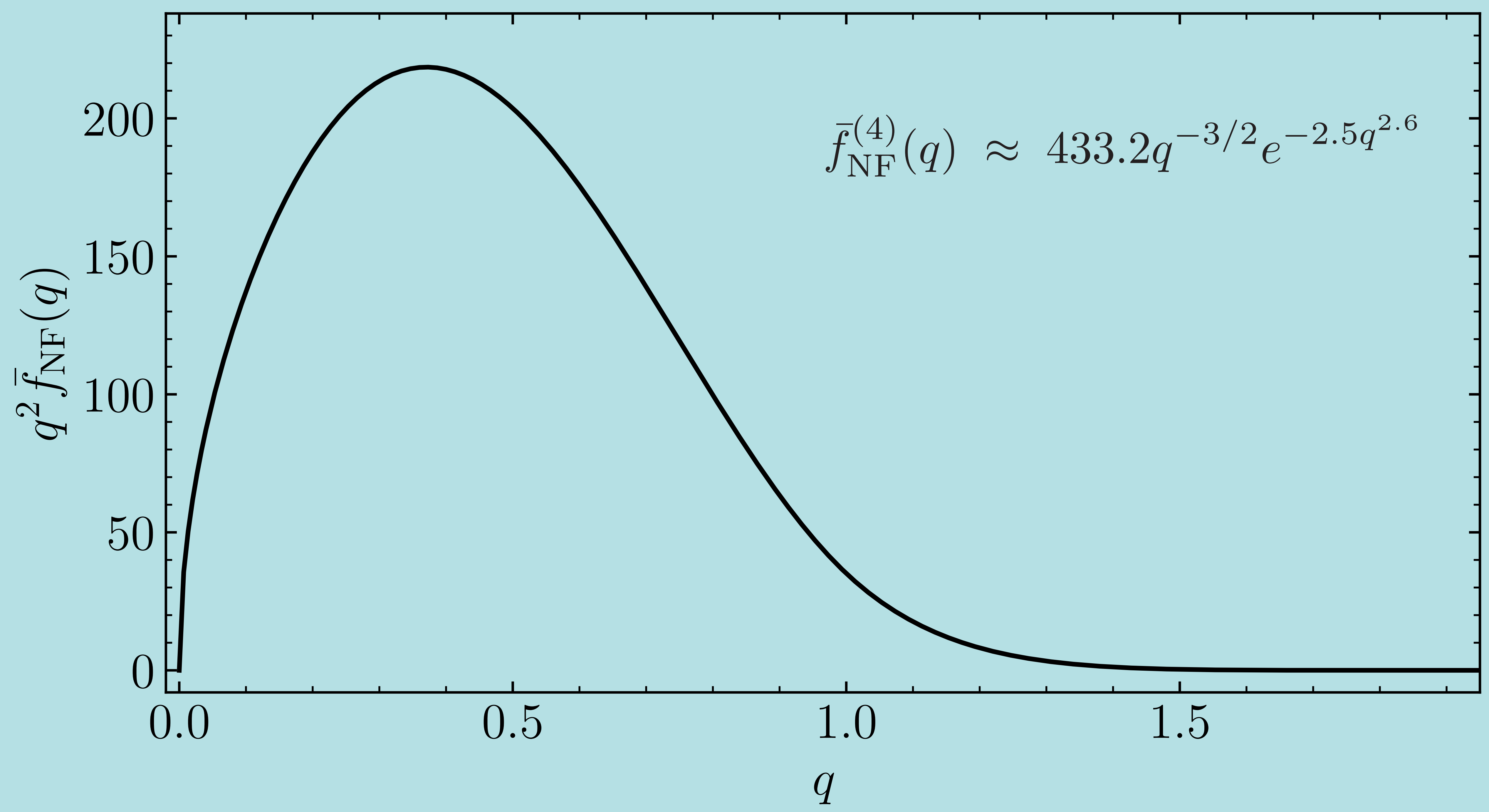


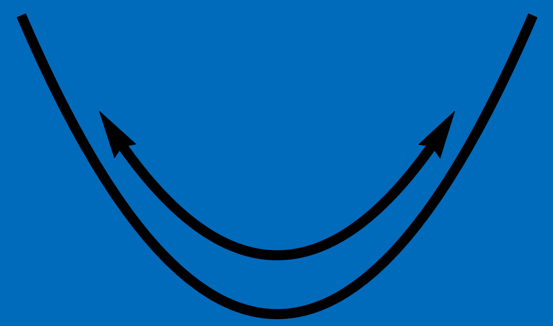
$$\Omega_\chi \propto \frac{m_\Phi^{n-2} M_P T_{\rm reh}^3}{M^{n+2}} \left(\Gamma_\Phi t_{\rm th}\right)$$

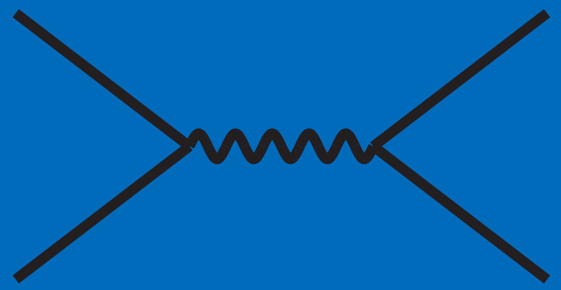
Non-thermal freeze-in $(n = 4)$

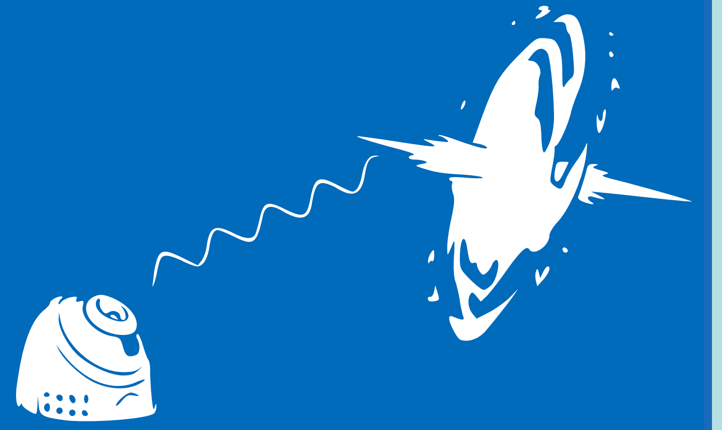
$$f_\chi(p,t)\,d^3\boldsymbol{p} \simeq \frac{256\pi^2 g_\psi}{15015\Lambda^6}\left(\frac{\pi^2 c g_{*s}^{\rm reh}}{24}\right)^{13/10}\left(\frac{\alpha_{\rm SM}^{16} T_{\rm reh}^{26} M_P^{13}}{m_\Phi^9}\right)^{1/5}\left(\frac{a_0}{a(t)}\right)^3 T_*^3\,\bar{f}_{\rm NF}^{(4)}(q)\,d^3\boldsymbol{q}$$

$$T_* = \frac{\alpha_{\rm SM}^{-32/15}}{2}\left(\frac{g_{*s}^0}{g_{*s}^{\rm reh}}\right)^{1/3}\left(\frac{\pi^2 c g_{*s}^{\rm reh}}{24}\right)^{2/15}\left(\frac{m_\Phi}{T_{\rm reh}}\right)^{7/15}\left(\frac{m_\Phi}{M_P}\right)^{16/15} T_0$$

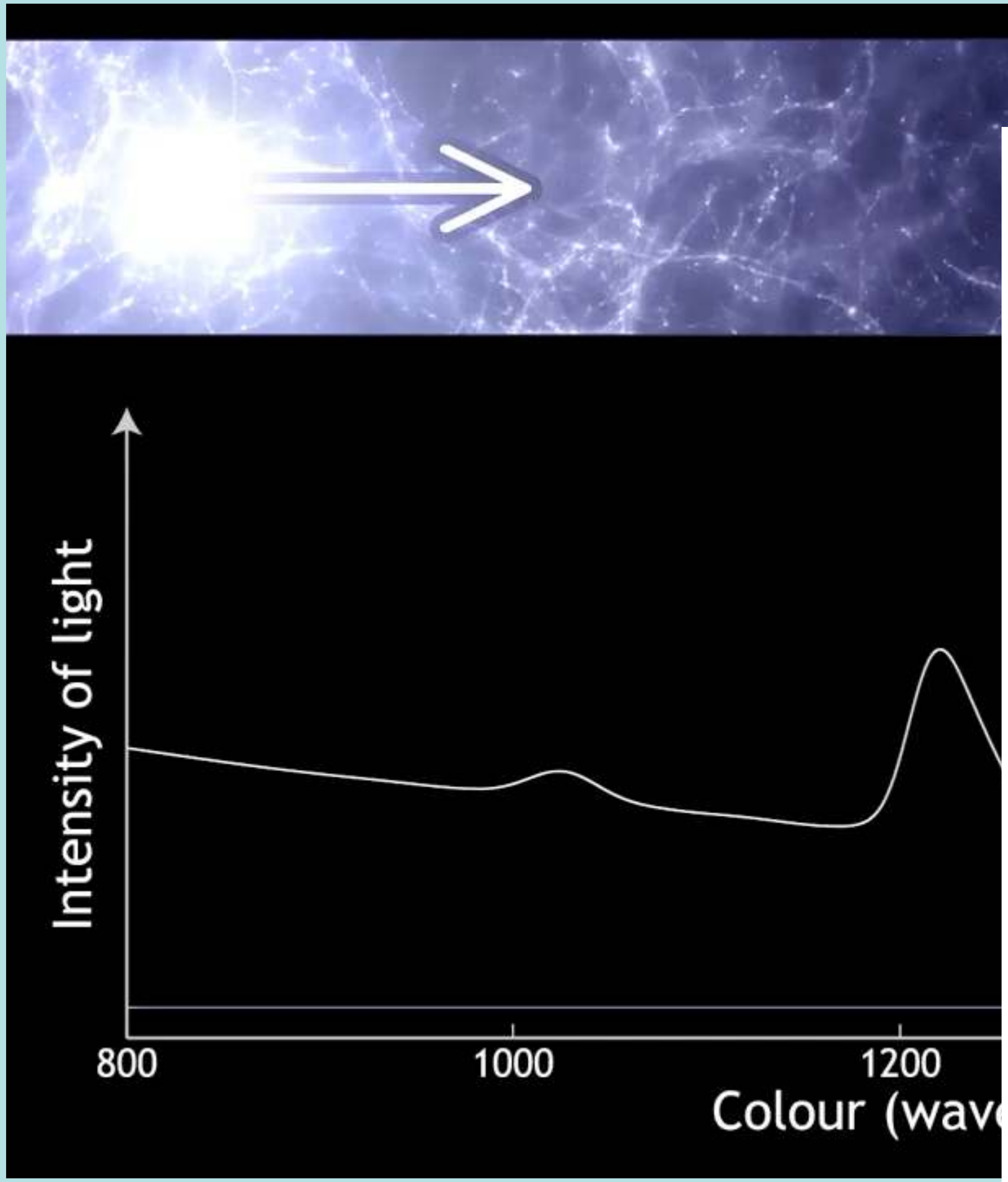


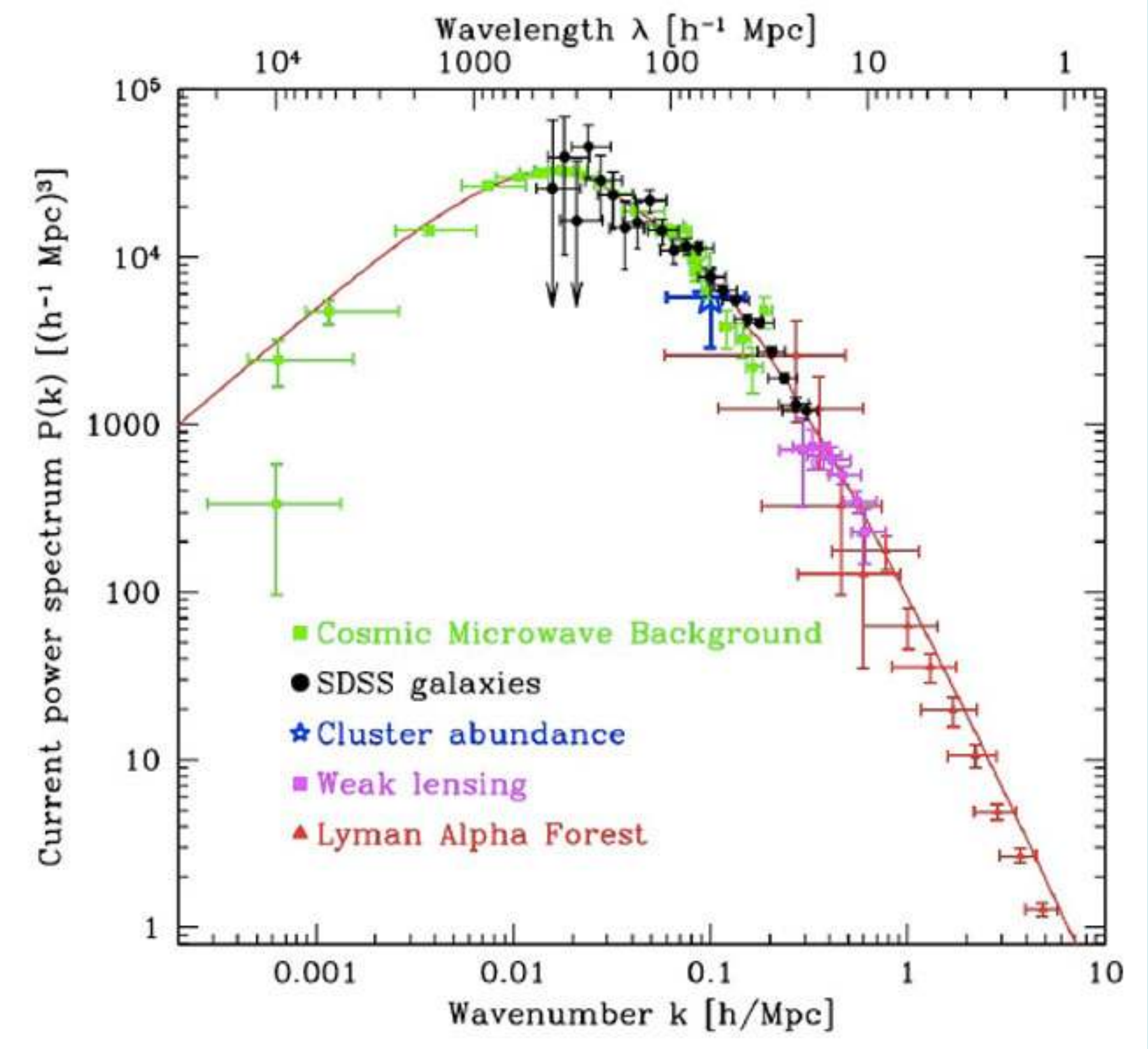






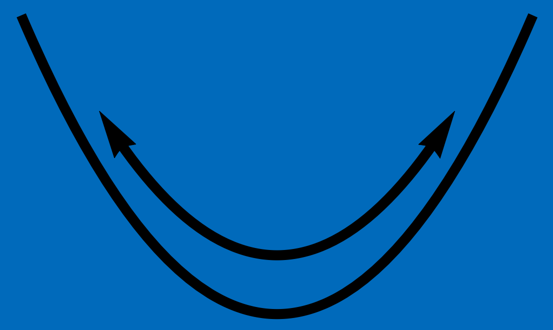
The Lyman-α constraint on Warm Dark Matter



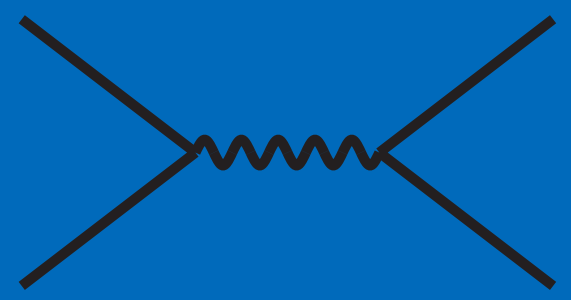


V. Iršič, et al., Mon.Not.Roy.Astron.Soc. 466 (2017) 4, 433
N. Palanque-Delabrouille et al., Astron. Astrphys. 559 (2013) A85; S. Chabanier et al., JCAP 07 (2019) 017

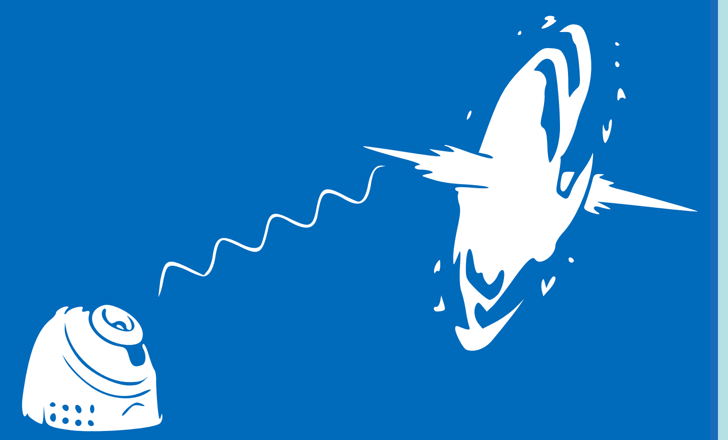
1. Reheating



2. Freeze-in



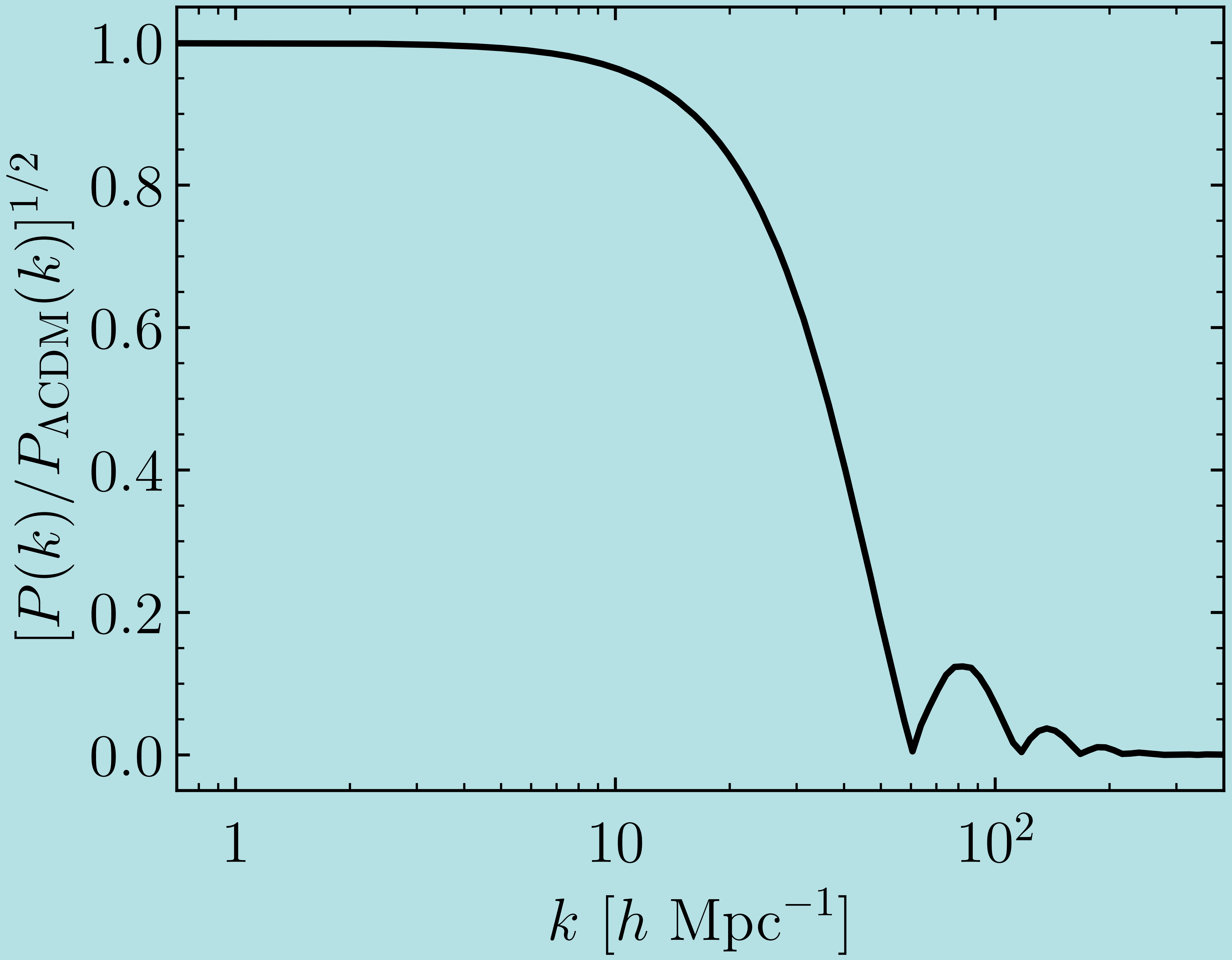
3. Lyman-α



4. The end?

The perturbed Boltzmann equation

$$f(\boldsymbol{x}, \boldsymbol{p}, \tau) = f_0(|\boldsymbol{p}|, \tau)[1 + \Psi(\boldsymbol{x}, \boldsymbol{p}, \tau)] \longrightarrow \ddot{\delta} + \mathcal{H}\dot{\delta} - \frac{3}{2}\mathcal{H}^2\left(1 - w\frac{10}{9}\frac{k^2}{\mathcal{H}^2}\right)\delta = 0$$



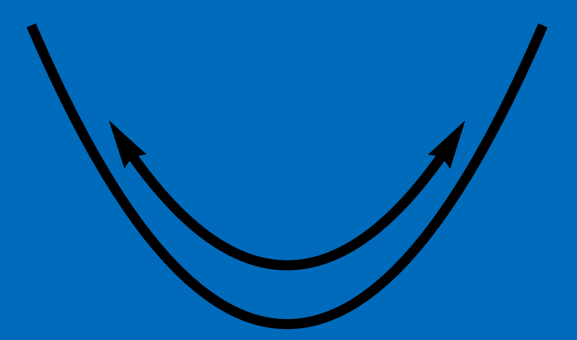
$$k_{\mathrm{FS}}(a) = \frac{9}{10}\frac{\mathcal{H}^2}{w}$$

$$k_H(a) \equiv \left[\int_0^a \frac{1}{k_{\mathrm{FS}}(\tilde{a})}\frac{d\tilde{a}}{\tilde{a}}\right]^{-1}$$

