A Grand-Unified Model for Cosmology

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Based on 1609.05849, with

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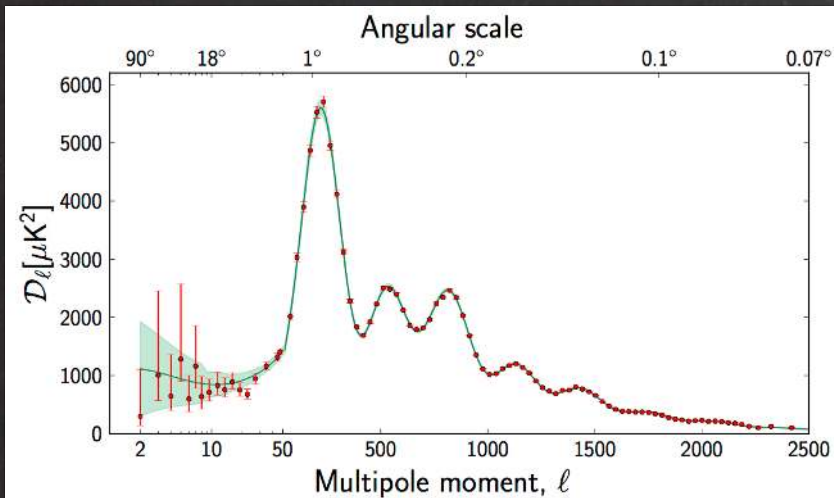
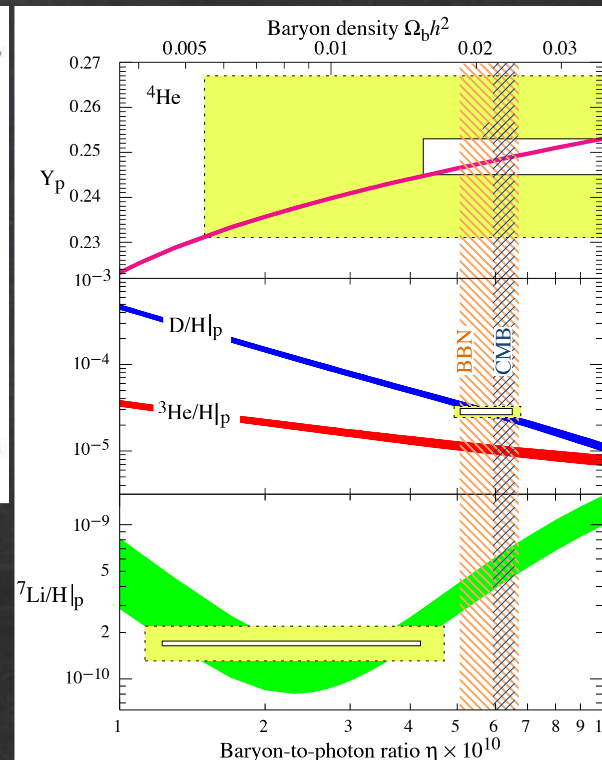
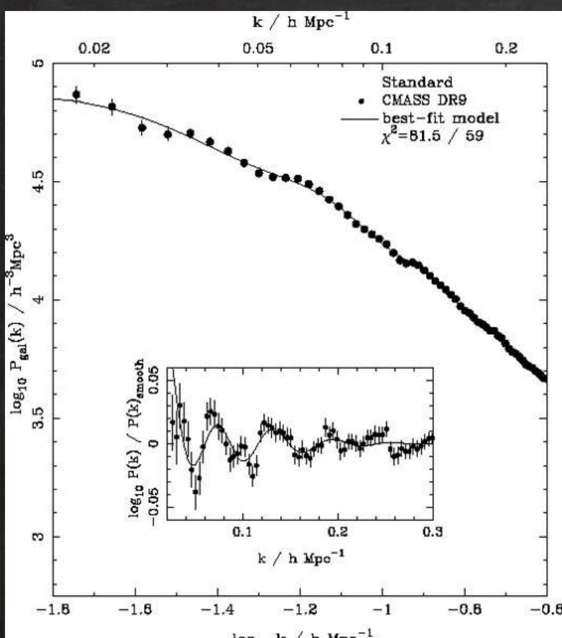
Cosmology is now a quantitative science:







- Initial conditions? (inflation)
- Matter-antimatter asymmetry?
- Dark matter?
- Neutrinos?
- Phase transitions?
- ...