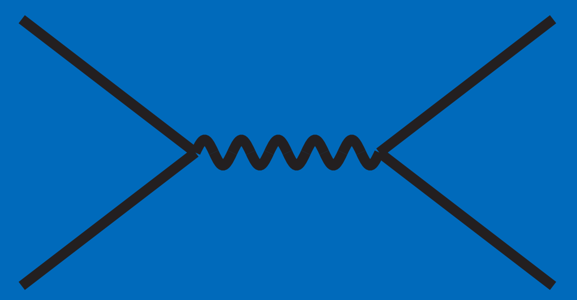
$$w \simeq \frac{\delta P}{\delta \rho} = \frac{T_*^2}{3m_{\mathrm{DM}}^2}\frac{\langle q^2\rangle}{a^2}$$

J. Lesgourgues, T. Tram, JCAP 09 (2011) 032

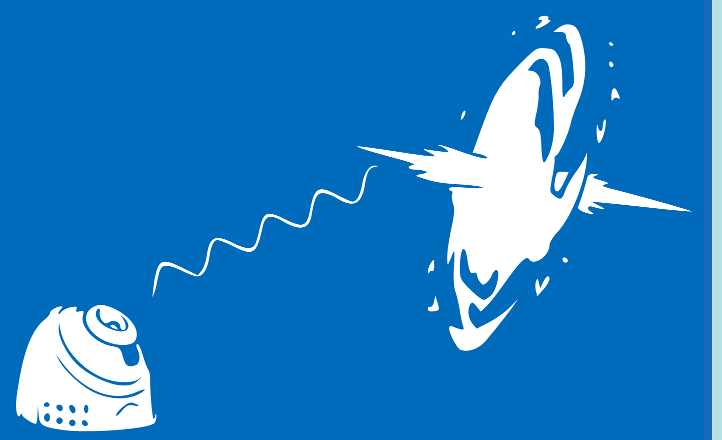
1. Reheating



2. Freeze-in

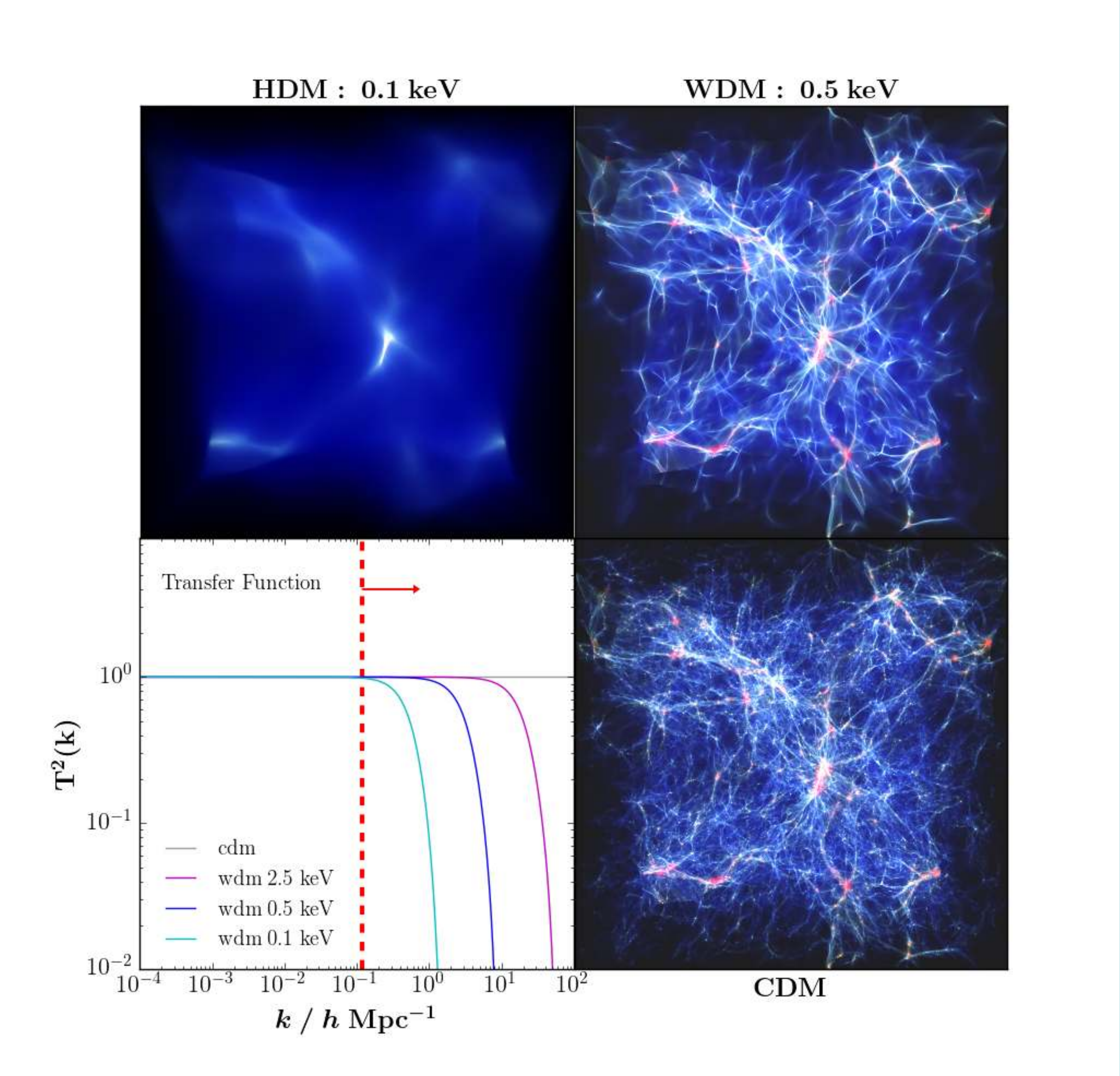


3. Lyman-α



4. The end?

The perturbed Boltzmann equation



$$\ddot{\delta} + \mathcal{H}\dot{\delta} - \frac{3}{2}\mathcal{H}^2 \left(1 - w\frac{10}{9}\frac{k^2}{\mathcal{H}^2}\right)\delta = 0$$

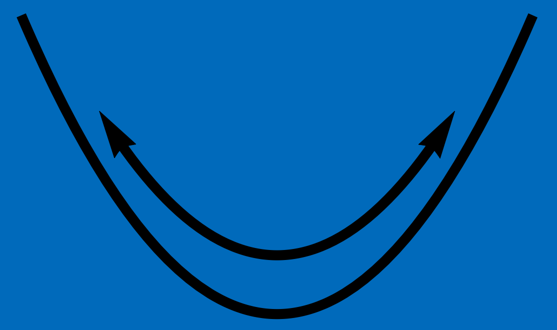
$$k_{\rm FS}(a) = \frac{9}{10}\frac{\mathcal{H}^2}{w}$$

$$k_H(a) \equiv \left[\int_0^a \frac{1}{k_{\rm FS}(\tilde{a})}\frac{d\tilde{a}}{\tilde{a}}\right]^{-1}$$

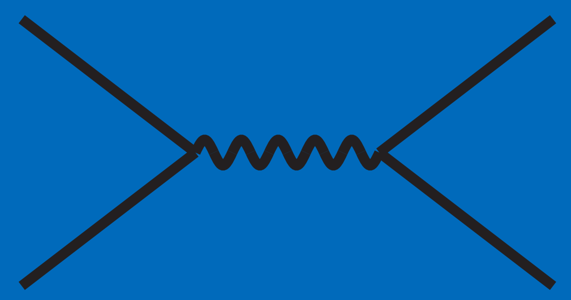
J. Baur et al., JCAP 08 (2016) 012

R. Murguia et al., JCAP 11 (2017) 046

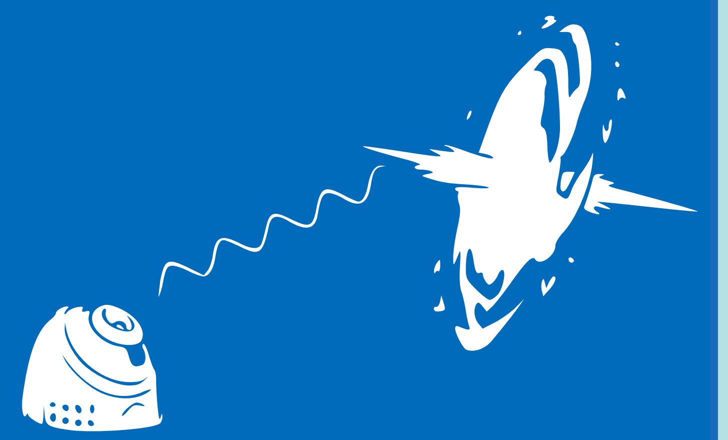
1. Reheating



2. Freeze-in



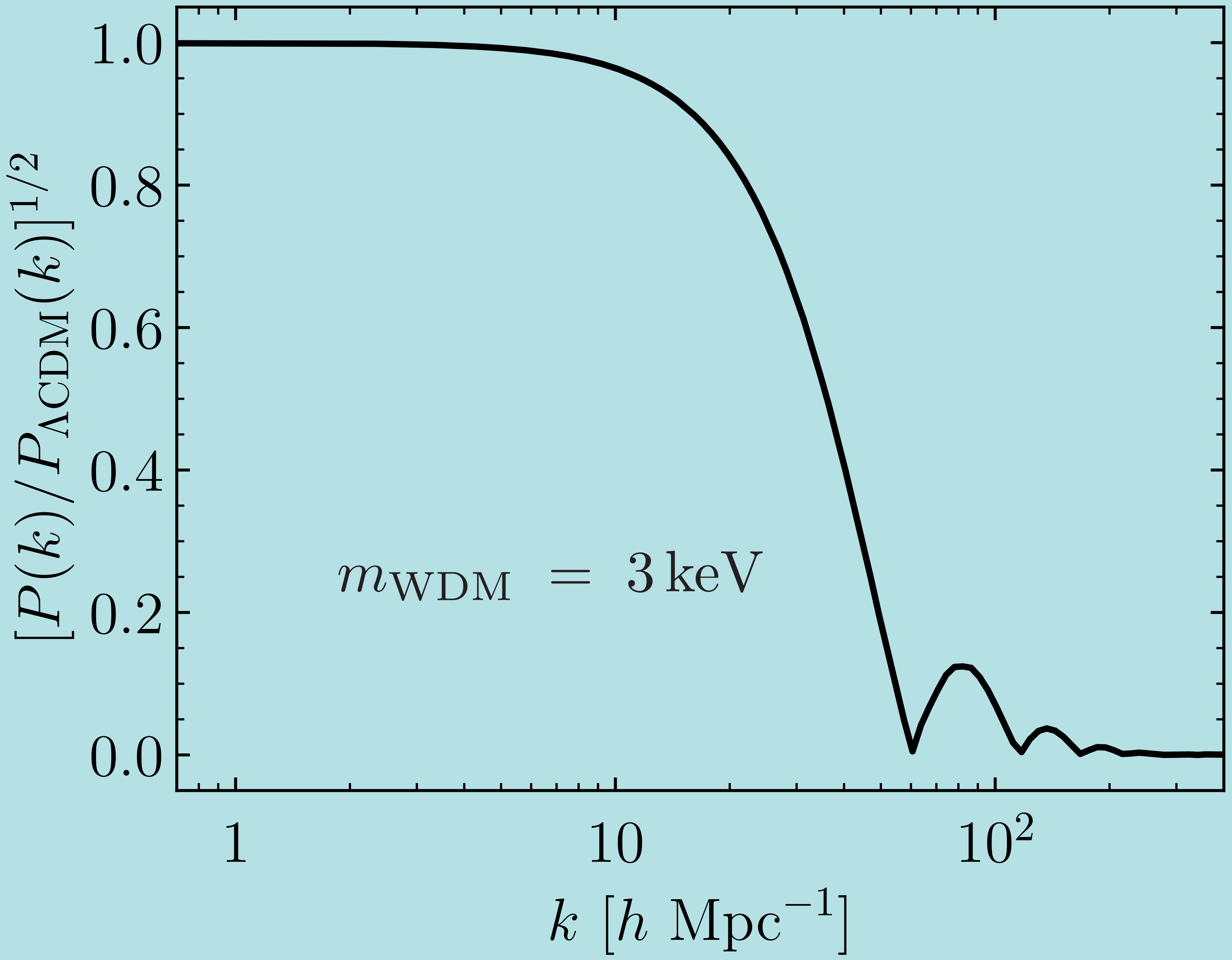
3. Lyman-α



4. The end?

The perturbed Boltzmann equation

$$f(\boldsymbol{x},\boldsymbol{p},\tau) = f_0(|\boldsymbol{p}|,\tau)[1+\Psi(\boldsymbol{x},\boldsymbol{p},\tau)] \longrightarrow \ddot{\delta} + \mathcal{H}\dot{\delta} - \frac{3}{2}\mathcal{H}^2\left(1 - w\frac{10}{9}\frac{k^2}{\mathcal{H}^2}\right)\delta = 0$$



$$k_{\mathrm{FS}}(a) = \frac{9}{10}\frac{\mathcal{H}^2}{w}$$

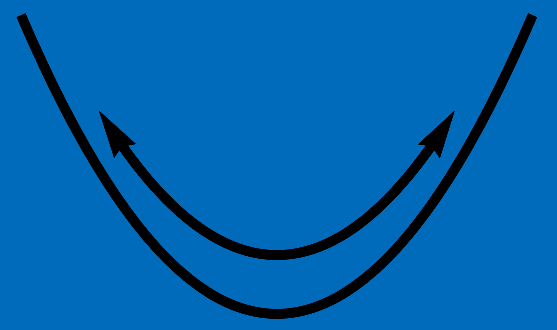
$$k_H(a) \equiv \left[\int_0^a \frac{1}{k_{\mathrm{FS}}(\tilde{a})}\frac{d\tilde{a}}{\tilde{a}}\right]^{-1}$$

$$w \simeq \frac{\delta P}{\delta\rho} = \frac{T_*^2}{3m_{\mathrm{DM}}^2}\frac{\langle q^2\rangle}{a^2}$$

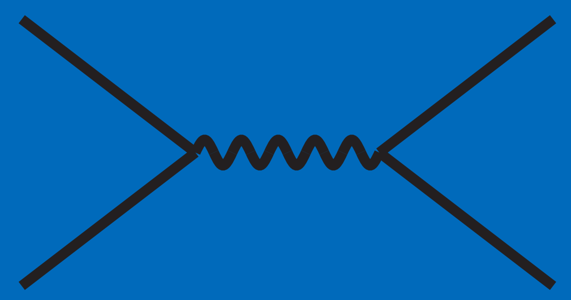
$$k_H(a=1) \simeq 3.5\,h\,\mathrm{Mpc}^{-1}$$

$$w_{\mathrm{WDM}}(a) \simeq 6\times10^{-15}\,a^{-2}\left(\frac{\mathrm{keV}}{m_{\mathrm{WDM}}}\right)^{8/3}$$

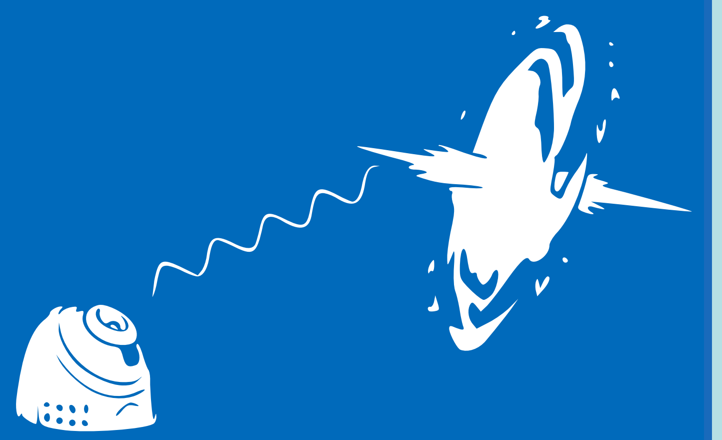
1. Reheating



2. Freeze-in



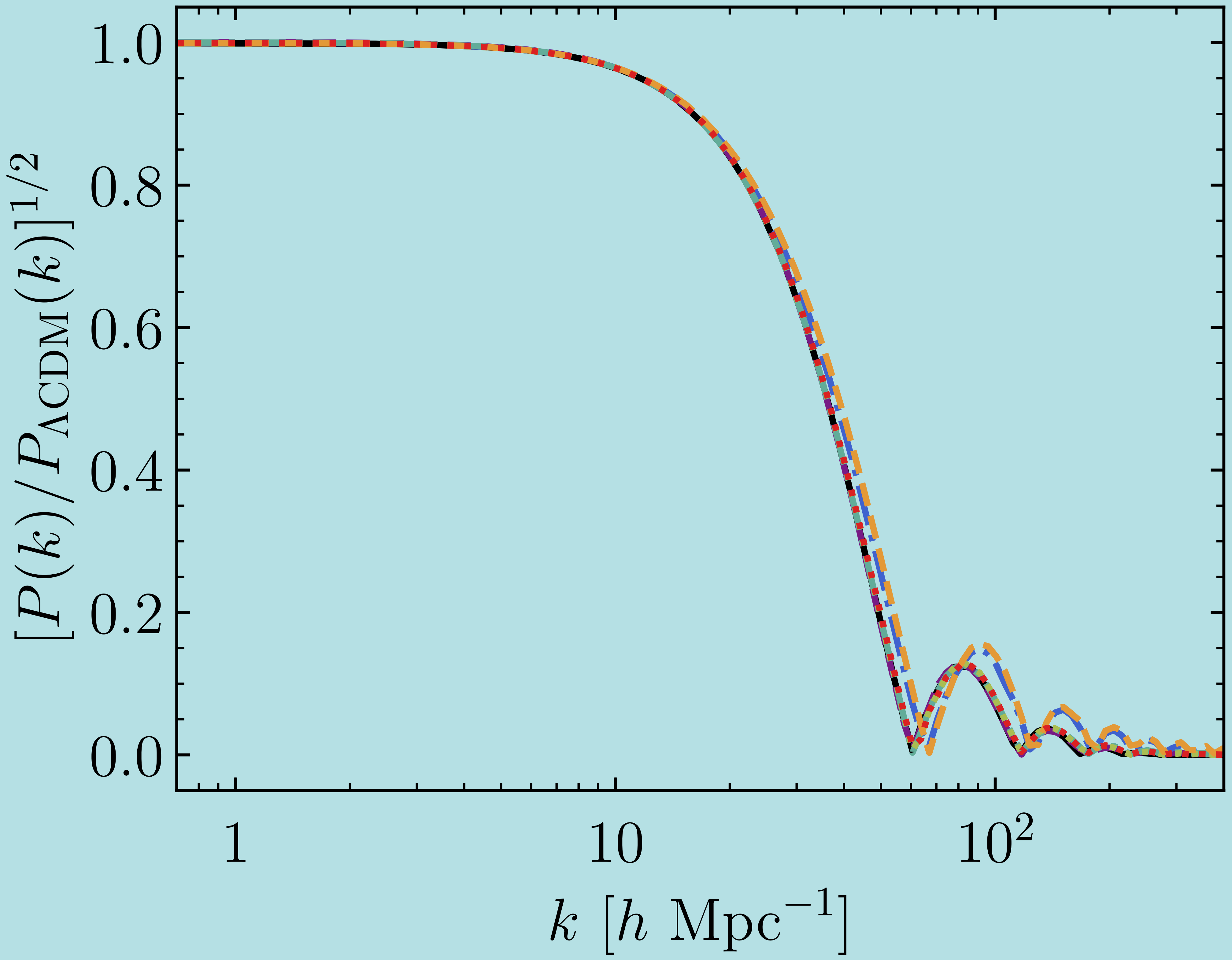
3. Lyman-α



4. The end?

The perturbed Boltzmann equation

$$f(\boldsymbol{x}, \boldsymbol{p}, \tau) = f_0(|\boldsymbol{p}|, \tau)[1 + \Psi(\boldsymbol{x}, \boldsymbol{p}, \tau)] \longrightarrow \ddot{\delta} + \mathcal{H}\dot{\delta} - \frac{3}{2}\mathcal{H}^2\left(1 - w\frac{10}{9}\frac{k^2}{\mathcal{H}^2}\right)\delta = 0$$



$$w(m_{\mathrm{DM}}) = w_{\mathrm{WDM}}(m_{\mathrm{WDM}})$$

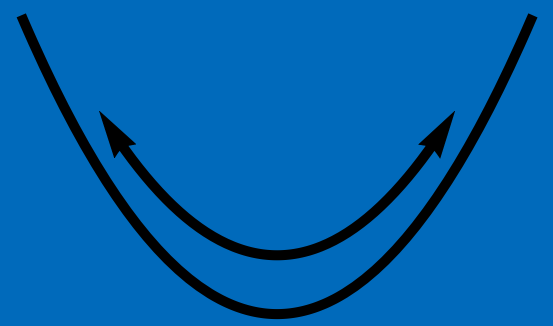
$$k_{\mathrm{FS}}(a) = \frac{9}{10}\frac{\mathcal{H}^2}{w}$$

$$k_H(a) \equiv \left[\int_0^a \frac{1}{k_{\mathrm{FS}}(\tilde{a})}\frac{d\tilde{a}}{\tilde{a}}\right]^{-1}$$

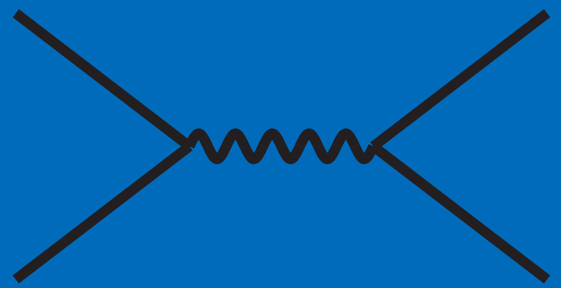
$$w \simeq \frac{\delta P}{\delta\rho} = \frac{T_*^2}{3m_{\mathrm{DM}}^2}\frac{\langle q^2\rangle}{a^2}$$

$$m_{\mathrm{DM}} = m_{\mathrm{WDM}}\left(\frac{T_*}{T_{\mathrm{WDM}}}\right)\sqrt{\frac{\langle q^2\rangle}{\langle q^2\rangle_{\mathrm{WDM}}}}$$

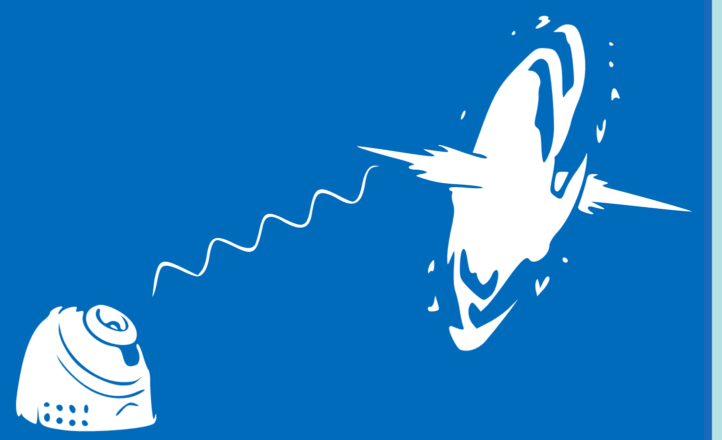
1. Reheating



2. Freeze-in

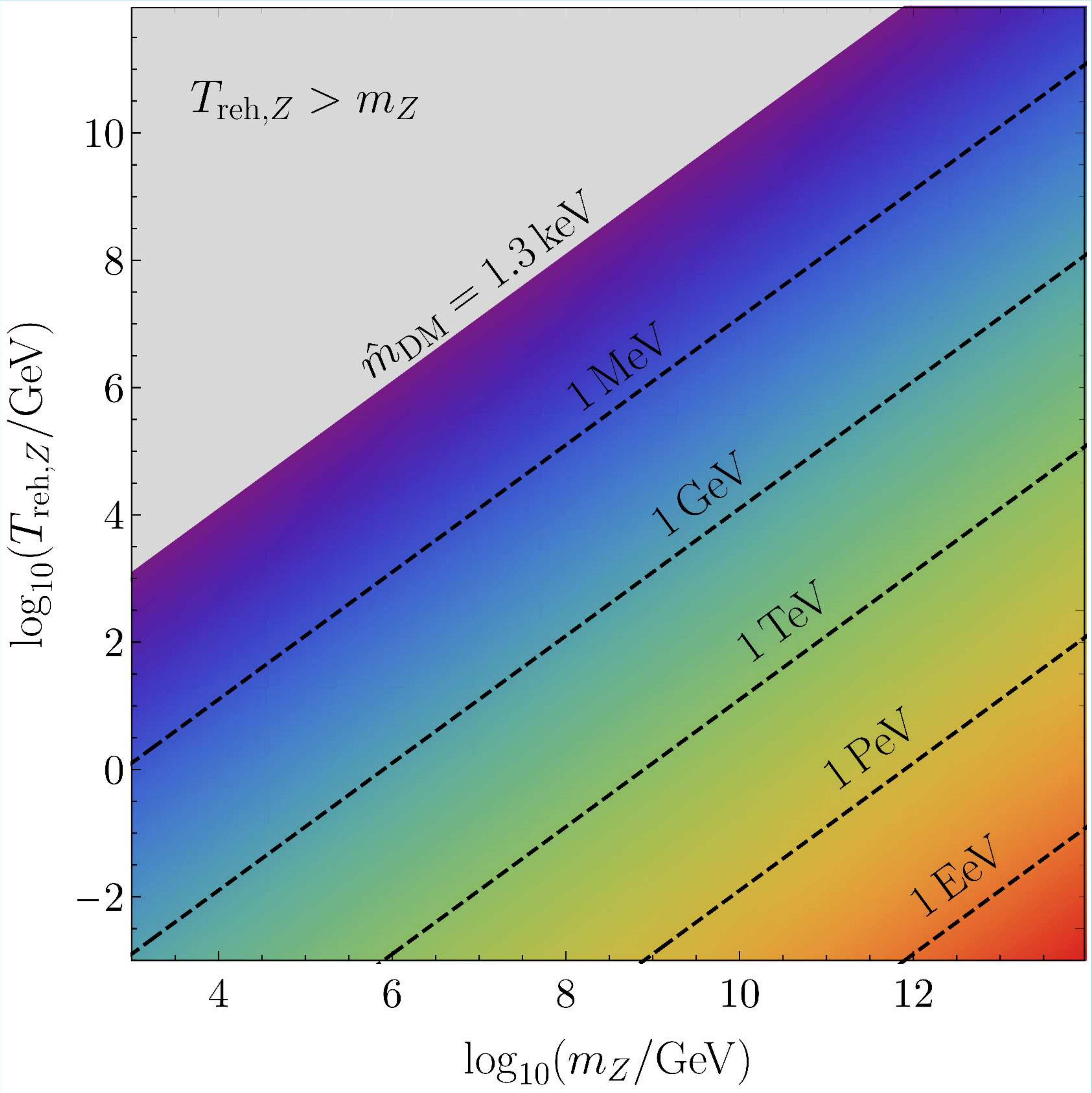


3. Lyman-α



4. The end?

Constraints on dark matter from condensate decay (inflaton, moduli)

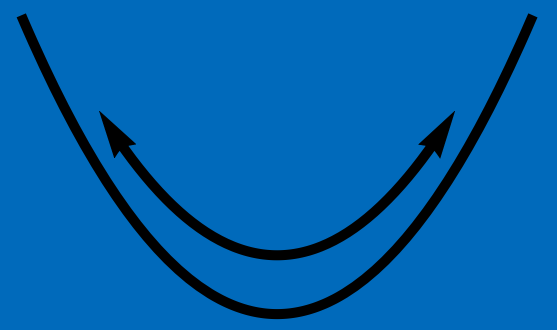


$$m_{\mathrm{DM}} > 3.78\ \mathrm{MeV} \left(\frac{m_{\mathrm{WDM}}}{3\ \mathrm{keV}}\right)^{4/3} \left(\frac{106.75}{g_{*s}^{\mathrm{reh}}}\right)^{1/3} \times \left(\frac{m_\Phi}{3\times 10^{13}\ \mathrm{GeV}}\right) \left(\frac{10^{10}\ \mathrm{GeV}}{T_{\mathrm{reh}}}\right)$$

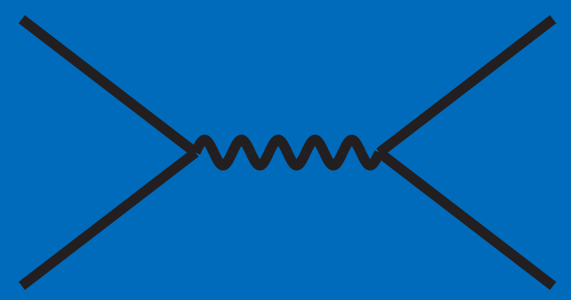
$$\mathrm{Br}_\chi \lesssim 1.5\times 10^{-4} \left(\frac{g_{*s}^{\mathrm{reh}}}{106.5}\right)^{1/3} \left(\frac{3\,\mathrm{keV}}{m_{\mathrm{WDM}}}\right)^{4/3}$$

(absolute)

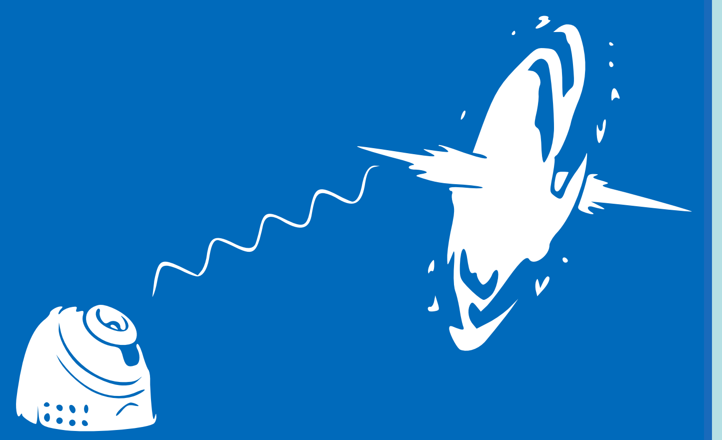
1. Reheating



2. Freeze-in



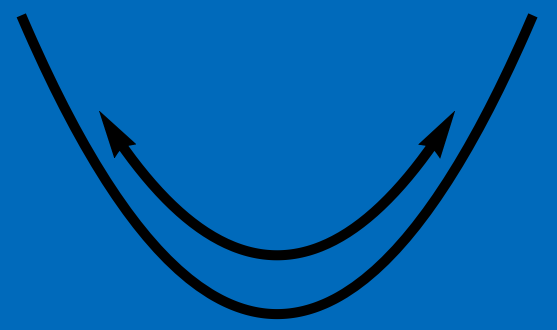
3. Lyman-α

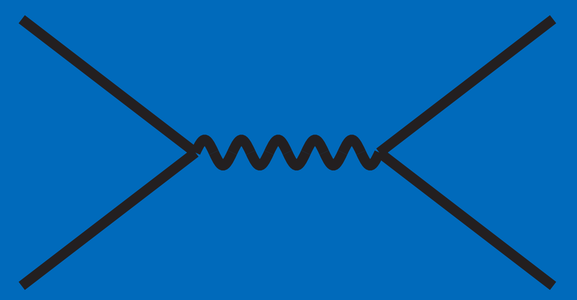


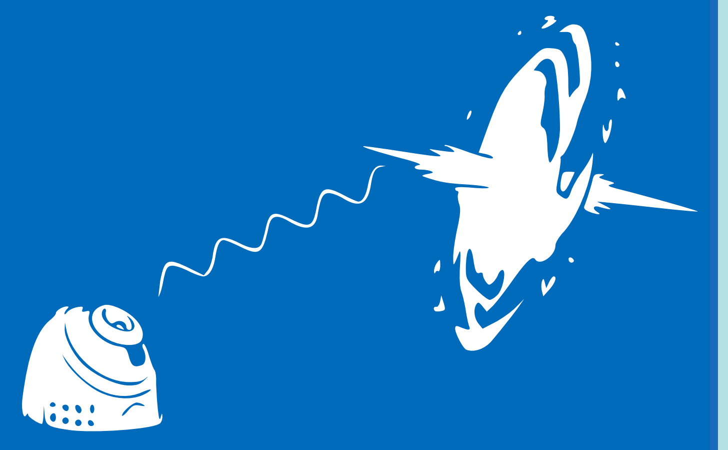
4. The end?

Constraints on dark matter from chain decay $\Phi \to A \to \chi$

$$
m_{\rm DM} > \left(\frac{m_{\rm WDM}}{3\ {\rm keV}}\right)^{4/3} \times \begin{cases} 1.23\ {\rm MeV} \left(\frac{106.75}{g_{*s}^{\rm reh}}\right)^{1/3} \left(\frac{m_\Phi}{3\times 10^{13}\ {\rm GeV}}\right)\left(\frac{10^{10}\ {\rm GeV}}{T_{\rm reh}}\right), & {\rm R} \\ 2.7\,{\rm MeV} \left(\frac{106.75}{g_{*s}^{\rm dec}}\right)^{1/3} \left(\frac{m_A}{3\times 10^{13}\ {\rm GeV}}\right)\left(\frac{10^{10}\ {\rm GeV}}{T_{\rm dec}}\right), & {\rm NR} \end{cases}
$$



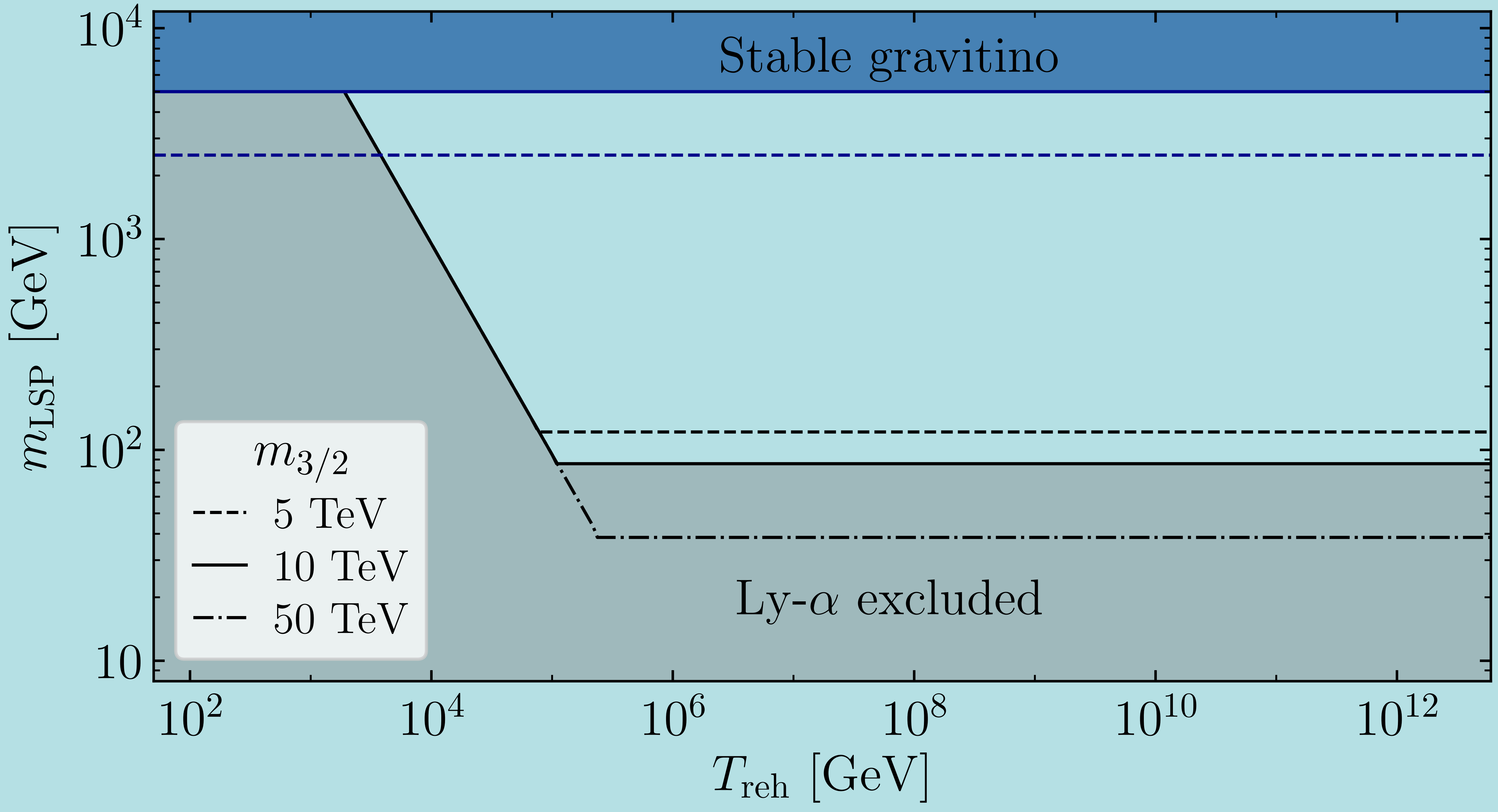




Constraints on dark matter from chain decay

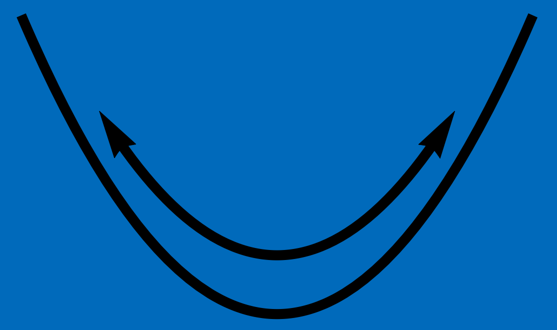
$$\Phi \rightarrow \Psi_{3/2} \rightarrow \mathrm{LSP}$$

$$m_{\mathrm{LSP}} > \begin{cases} 86\,\mathrm{GeV}\left(\dfrac{m_{\mathrm{WDM}}}{3\,\mathrm{keV}}\right)^{4/3}\left(\dfrac{10\,\mathrm{TeV}}{m_{3/2}}\right)^{1/2}, & T_{\mathrm{reh}} \gg 10^5\,\mathrm{GeV}\left(\dfrac{m_{3/2}}{10\,\mathrm{TeV}}\right)^{1/2} \\ 95\,\mathrm{GeV}\left(\dfrac{m_{\mathrm{WDM}}}{3\,\mathrm{keV}}\right)^{4/3}\left(\dfrac{10^5\,\mathrm{GeV}}{T_{\mathrm{reh}}}\right), & T_{\mathrm{reh}} \ll 10^5\,\mathrm{GeV}\left(\dfrac{m_{3/2}}{10\,\mathrm{TeV}}\right)^{1/2} \end{cases}$$

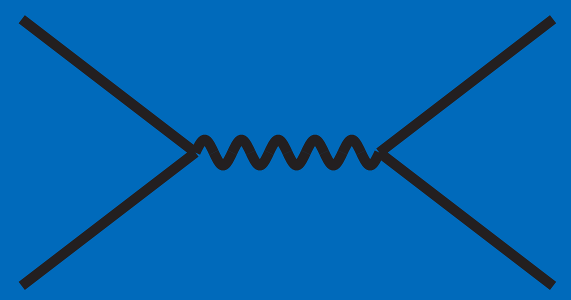


$$\Gamma_{3/2} = \frac{193}{384\pi}\frac{m_{3/2}^3}{M_P^2}$$

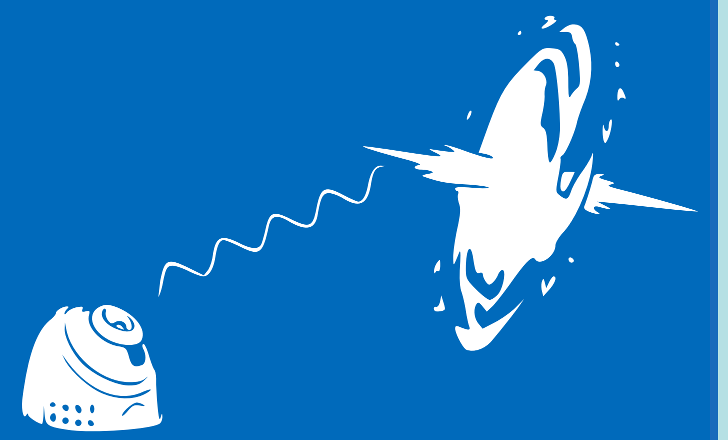
1. Reheating



2. Freeze-in



3. Lyman-α

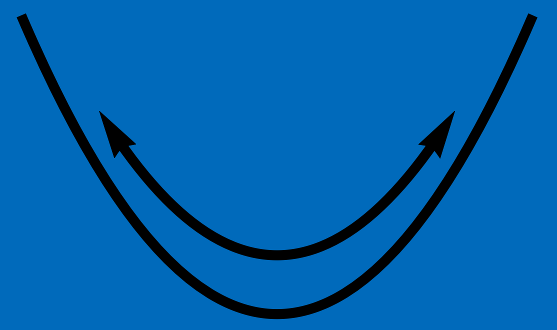


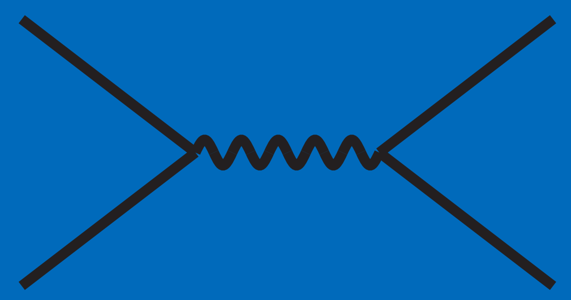
4. The end?