Goal:

A particle physics basis for cosmology (inflation)

INTRODUCTION

Why Inflation?

Cosmological inflation is the early period of accelerated expansion, $a \sim e^{Ht}$
It provides a solution to the problem of initial conditions in standard cosmology:

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- The Flatness Problem

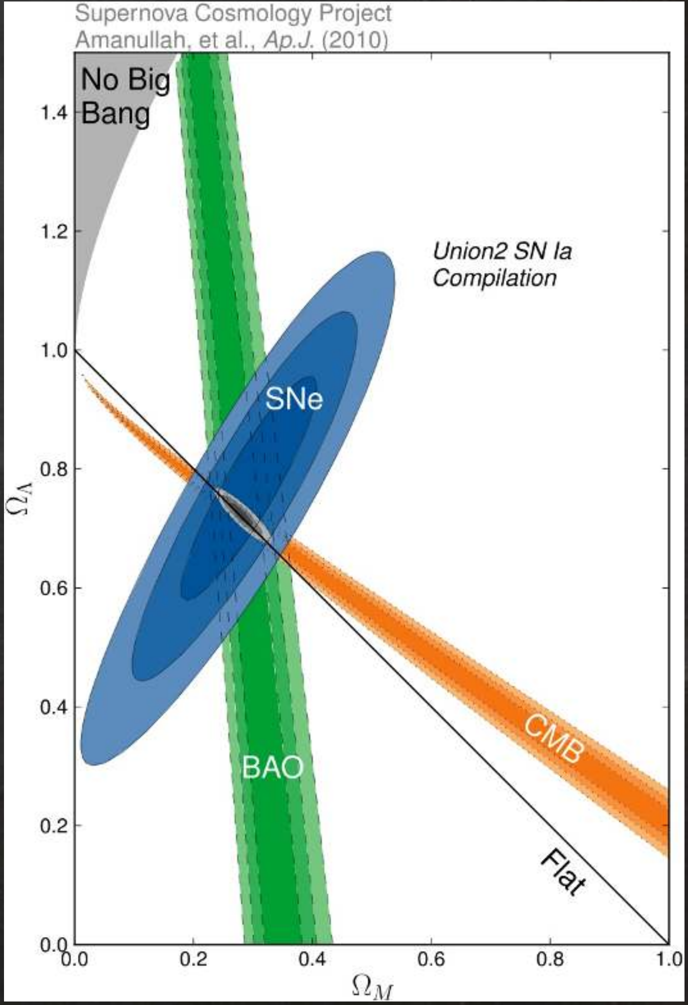
For a density $\rho_c = 3H^2 M_P^2$, $k = 0$

$$\Omega = \rho/\rho_c = 1.000 \pm 0.005$$ (Planck, 2015)

But the condition $\Omega \approx 1$ is unstable!