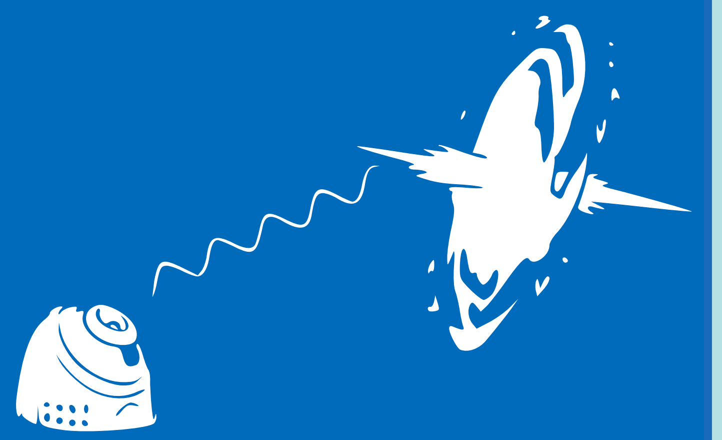
Constraints on dark matter from thermal freeze-in

$$m_{\rm DM} > \left(\frac{m_{\rm WDM}}{3\ {\rm keV}}\right)^{4/3} \left(\frac{106.75}{g_{*s}^{\rm reh}}\right)^{1/3} \begin{cases} 7.3\ {\rm keV}\,, & (n=0\,,\ {\rm FF}) \\ 8.5\ {\rm keV}\,, & (n=2\,,\ {\rm FF}) \\ 8.5\ {\rm keV}\,, & (n=4\,,\ {\rm FF}) \end{cases}$$

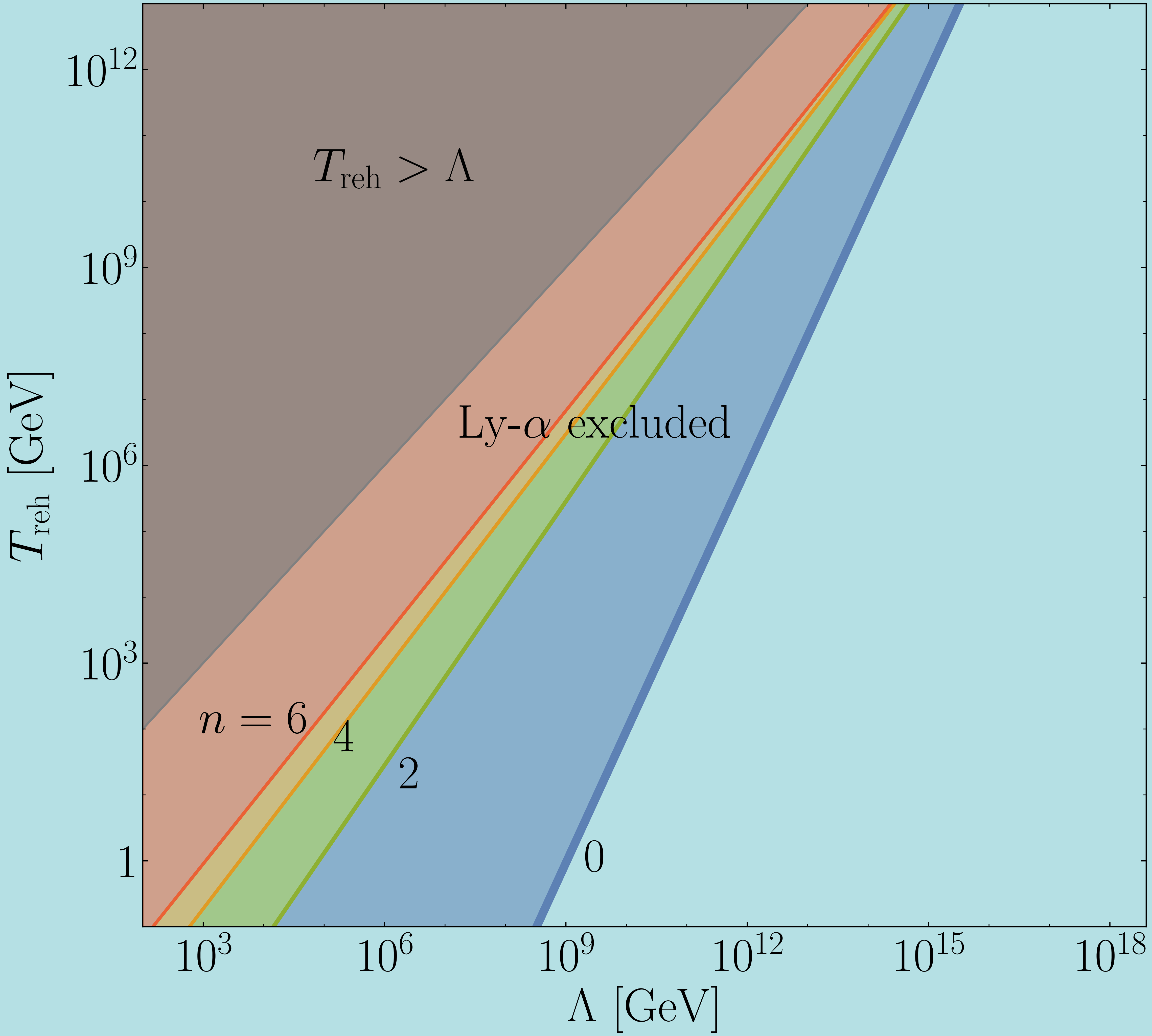
$$m_{\rm DM} > \left(\frac{m_{\rm WDM}}{3\ {\rm keV}}\right)^{4/3} \left(\frac{106.75}{g_{*s}^{\rm reh}}\right)^{1/3} \begin{cases} 9\,{\rm keV}\ \ln^{-1/2}\left(\frac{T_{\rm max}}{T_{\rm reh}}\right)\,, & n=6\,, \\ 23\,{\rm keV}\left(\frac{T_{\rm reh}}{T_{\rm max}}\right)\,, & n=8\,, \\ 52\,{\rm keV}\left(\frac{T_{\rm reh}}{T_{\rm max}}\right)^{5/3}\,, & n\geq 10\,. \end{cases}$$





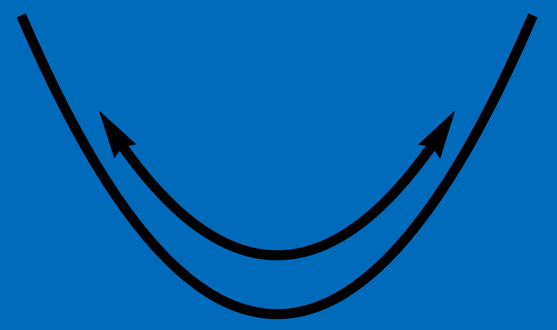


Constraints on dark matter from thermal freeze-in

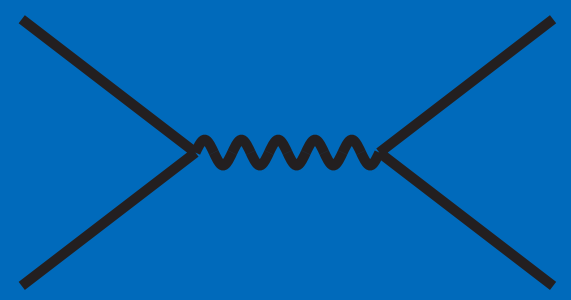


$$\sigma(s) = \frac{s^{n/2}}{\Lambda^{n+2}}$$

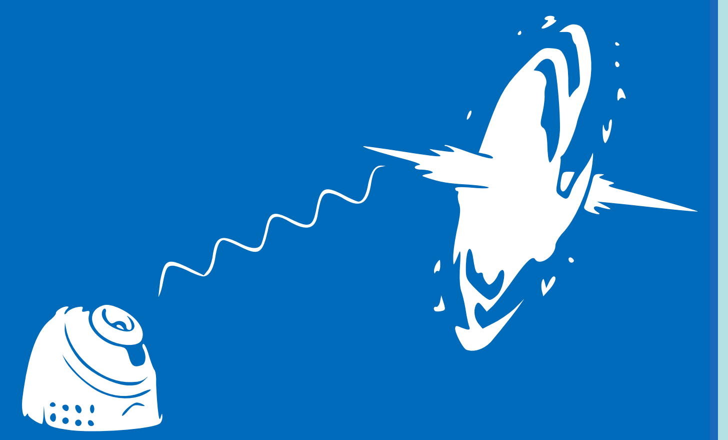
1. Reheating



2. Freeze-in



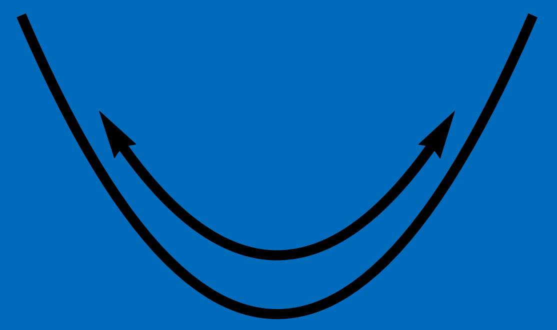
3. Lyman-α

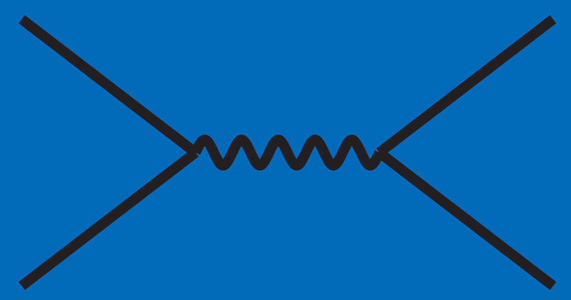


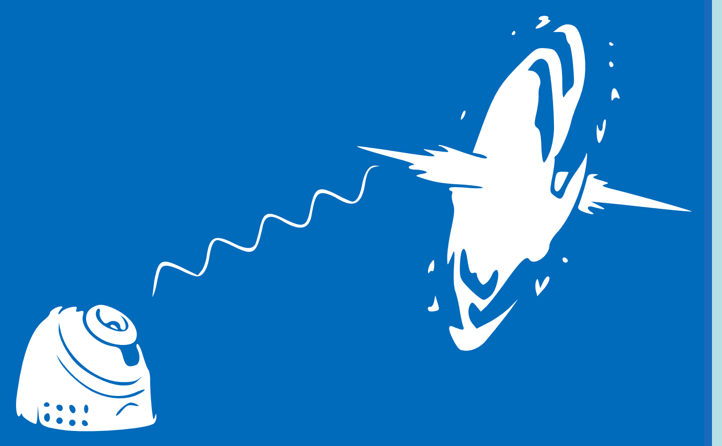
4. The end?

Constraints on dark matter from non-thermal freeze-in (n=4)

$$m_{3/2} \lesssim 0.44\ \mathrm{keV} \left(\frac{m_{\mathrm{WDM}}}{3\ \mathrm{keV}}\right)^{4/3} \left(\frac{\alpha_{\mathrm{SM}}}{0.03}\right)^{-32/15} \left(\frac{c}{3/5}\right)^{2/15} \times \left(\frac{106.75}{g_{*s}^{\mathrm{reh}}}\right)^{1/5} \left(\frac{10^{10}\ \mathrm{GeV}}{T_{\mathrm{reh}}}\right)^{7/15} \left(\frac{m_{\Phi}}{3\times 10^{13}\ \mathrm{GeV}}\right)^{23/15}$$



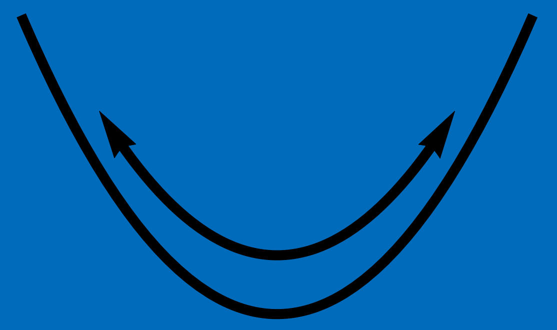




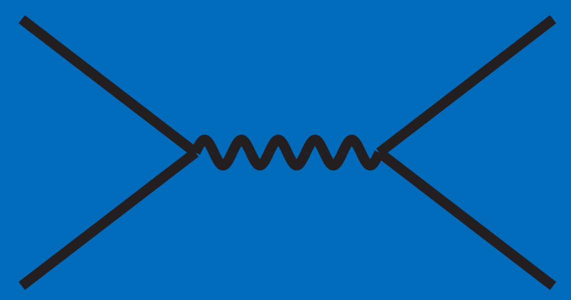
Non-supersymmetric decaying spin-3/2 particle

$$\mathcal{L} = i\frac{\alpha_1}{2M_P}\bar{\nu}_R\gamma^\mu[\gamma^\rho, \gamma^\sigma]\Psi_\mu F_{\rho\sigma} + i\frac{\alpha_2}{2M_P} i\sigma_2(D^\mu H)^*\bar{L}\Psi_\mu + \text{h.c.}$$

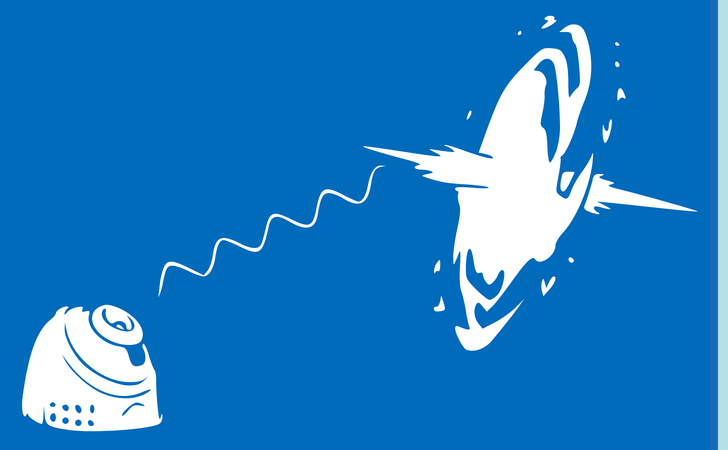
1. Reheating



2. Freeze-in



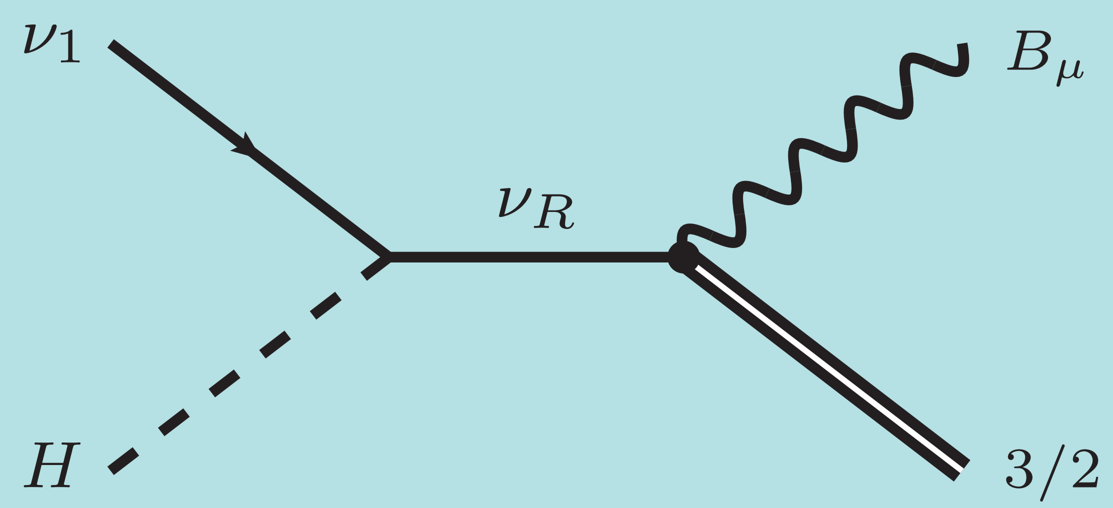
3. Lyman-α

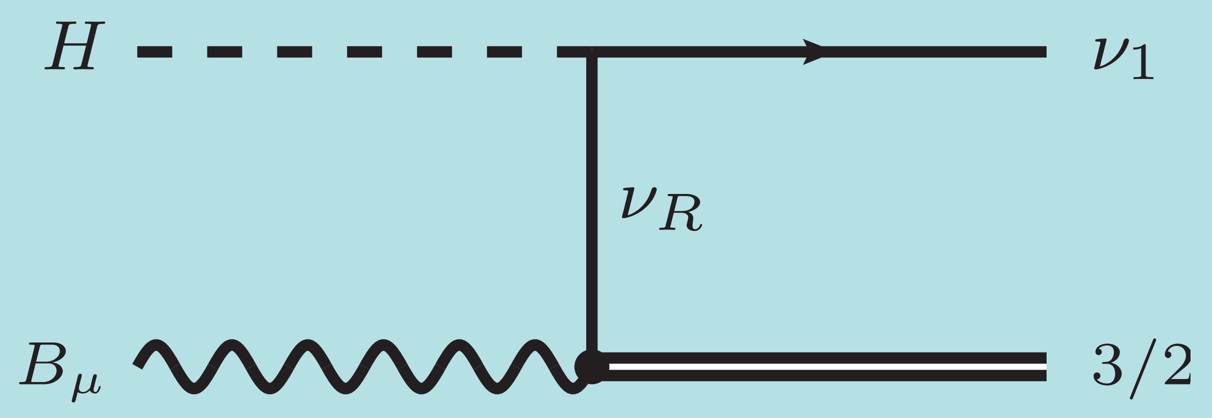


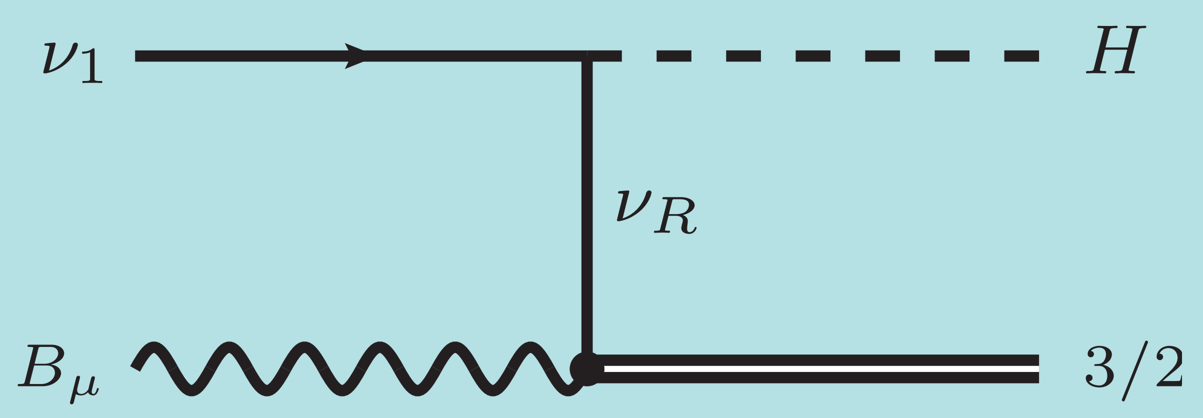
4. The end?

Non-supersymmetric decaying spin-3/2 particle

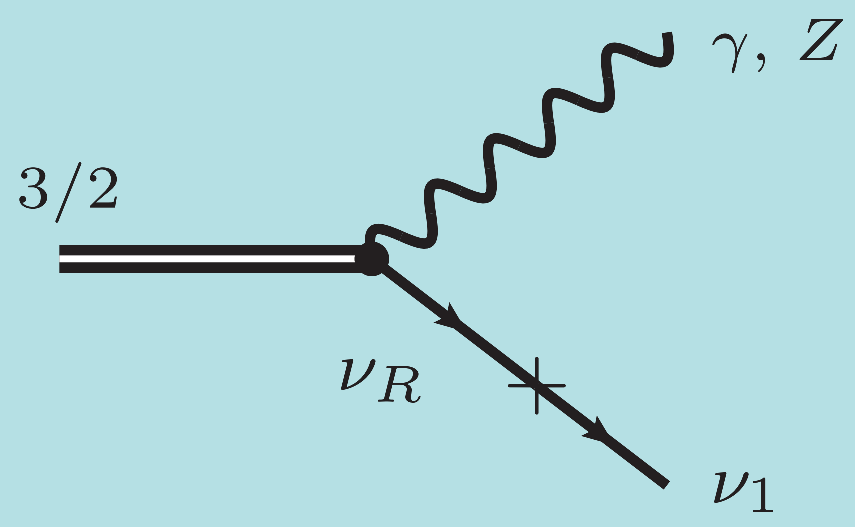
$$\mathcal{L} = i\frac{\alpha_1}{2M_P}\bar{\nu}_R\gamma^\mu[\gamma^\rho,\gamma^\sigma]\Psi_\mu F_{\rho\sigma} + i\frac{\alpha_2}{2M_P} i\sigma_2 (D^\mu H)^* \bar{L}\Psi_\mu + \text{h.c.}$$

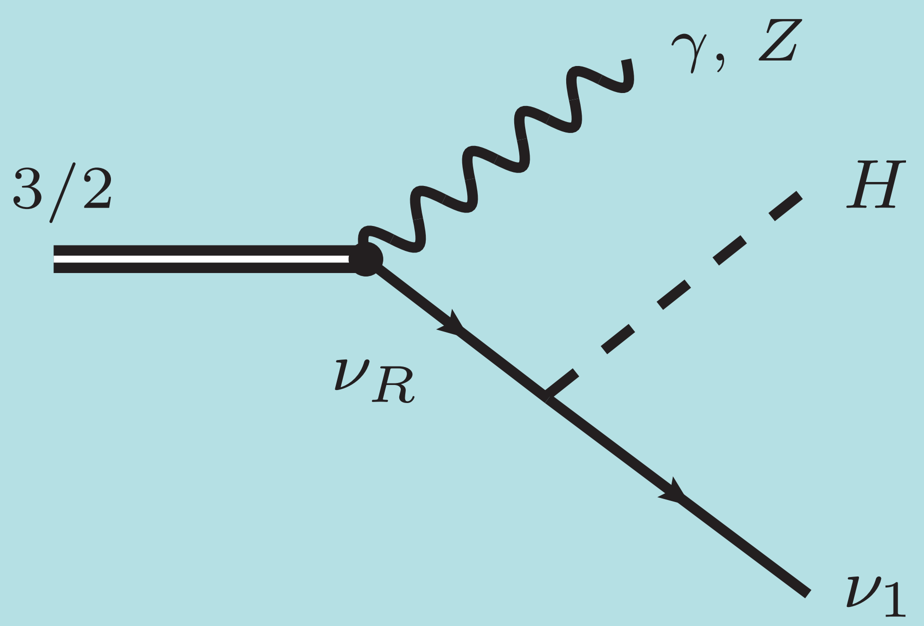






$$\sigma(s) = \frac{11\alpha_1^2 y^2 s^2}{72\pi m_{3/2}^2 M_R^2 M_P^2}$$

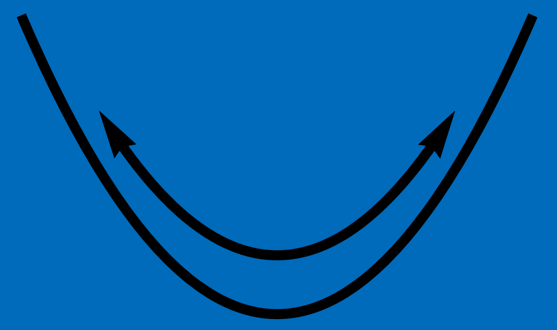




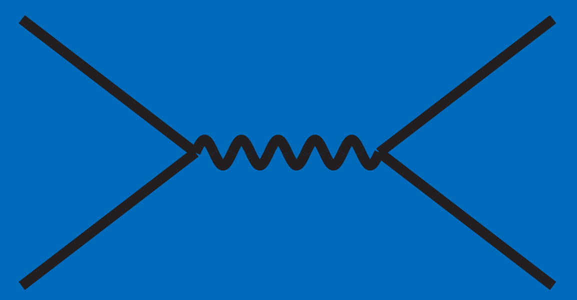
$$\tau_{3/2}^{2b} \simeq 1.6\times 10^{29}\left(\frac{10^{-2}}{y\ \alpha_1}\right)^2\left(\frac{M_R}{10^{14}\,\text{GeV}}\right)^2\left(\frac{10^4\,\text{GeV}}{m_{3/2}}\right)^3\ \text{s}$$

$$\tau_{3/2}^{3b} \simeq 5.6\times 10^{28}\left(\frac{10^{-2}}{y\ \alpha_1}\right)^2\left(\frac{M_R}{10^{14}\,\text{GeV}}\right)^2\left(\frac{10^4\,\text{GeV}}{m_{3/2}}\right)^5\ \text{s}$$

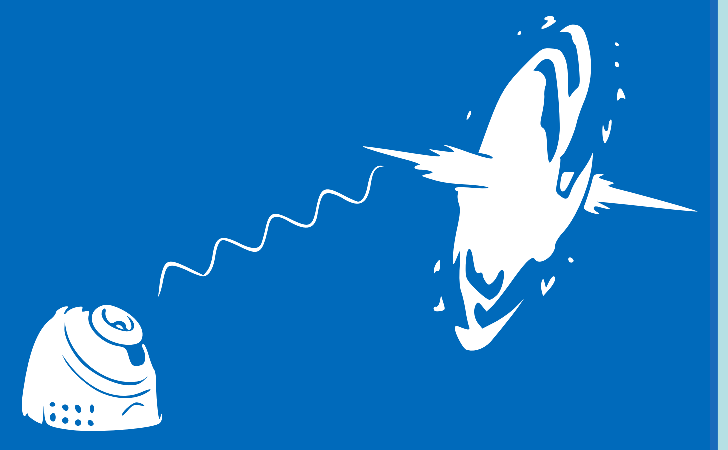
1. Reheating



2. Freeze-in



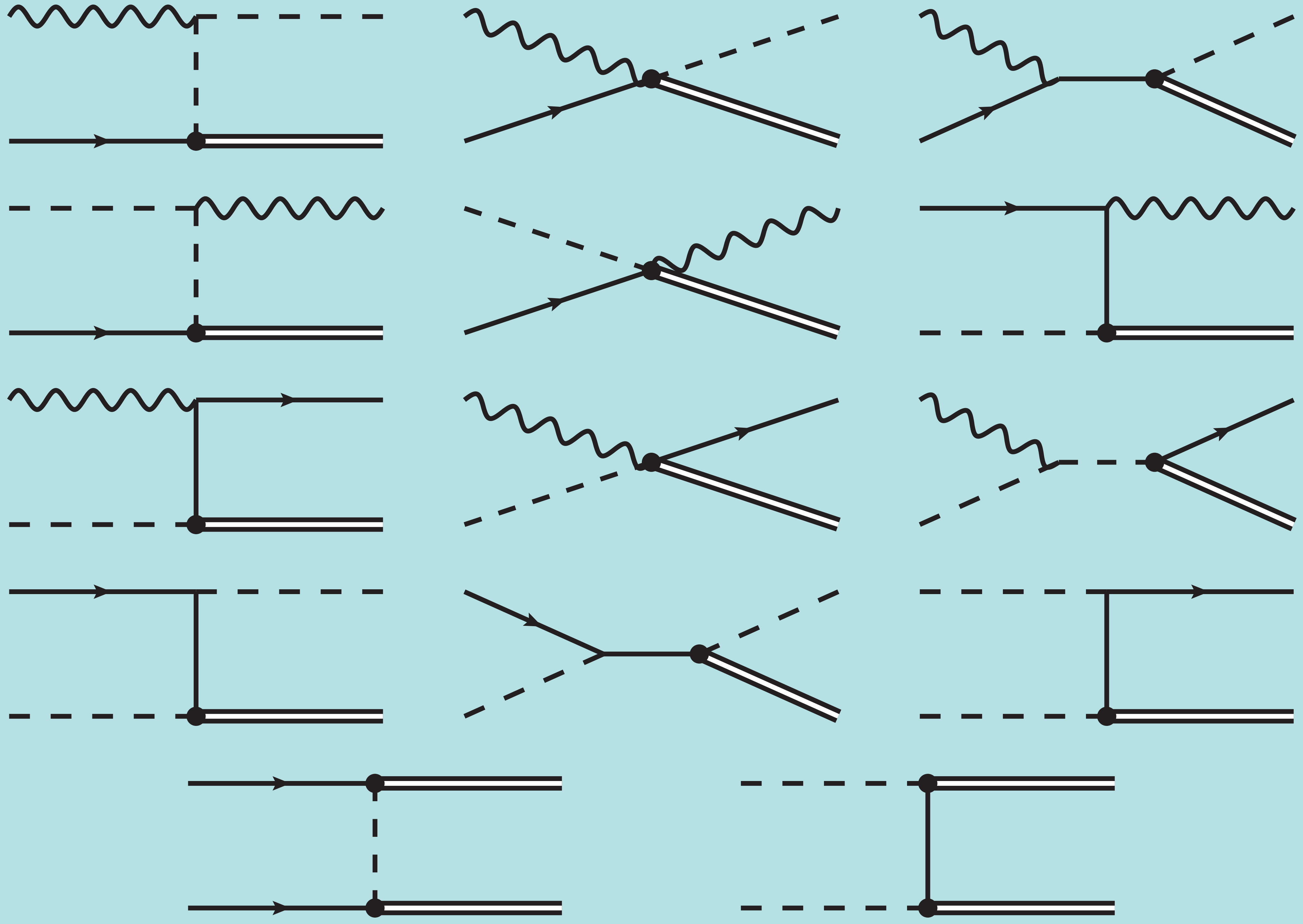
3. Lyman-α

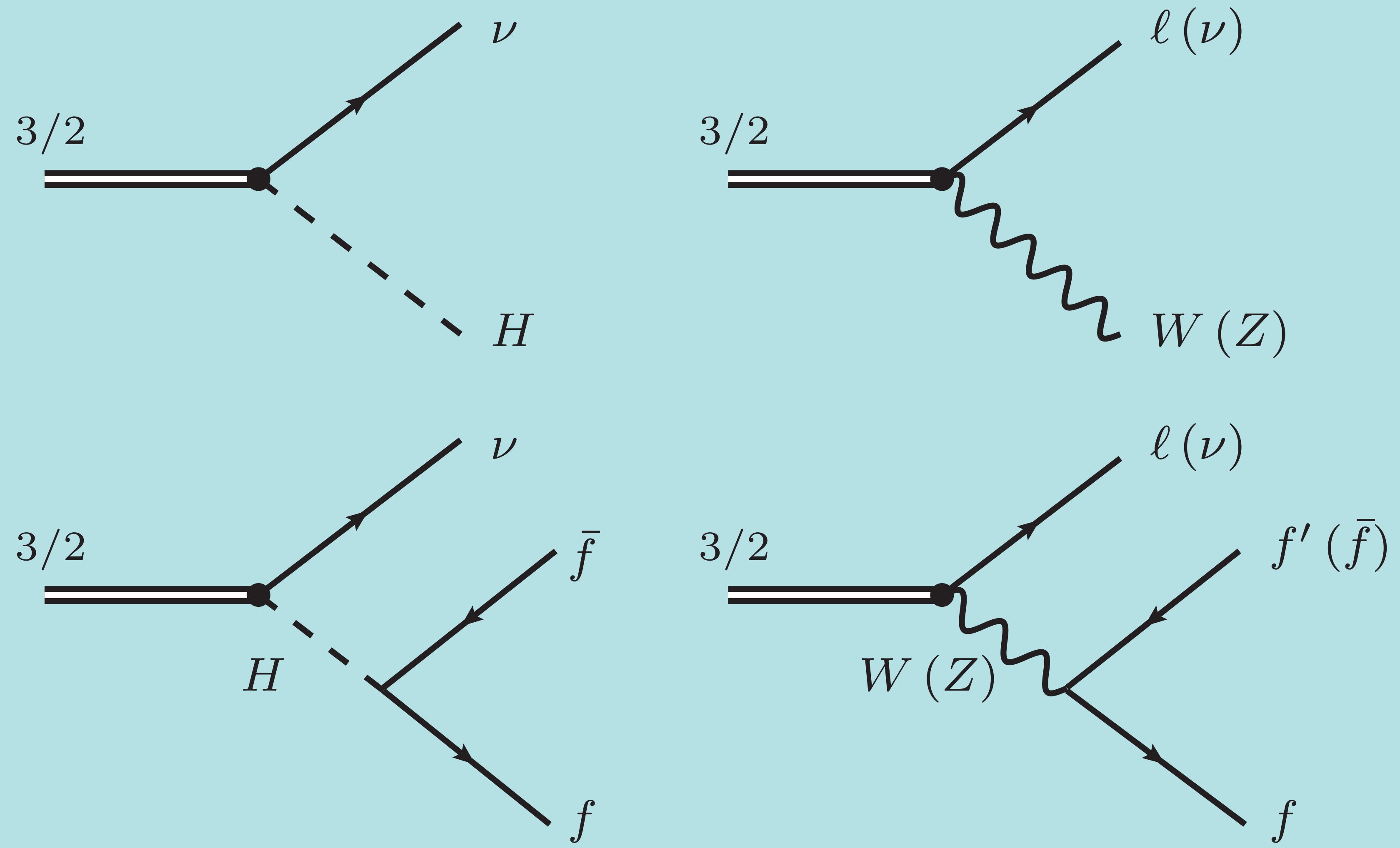


4. The end?

Non-supersymmetric decaying spin-3/2 particle

$$\mathcal{L} = i\frac{\alpha_1}{2M_P}\bar{\nu}_R\gamma^\mu[\gamma^\rho,\gamma^\sigma]\Psi_\mu F_{\rho\sigma} + i\frac{\alpha_2}{2M_P}i\sigma_2(D^\mu H)^*\bar{L}\Psi_\mu + \text{h.c.}$$

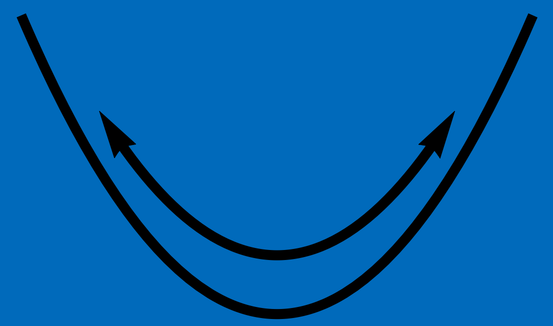




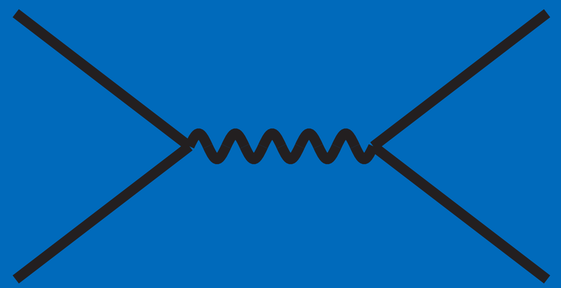
$$\sigma(s) = \frac{\alpha_2^2 s}{9216\pi m_{3/2}^2 M_P^2}(639g^2 + 87g'^2 + 144h_t^2 + 32h_\tau^2)$$

$$\frac{\tau_{3/2}}{10^{28}\text{s}} \simeq \begin{cases} 14.8\left(\frac{10^{-7}}{\alpha_2}\right)^2\left(\frac{1\,\text{GeV}}{m_{3/2}}\right)^3, & m_{3/2} > m_H \\ 0.6\left(\frac{10^{-3}}{\alpha_2}\right)^2\left(\frac{1\,\text{GeV}}{m_{3/2}}\right)^{5.28}, & m_e < m_{3/2} < m_W \\ 4.8\left(\frac{10^{-3}}{\alpha_2}\right)^2\left(\frac{1\,\text{GeV}}{m_{3/2}}\right)^5, & m_{3/2} < m_e \end{cases}$$

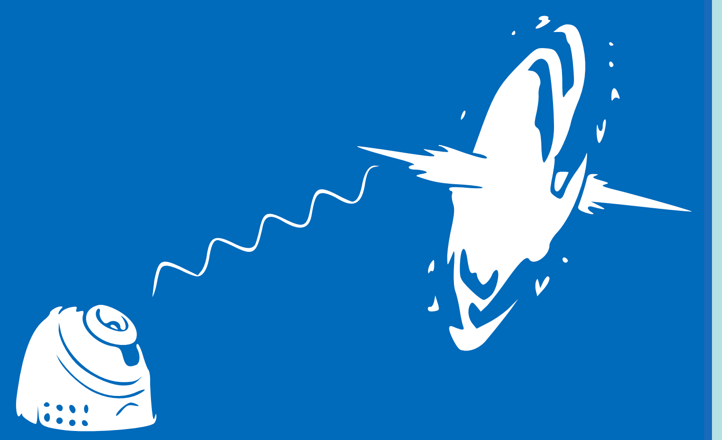
1. Reheating



2. Freeze-in



3. Lyman-α



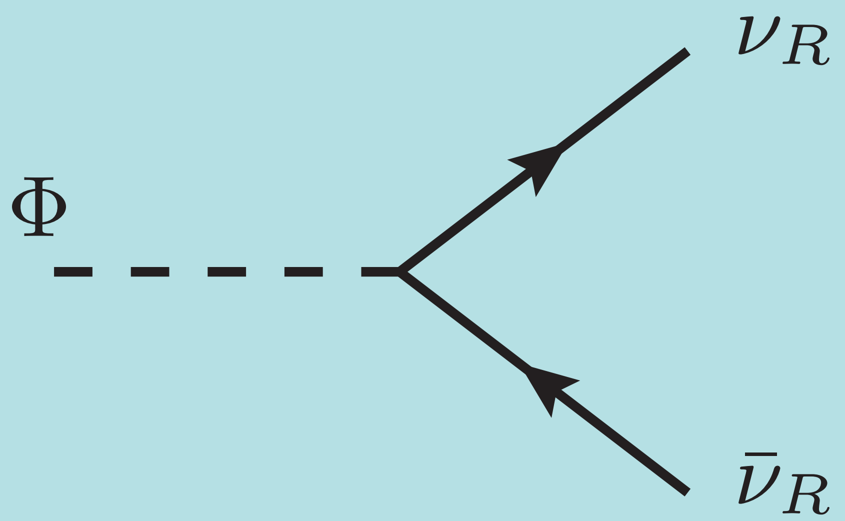
4. The end?

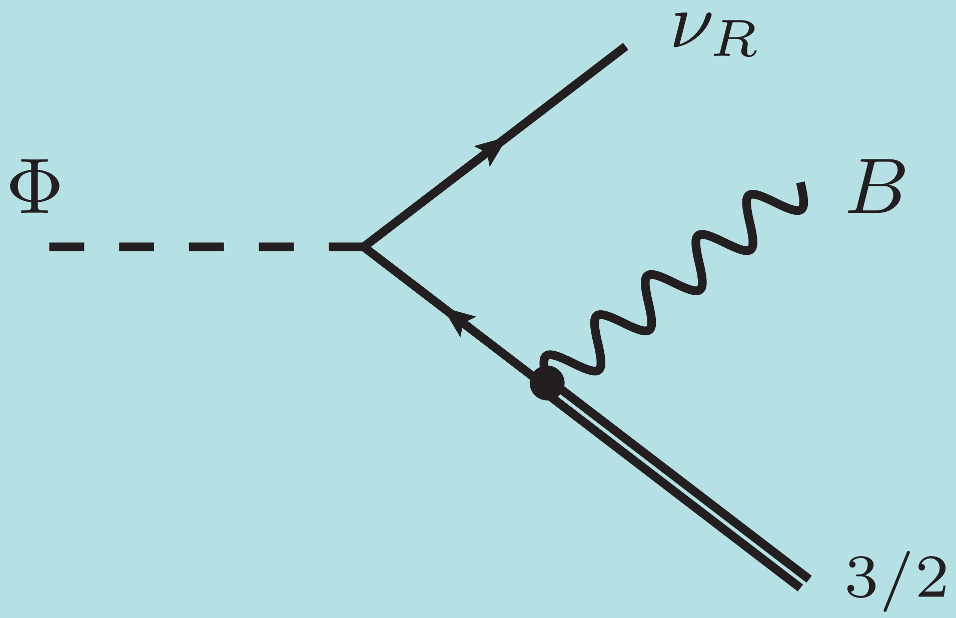
Non-supersymmetric decaying spin-3/2 particle

$$\mathcal{L} = i\frac{\alpha_1}{2M_P}\bar{\nu}_R\gamma^\mu[\gamma^\rho,\gamma^\sigma]\Psi_\mu F_{\rho\sigma} + i\frac{\alpha_2}{2M_P} i\sigma_2 (D^\mu H)^* \bar{L}\Psi_\mu + \text{h.c.}$$

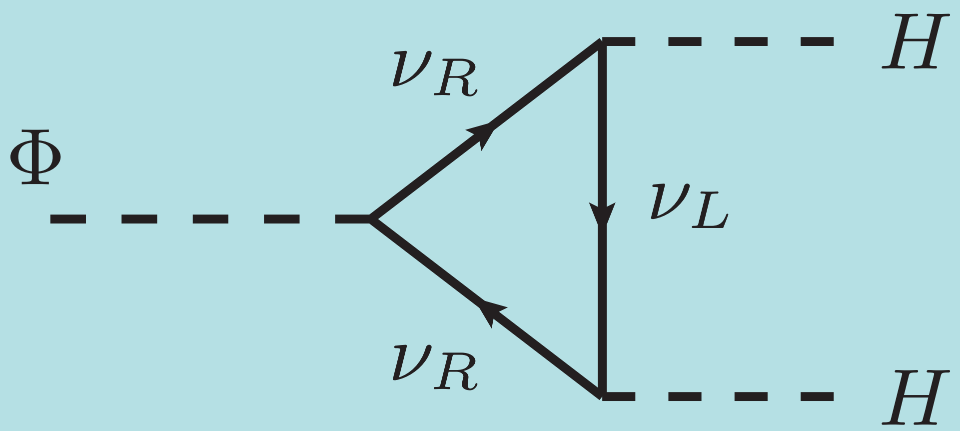
$$+ y_\nu \Phi \bar{\nu}_R \nu_R$$

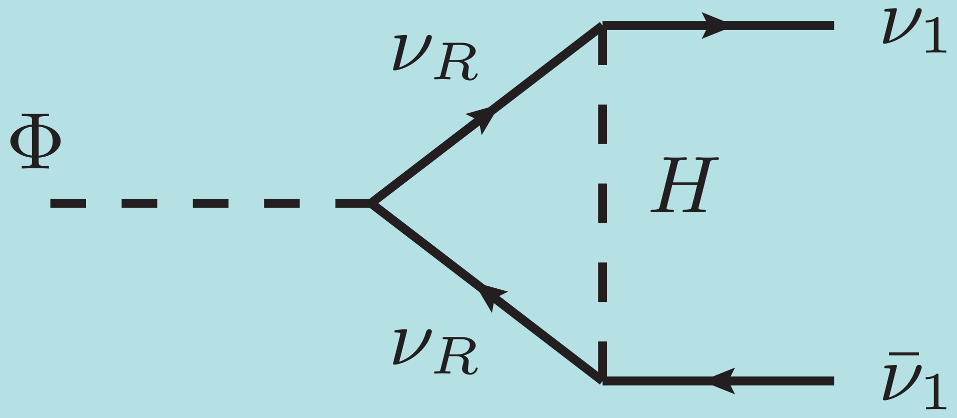
$M_R \ll m_\Phi$:

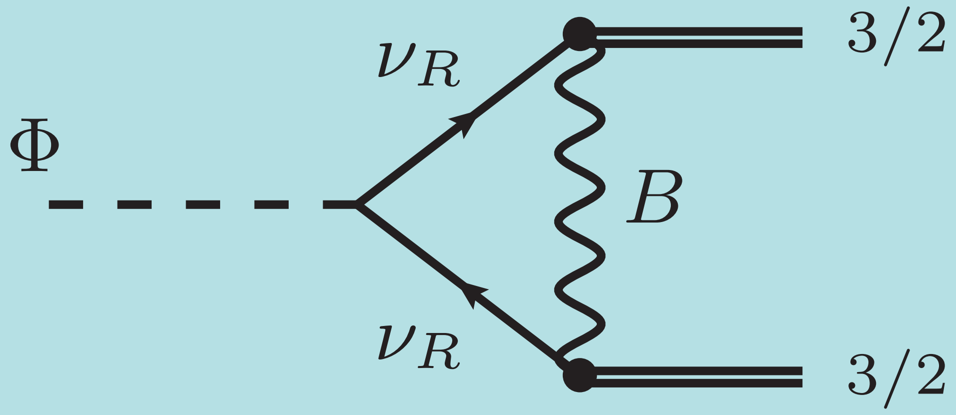




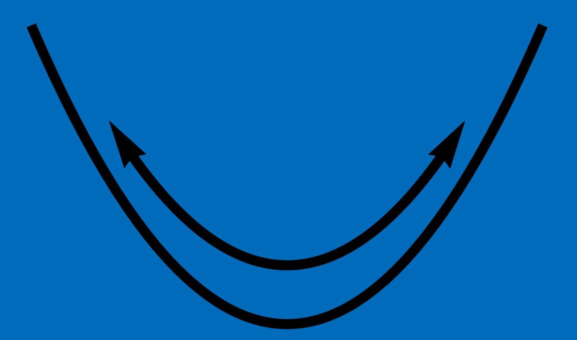
$M_R \gg m_\Phi$:



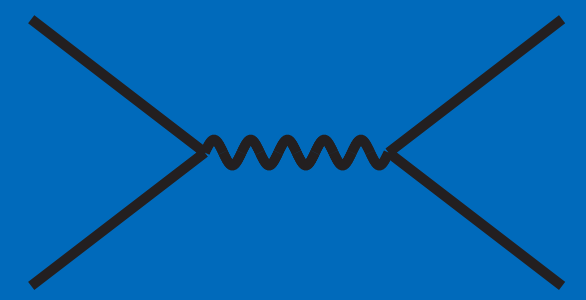




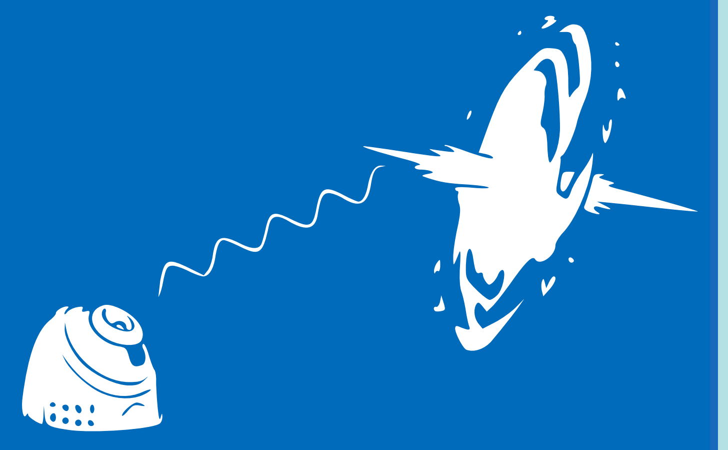
1. Reheating



2. Freeze-in



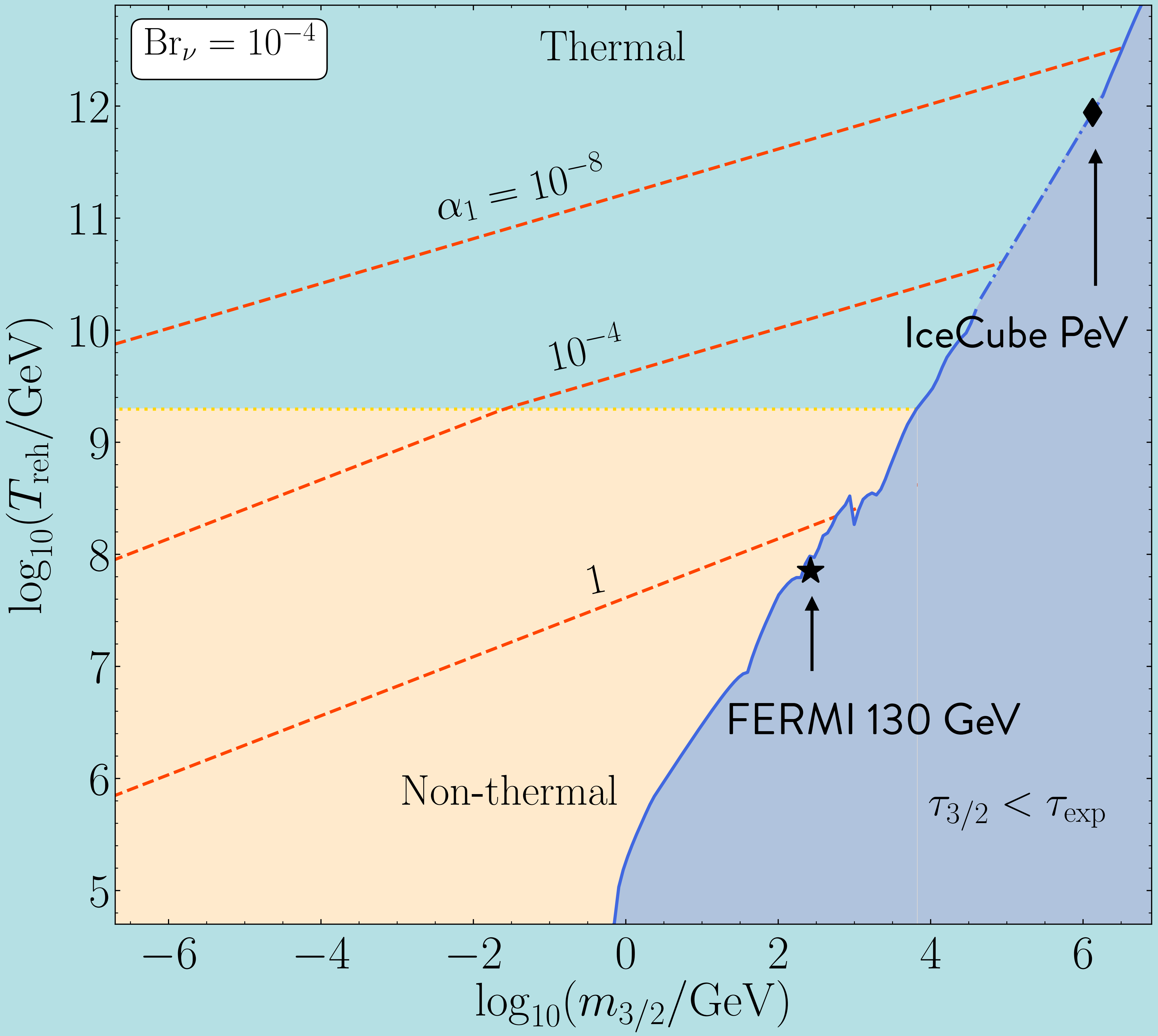
3. Lyman-α



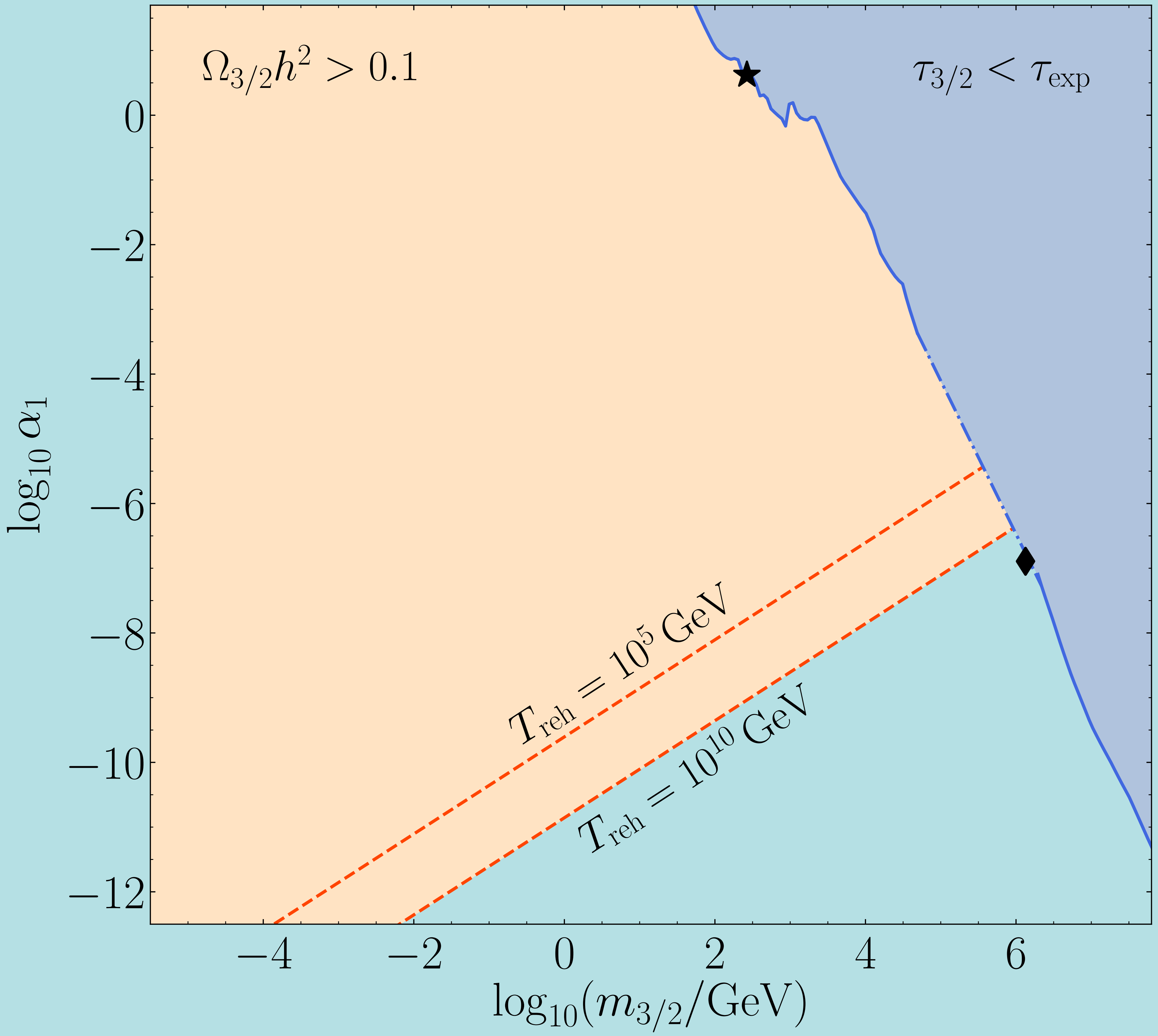
4. The end?

Non-supersymmetric decaying spin-3/2 particle

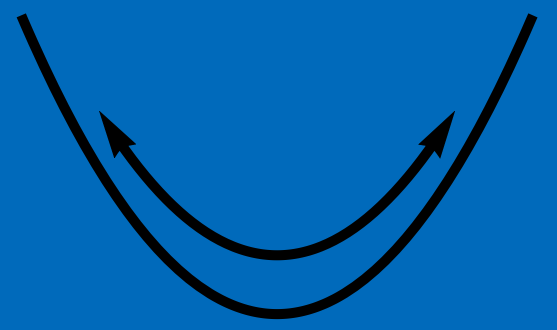
Scattering



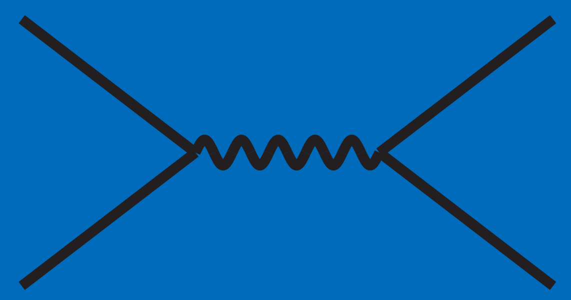
Inflaton decay



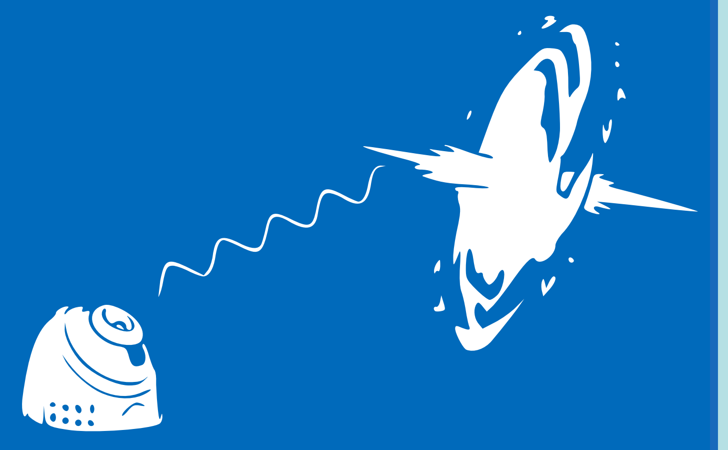
1. Reheating



2. Freeze-in



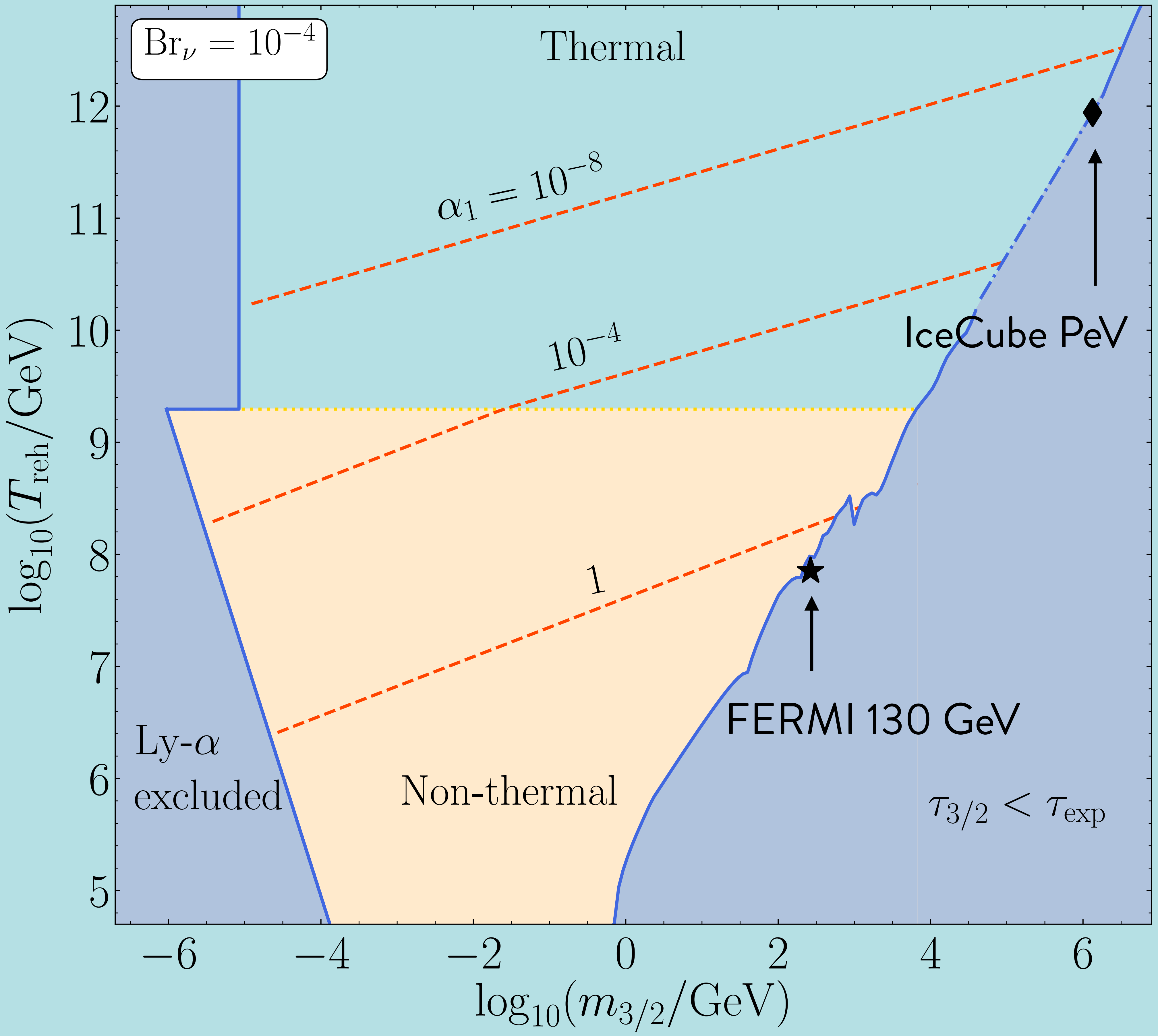
3. Lyman-α



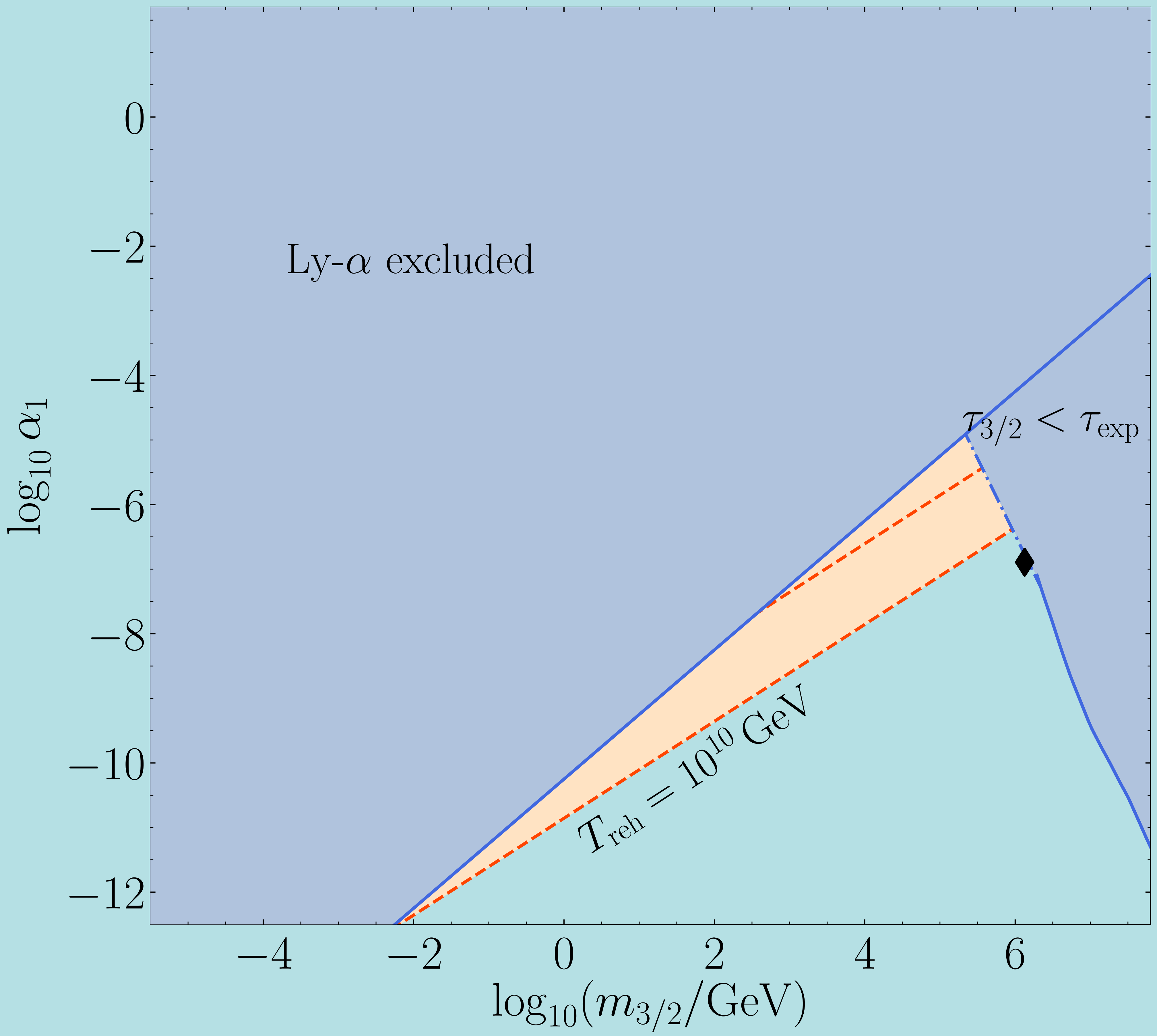
4. The end?

Non-supersymmetric decaying spin-3/2 particle

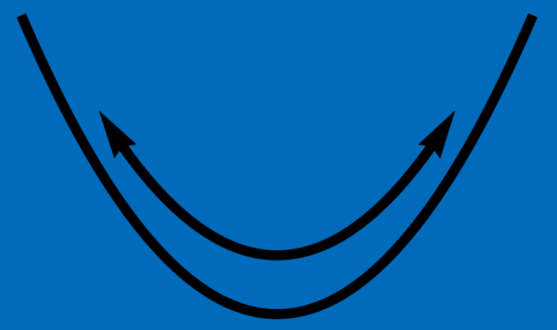
Scattering



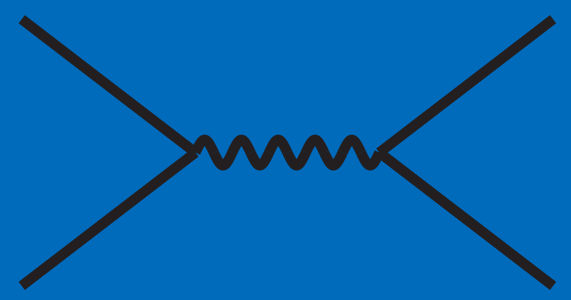
Inflaton decay



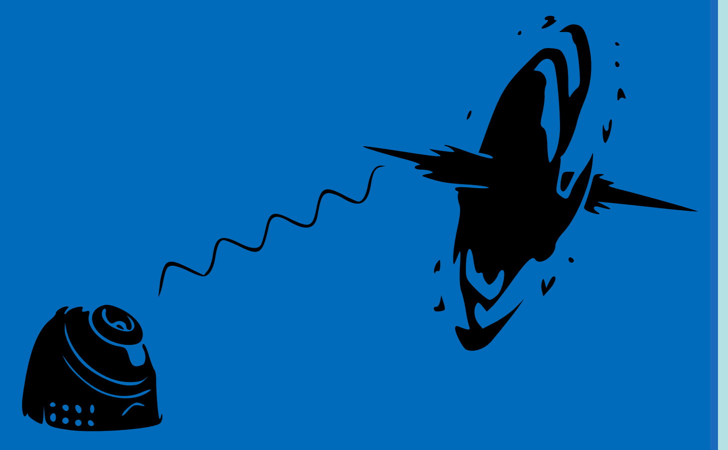
1. Reheating



2. Freeze-in

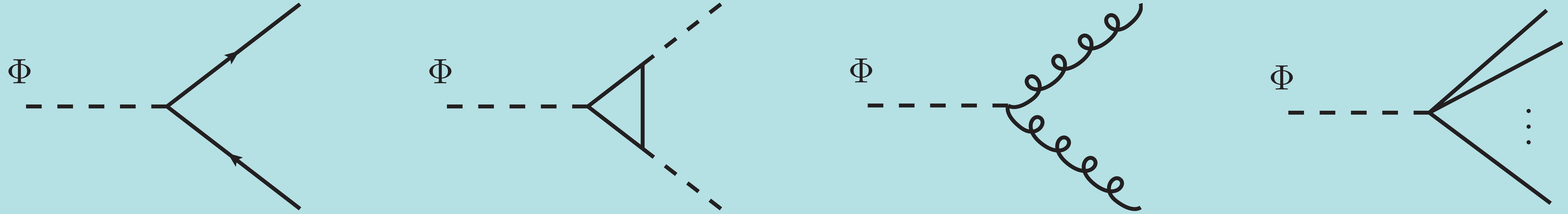


3. Lyman-α



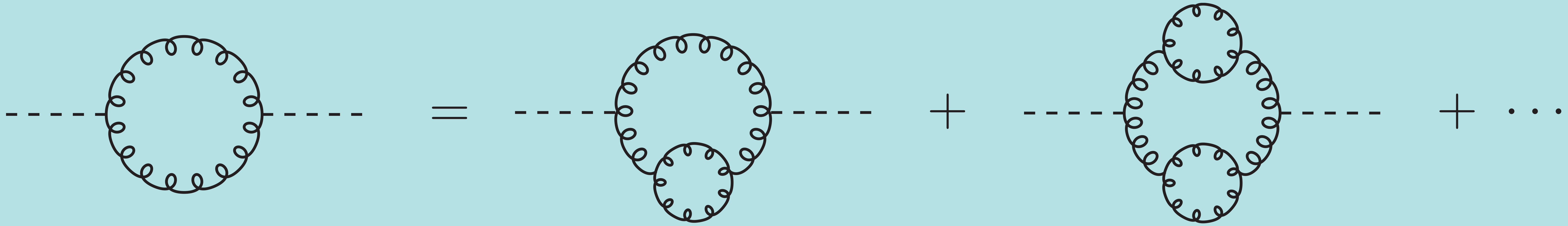
4. The end?