$$|\Omega - 1|_{\mathrm{GUT}} < 10^{-57}$$

Inflation $\Rightarrow$ $|\Omega - 1| \propto e^{-2Ht}$ ✓



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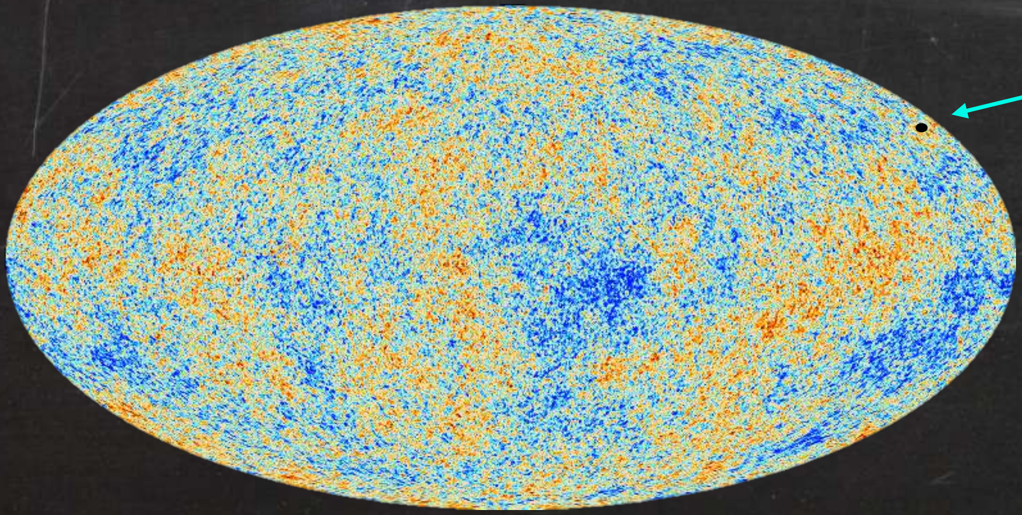
- The Flatness Problem
- The Horizon Problem

If the Universe is dominated by matter or radiation, the particle horizon is

$$L_p = a(t)\int_0^t \frac{dt'}{a(t')} \sim H^{-1}$$



Size of particle horizon at recombination

But $\frac{\Delta T}{T} \sim 10^{-5}$

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Inflation

$$\Downarrow$$

$$L_p(t) = H^{-1}(e^{Ht} - 1)$$ ✓

Why Inflation?

Cosmological inflation is the early period of accelerated expansion, $a \sim e^{Ht}$
It provides a solution to the problem of initial conditions in standard cosmology:

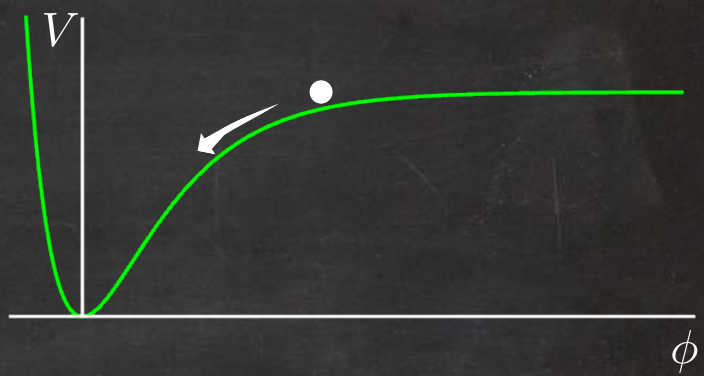
- The Flatness Problem
- The Horizon Problem
- Small Scale Inhomogeneities

There is a plethora of structure on smaller scales (stars, galaxies, clusters,…)

Quantum fluctuations during inflation are stretched into macroscopic scales; they are "frozen in". At the later stages of radiation and matter domination, these fluctuations set the initial conditions for structure formation

A scalar field slowly rolling in its potential

$$\mathcal{S} = \int d^4x\,\sqrt{-g}\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]$$



$$\Rightarrow \left[\begin{array}{l} \ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \\ H^2 = \dfrac{1}{3M_P^2}\left[\dfrac{1}{2}\dot{\phi}^2 + V(\phi)\right] \end{array}\right. \qquad \Rightarrow \qquad \ln\left(\frac{a_f}{a_i}\right) \gg 1$$

Easy to realize:

$$V(\phi) \sim \begin{cases} \phi & ✔ \\ \phi^2 & ✔ \\ \phi^4 & ✔ \\ \vdots & \\ e^{\phi} & ✘ \end{cases}$$

The power spectrum of perturbations

$$\phi(t, x_i) \;=\; \phi(t) + \delta\phi(t, x_i)$$

$$ds^2 = (1+2\Phi)dt^2 - 2a(t)B_i dx^i dt - a^2(t)[(1+2\Psi)\delta_{ij} + 2E_{ij}]dx^i dx^j$$