For $k \neq 2$ the nature of the final state matters



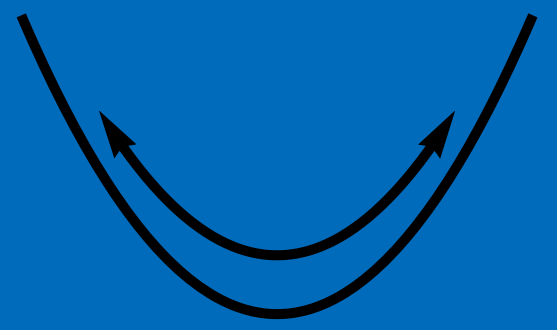
$$\frac{\partial f_\chi}{\partial t} - H|\boldsymbol{p}|\frac{\partial f_\chi}{\partial |\boldsymbol{p}|} = \sum_{n=1}^{\infty} \frac{2\pi}{n^2\omega^2(t)}|\mathcal{M}_n|^2(1 \pm 2f_\chi)\delta(n\omega(t) - 2p_0)$$

In-medium effects cannot be neglected

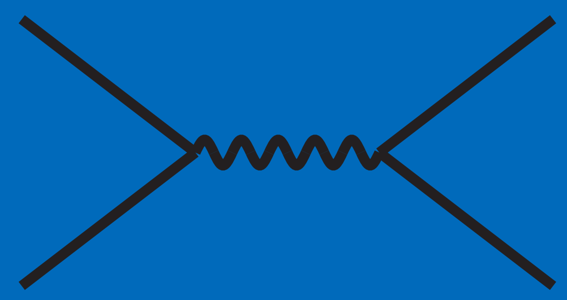


$$\frac{dN}{dV\,dt} = -2\int \frac{d^3\boldsymbol{p}}{(2\pi)^3 2E} f(E)\,\mathrm{Im}\,\Pi$$

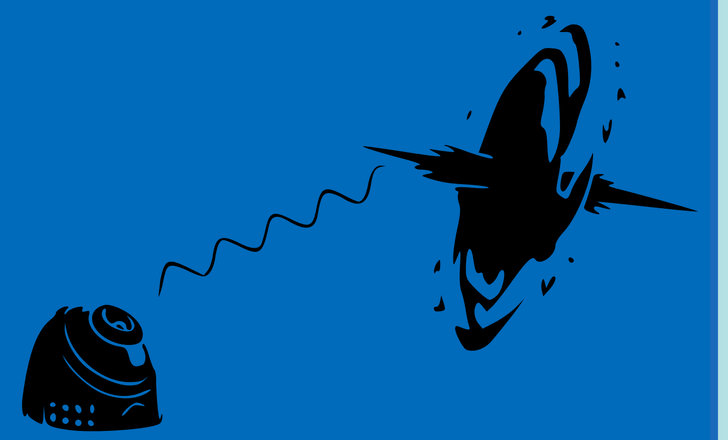
1. Reheating



2. Freeze-in

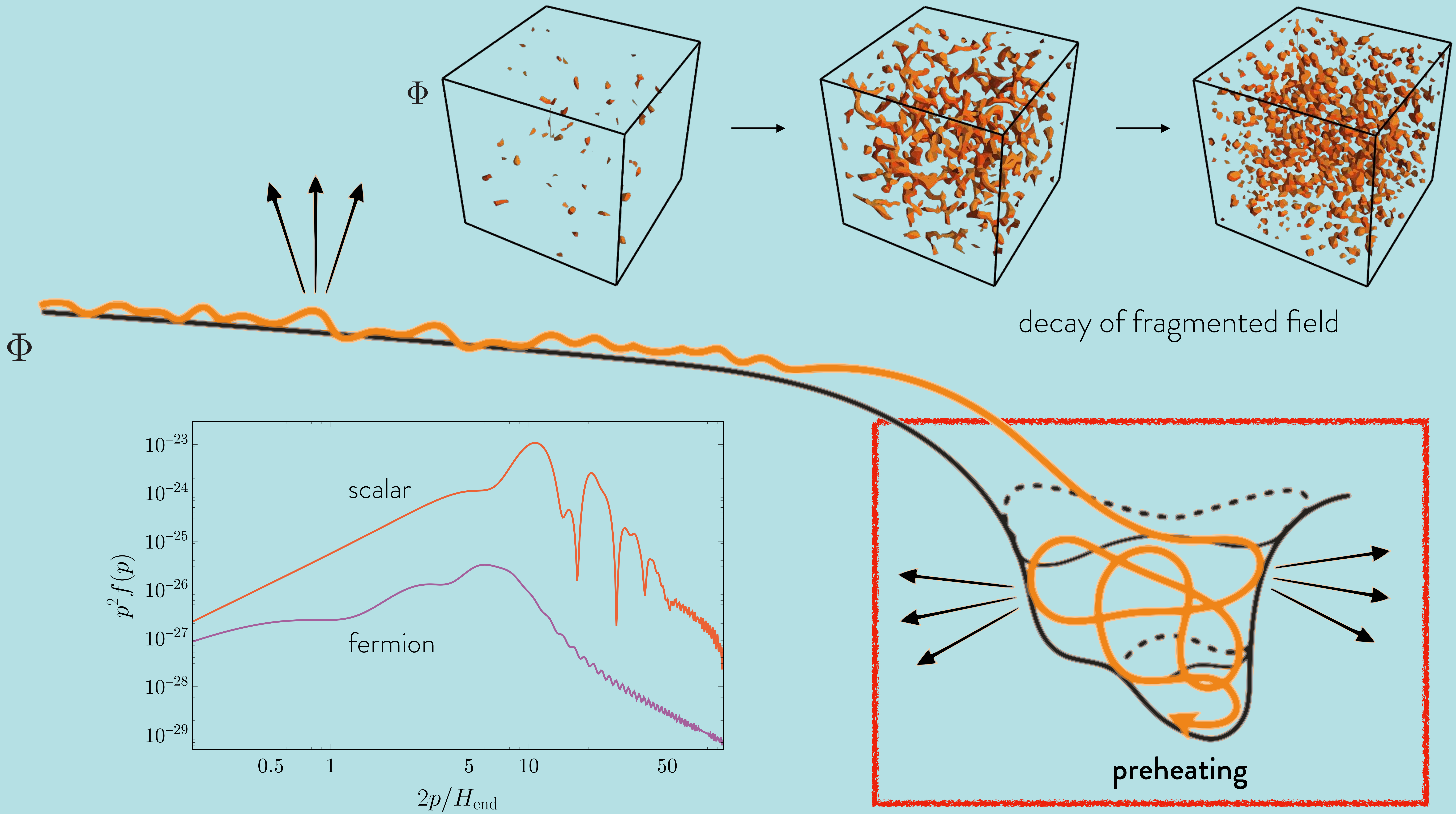


3. Lyman-α

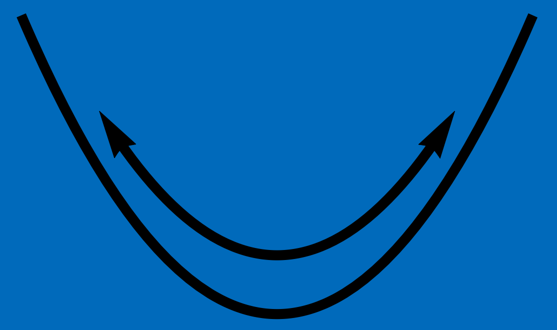


4. The end?

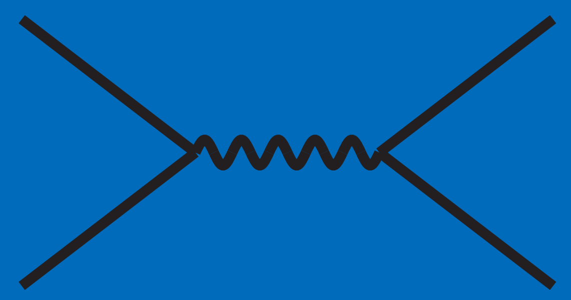
Non-perturbative particle production



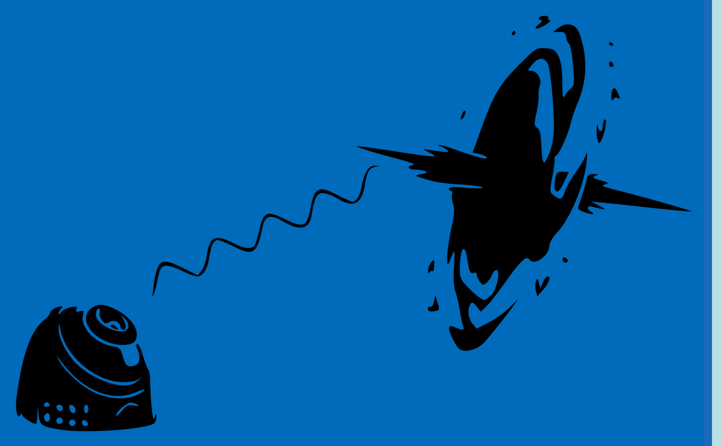
1. Reheating



2. Freeze-in

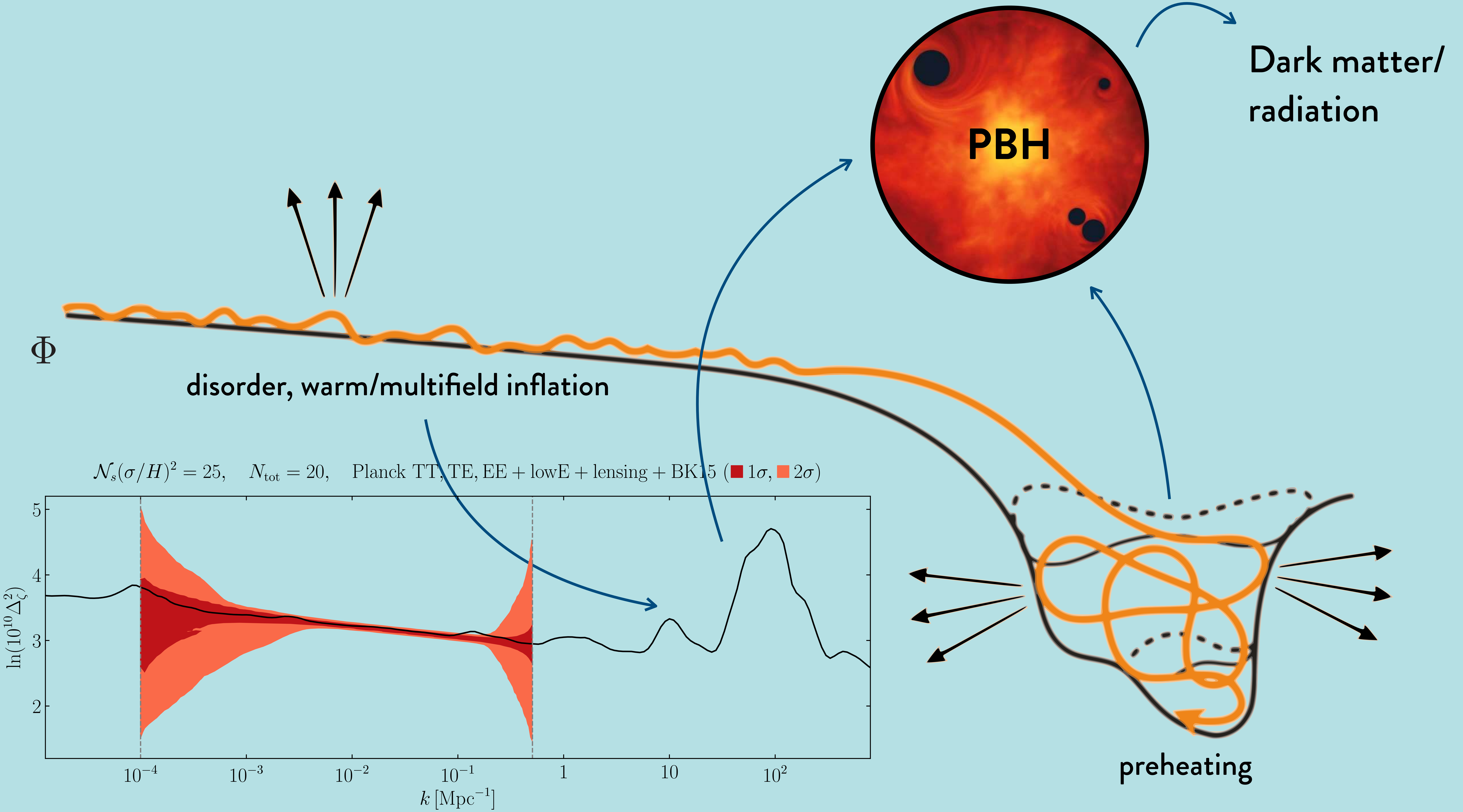


3. Lyman-α



4. The end?

Non-perturbative particle production



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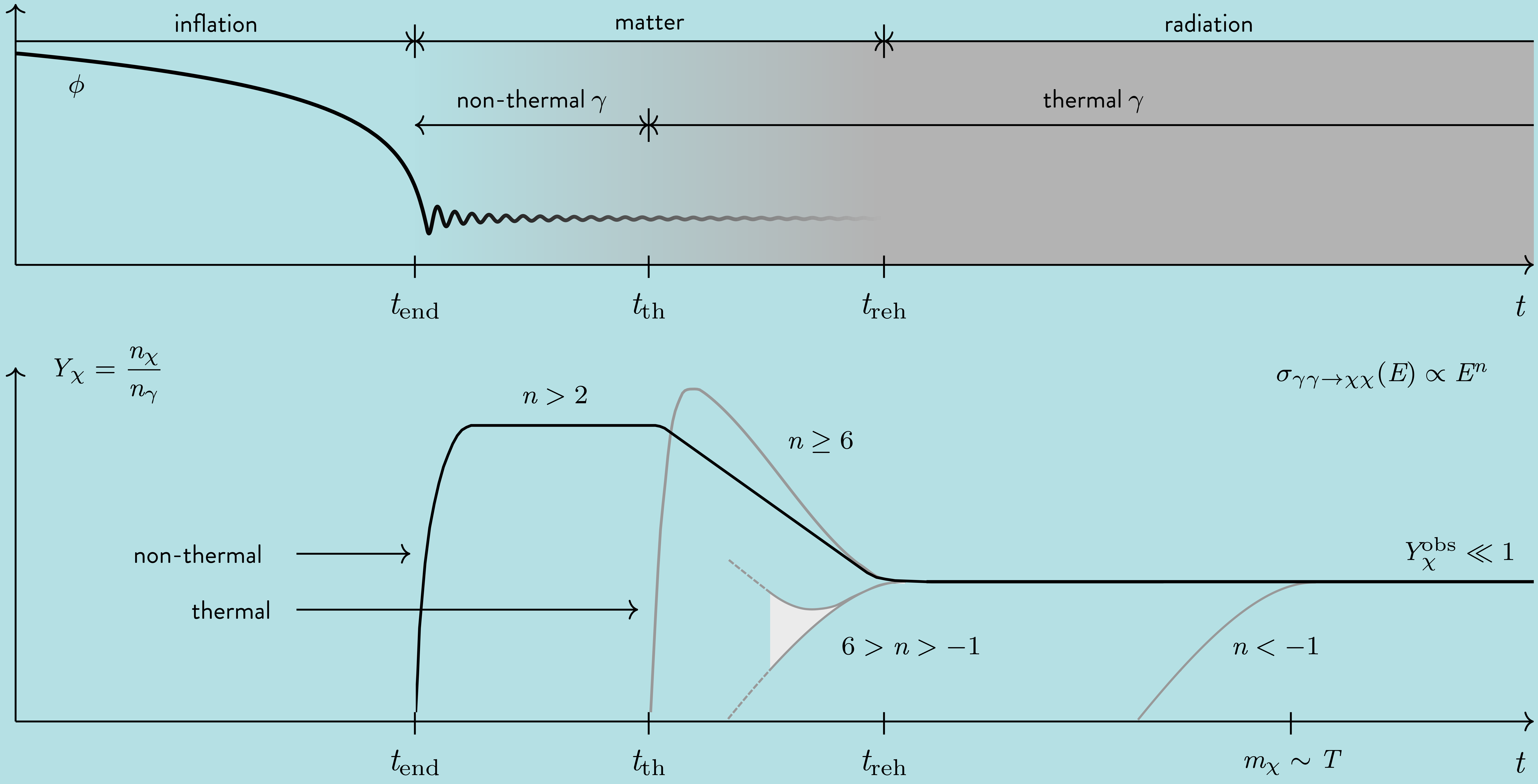
1. Reheating

2. Freeze-in

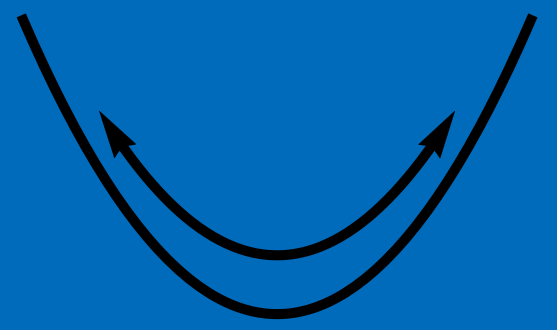
3. Lyman-α

4. The end?

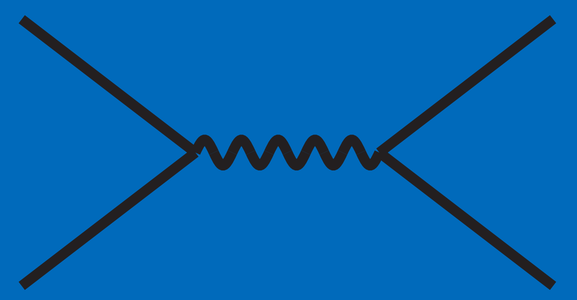
So, how warm are non-thermal relics?



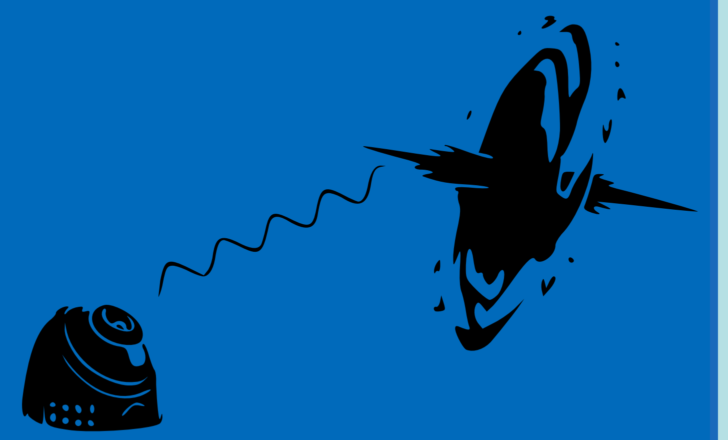
1. Reheating



2. Freeze-in

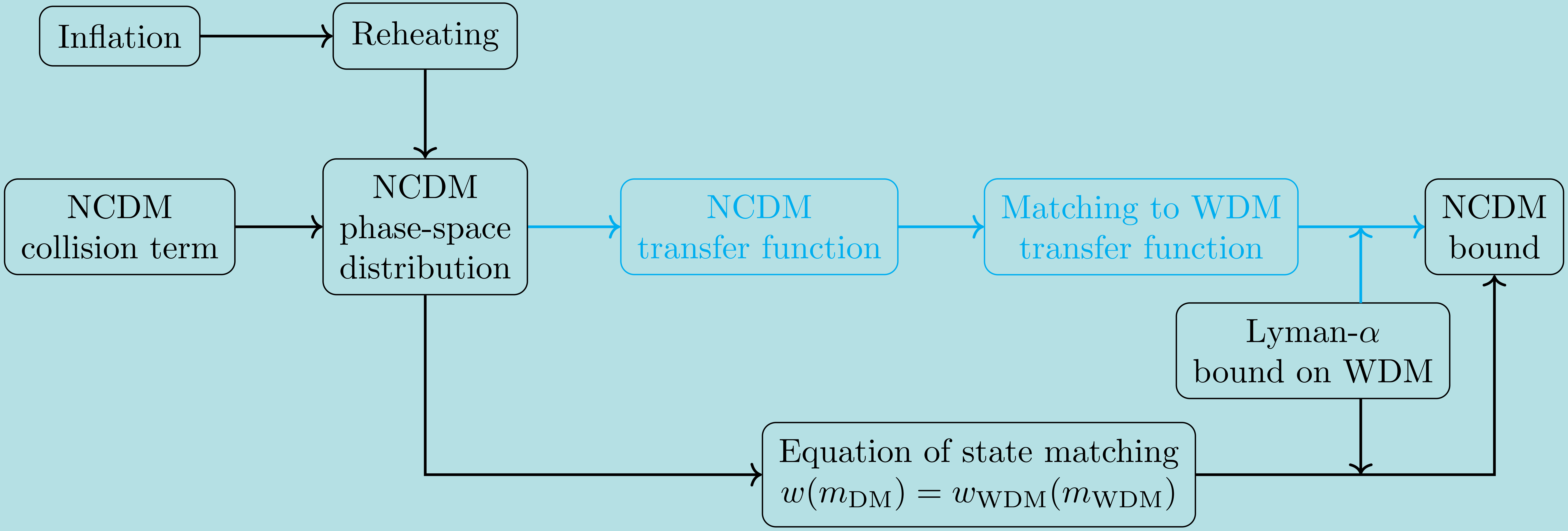


3. Lyman-α



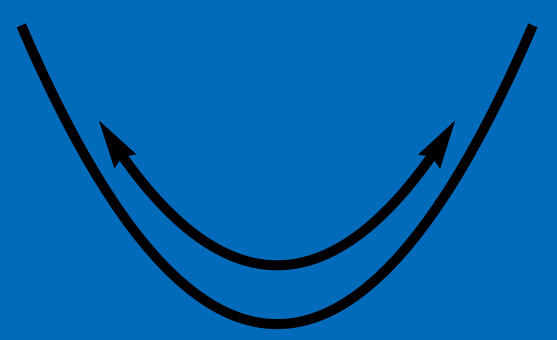
4. The end?

So, how warm are non-thermal relics?

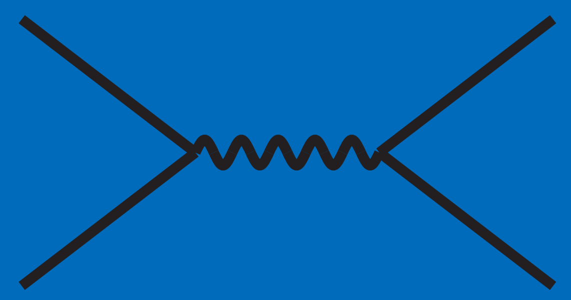


谢谢!

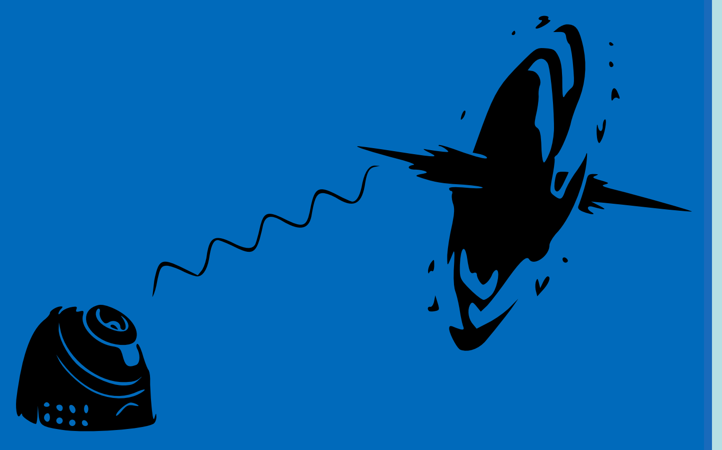
1. Reheating



2. Freeze-in



3. Lyman-α

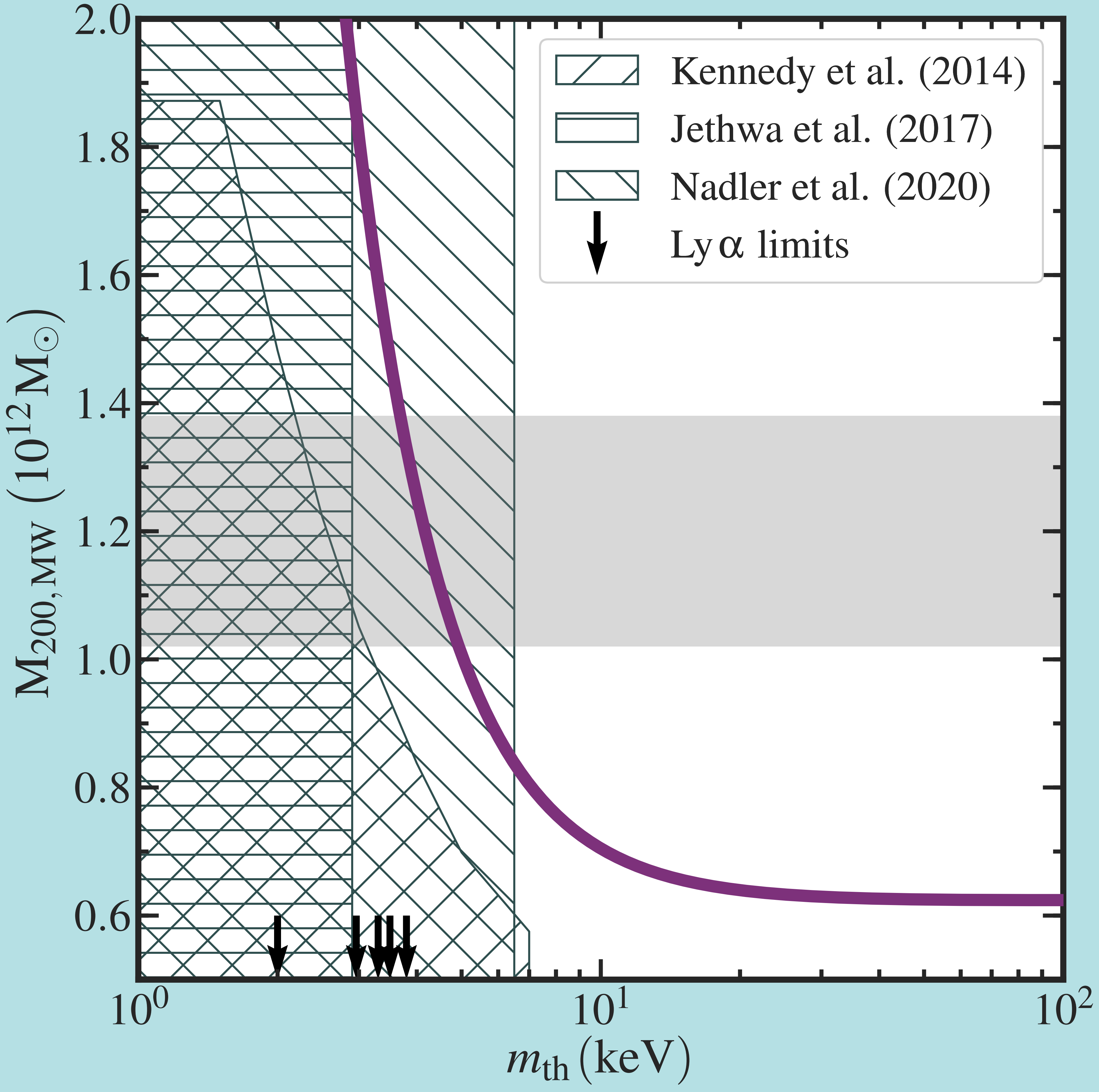


4. The end?

Constraints on the properties of warm dark matter using the satellite galaxies of the Milky Way

O. Newton et al., arXiv:2011.08865

Milky Way DM halo mass

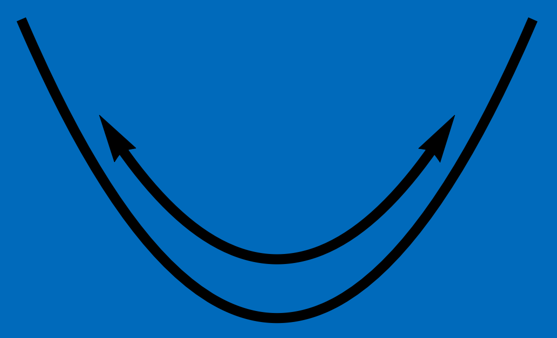


(m_{WDM})

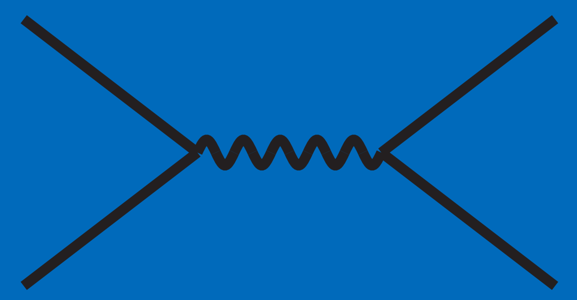
GALFORM galaxy formation model

$(z_{\mathrm{reion}} = 7)$

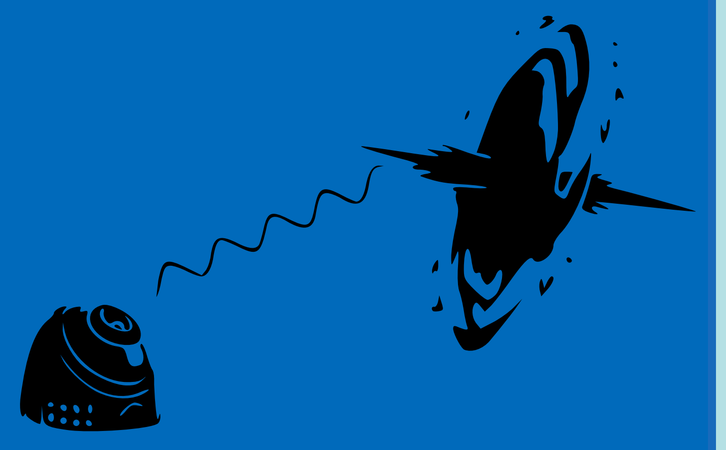
1. Reheating



2. Freeze-in



3. Lyman-α



4. The end?

Out-of-equilibrium decay chain $\Phi \to A \to \chi$

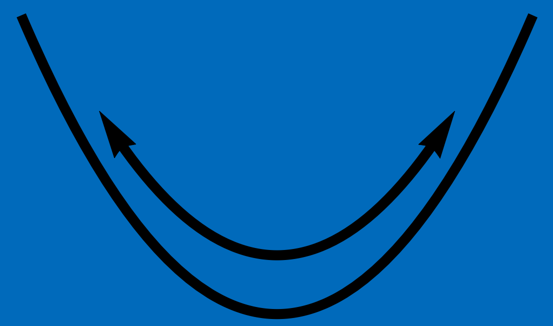
$$\Omega_\chi h^2 \simeq 0.1 \left(\frac{\mathrm{Br}_\chi}{5.5\times 10^{-4}}\right)\left(\frac{m_{\mathrm{DM}}}{1\,\mathrm{MeV}}\right)\left(\frac{T_{\mathrm{reh}}}{10^{10}\,\mathrm{GeV}}\right)\left(\frac{3\times 10^{13}\,\mathrm{GeV}}{m_\Phi}\right)$$

$$\frac{\partial f_A}{\partial t} - Hp\frac{\partial f_A}{\partial p} = -\frac{m_A \Gamma_A}{\sqrt{m_A^2+p^2}} f_A$$

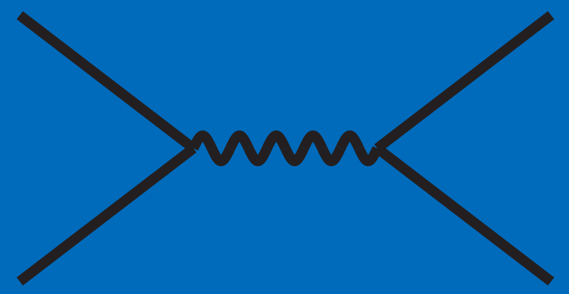
$$t_{\mathrm{dec}} \simeq \begin{cases} \Gamma_A^{-1}\,, & \dfrac{\Gamma_A}{H_A} \ll 1\,, \\ \left(\dfrac{m_\phi \langle q_A\rangle}{2 m_A \Gamma_A \Gamma_\Phi^{1/2}}\right)^{2/3}\,, & \dfrac{\Gamma_A}{H_A} \gg 1\,. \end{cases}$$

$$\mathcal{C}[f_\chi(p,t)] = \frac{4\pi^4 g_{*s}^{\mathrm{reh}} \mathrm{Br}_\chi \mathrm{Br}_A \Gamma_A m_A}{5 g_A p^2}\left(\frac{T_{\mathrm{reh}}}{m_\phi}\right)^4\left(\frac{m_\Phi}{2}\right)\left(\frac{a_{\mathrm{reh}}}{a(t)}\right) \times \int_{\left|\frac{2p}{m_\Phi}\frac{a(t)}{a_{\mathrm{reh}}} - \frac{m_A^2}{2pm_\Phi}\frac{a(t)}{a_{\mathrm{reh}}}\right|}^{\infty} \frac{z\,dz}{\sqrt{z^2+\left(\frac{2m_A a(t)}{m_\Phi a_{\mathrm{reh}}}\right)^2}}\,\bar{f}_{\mathrm{R}}(z)$$

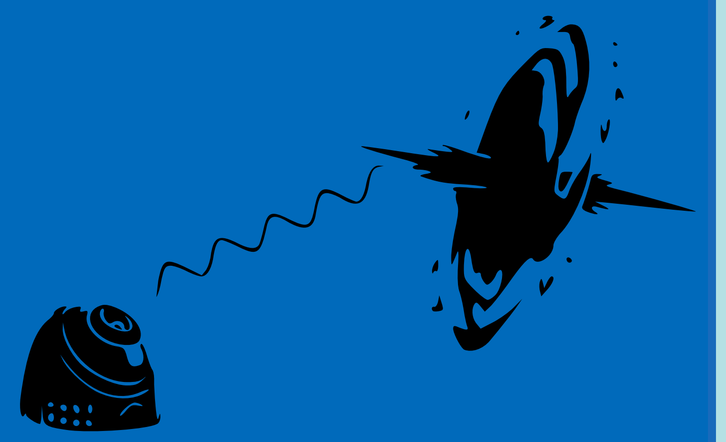
1. Reheating



2. Freeze-in



3. Lyman-α



4. The end?

Light, but not too light DM

Split energy density into relativistic and non-relativistic parts,

$$\rho_\chi = (\rho_\chi - m_{\rm DM} n_\chi) + m_{\rm DM} n_\chi \quad \Rightarrow \quad \Delta N_{\rm eff} = \frac{8}{7}\left(\frac{T}{T_\nu}\right)^4 \frac{\rho_\chi - m_{\rm DM} n_\chi}{\rho_\gamma}$$

Saturating the Ly-α bound,

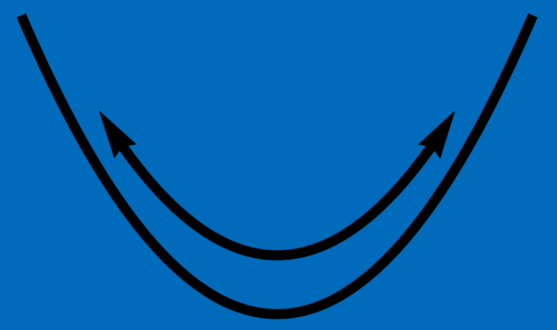
$$\Delta N_{\rm eff} < \frac{1.4\times 10^{-4}}{\sqrt{\langle q^2\rangle}} \left(\frac{g_{*s}(T)}{g^0_{*s}}\right)^{4/3} \left(\frac{\Omega_\chi h^2}{0.1}\right) \left(\frac{3\,\mathrm{keV}}{m_{\rm WDM}}\right)^{4/3} \left(\frac{T}{T_\nu}\right)^4 \left[\left\langle \sqrt{q^2 + \mu_*(T)^2}\right\rangle - \mu_*(T)\right]$$

$$\mu_*(T) \equiv \sqrt{\langle q^2\rangle} \left(\frac{g^0_{*s}}{g_{*s}(T)}\right)^{1/3} \left(\frac{3\,\mathrm{keV}}{m_{\rm WDM}}\right)^{4/3} \left(\frac{7.56\,\mathrm{keV}}{T}\right)$$

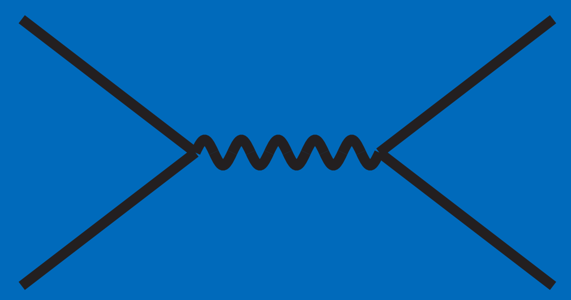
$$\Rightarrow \quad \Delta N_{\rm eff}(T_{\rm BBN}) < 5.4\times 10^{-5} \left(\frac{\langle q\rangle}{\sqrt{\langle q^2\rangle}}\right) \left(\frac{3\,\mathrm{keV}}{m_{\rm WDM}}\right)^{4/3}$$

$$\Delta N_{\rm eff}(T_{\rm CMB}) < 9\times 10^{-9} \left(\frac{3\,\mathrm{keV}}{m_{\rm WDM}}\right)^{4/3}$$

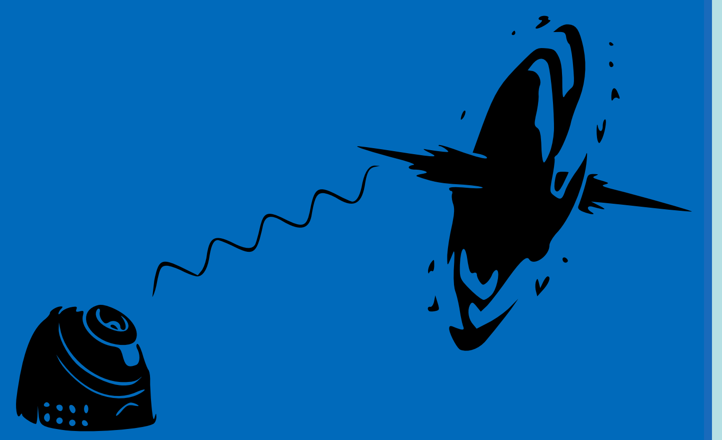
1. Reheating



2. Freeze-in

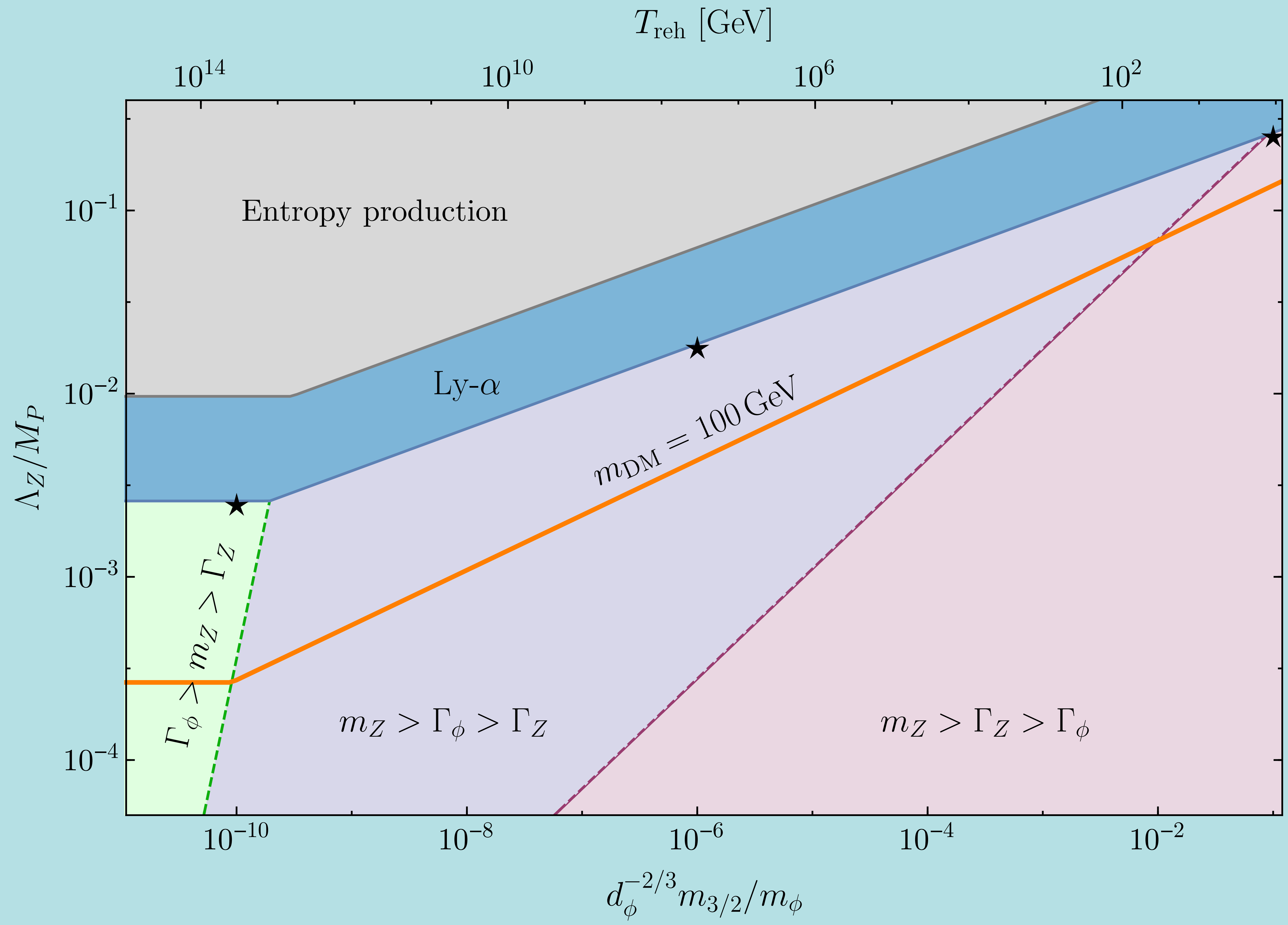


3. Lyman-α



4. The end?

Strongly stabilized moduli



$$\Delta K = -\frac{(Z\bar{Z})^2}{\Lambda_Z^2} \quad \Rightarrow \quad m_Z = \sqrt{12}\, m_{3/2}\left(\frac{M_P}{\Lambda_Z}\right), \quad Z_0 = \frac{\Lambda_Z^2}{\sqrt{6} M_P}, \quad \Gamma_Z = \frac{3\sqrt{3}\, m_{3/2}^3 M_P^3}{\pi \Lambda_Z^5}$$