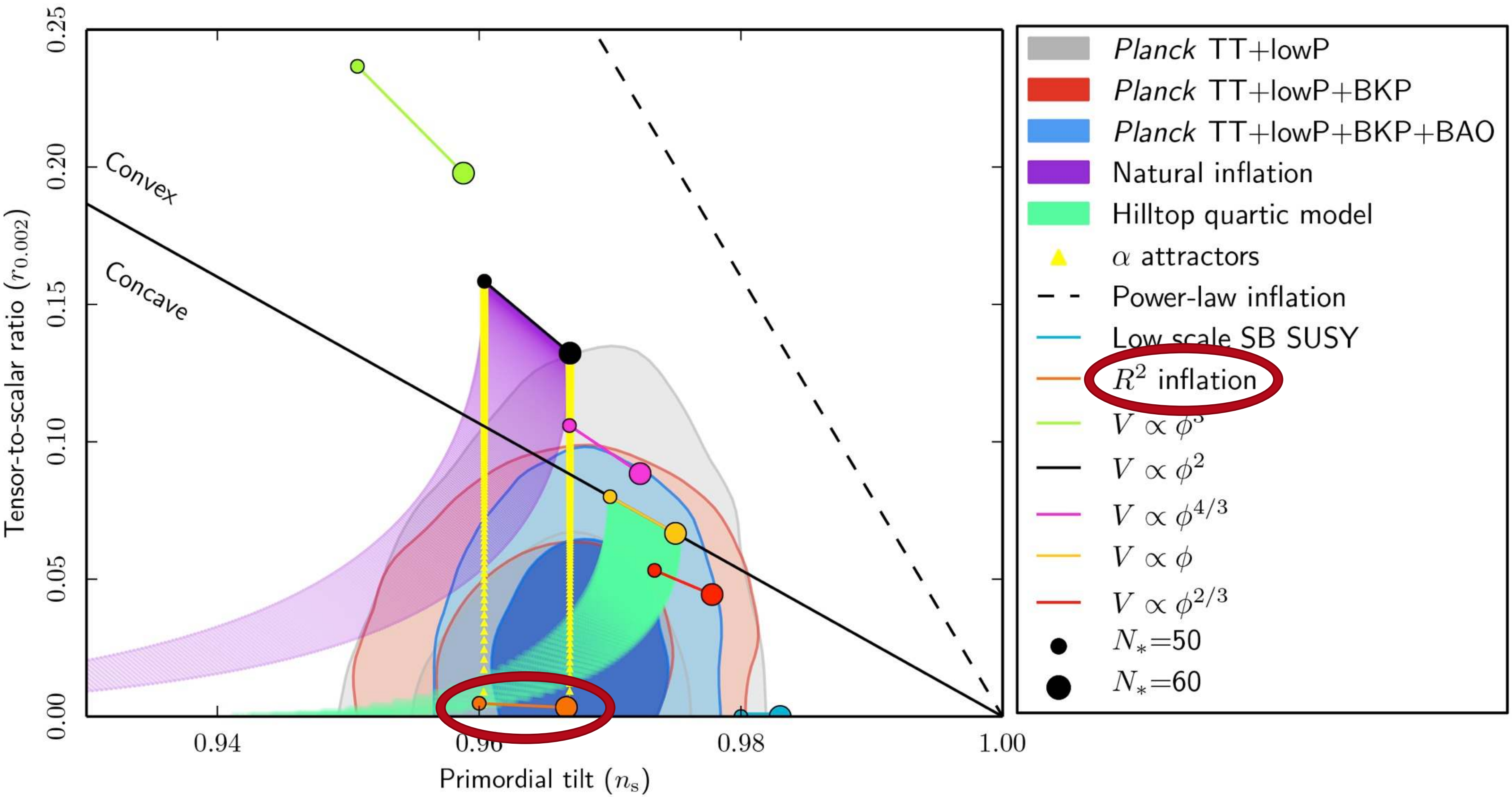
- Scalar

$$\langle \mathcal{R}(k)\mathcal{R}(k')\rangle = \frac{2\pi^2}{k^3}\mathcal{P}_{\mathcal{R}}\,\delta(k-k') \longrightarrow \mathcal{P}_{\mathcal{R}} = \underbrace{\frac{H^4}{4\pi^2\dot{\phi}^2}}_{A_S}\left(\frac{k}{aH}\right)^{n_s-1}$$

- Tensor

$$\sum_{\gamma=+,\times}\langle h_{\mathbf{k},\gamma}\bar{h}_{\mathbf{k}',\gamma}\rangle = \frac{2\pi^2}{k^3}\mathcal{P}_{\mathcal{T}}\,\delta(k-k') \longrightarrow \mathcal{P}_{\mathcal{T}} = \underbrace{\frac{2}{\pi^2}H^2}_{A_T}\left(\frac{k}{aH}\right)^{n_T}$$

$$\ln(10^{10} A_S) = 3.094 \pm 0.034, \qquad n_s = 1 + \frac{d \log \mathcal{P}_{\mathcal{R}}}{d \log k} \qquad r = \frac{\mathcal{P}_T}{\mathcal{P}_{\mathcal{R}}}$$



$$S = \frac{1}{2}\int d^4x\,\sqrt{-g}\left(-R + \frac{R^2}{6m^2}\right)$$

(Starobinsky, 1980)

"change of variables" (conformal transformation)

$$= \int d^4x\,\sqrt{-\tilde{g}}\left[-\frac{\tilde{R}}{2} + \frac{1}{2}(\partial_\mu\phi')^2 - \underbrace{\frac{3}{4}m^2\left(1 - e^{-\sqrt{2/3}\phi'}\right)^2}_{\text{Starobinsky potential}}\right]$$

$$n_s \approx 0.961\ -\ 0.968$$

$$r \approx 0.0030\ -\ 0.0042$$

$$m \simeq 10^{-5}M_P$$

The Standard Model

$SU(3)_C \times SU(2)_L \times U(1)_Y$

Q: u_L, d_L

L: e_L, ν_L

u_R, d_R

e_R

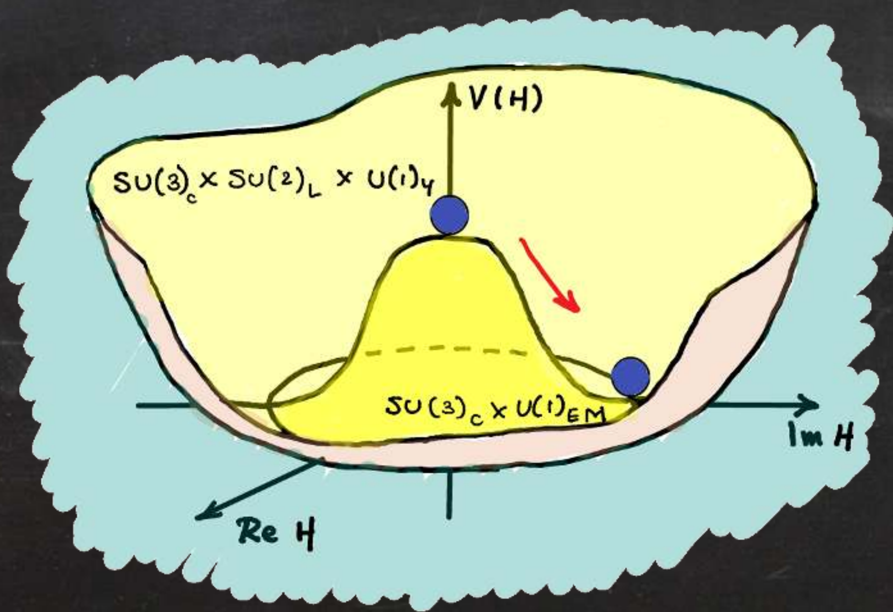
(×3)

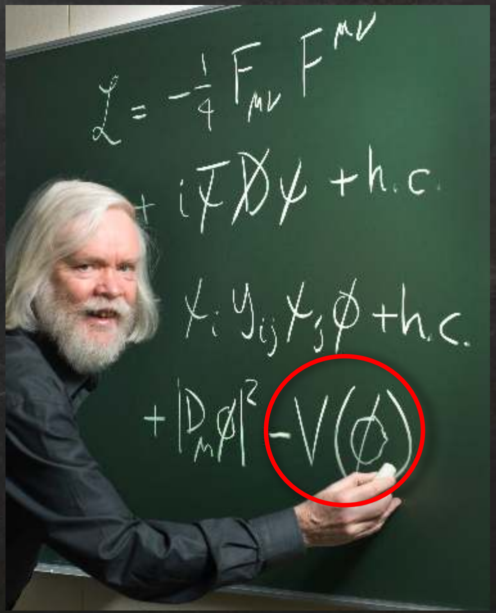
$g_1 \cdots g_8$

$W^{\pm}$ W^0

B^0

H





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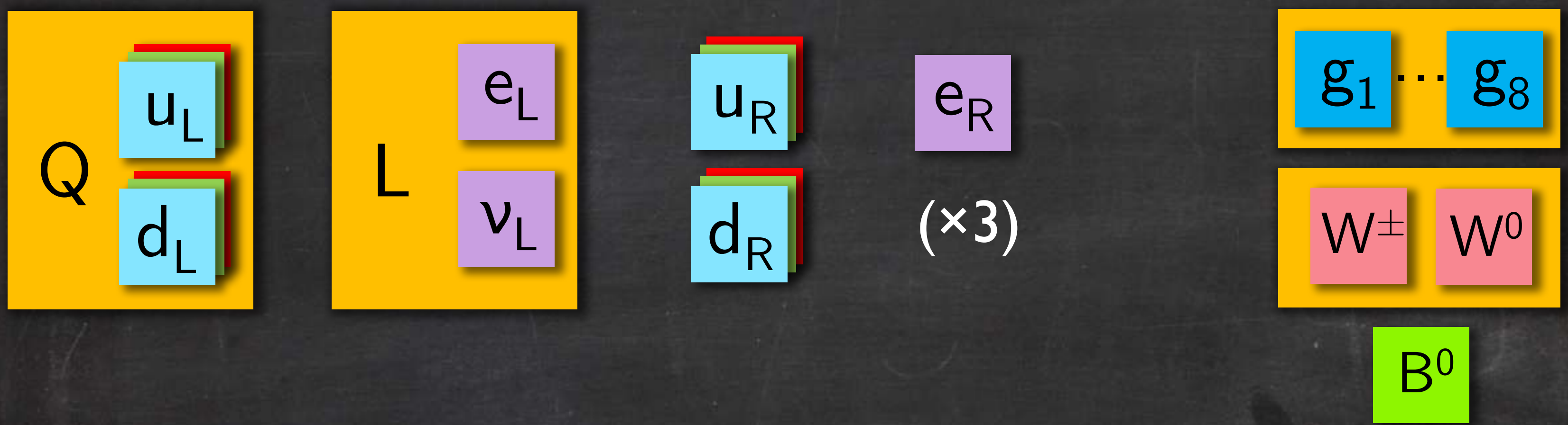
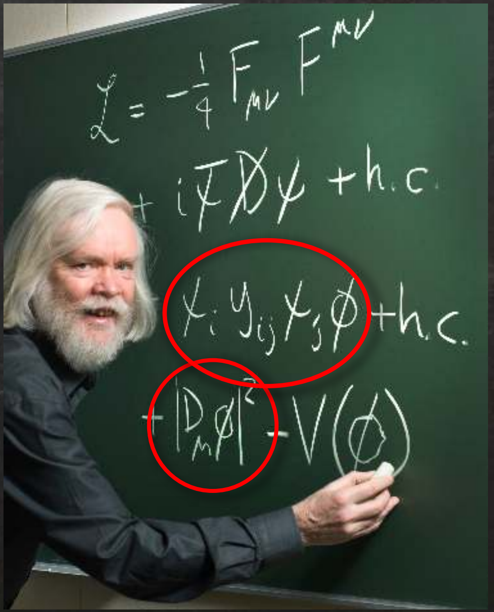


H

$$-\mathcal{L}_{\rm mass} \sim \underbrace{y\langle H\rangle}_{m_\psi} \left(\bar\psi_L\psi_R + \bar\psi_R\psi_L\right)$$

$$+ \underbrace{g^2\langle H\rangle}_{m_A^2} A_\mu A^\mu$$



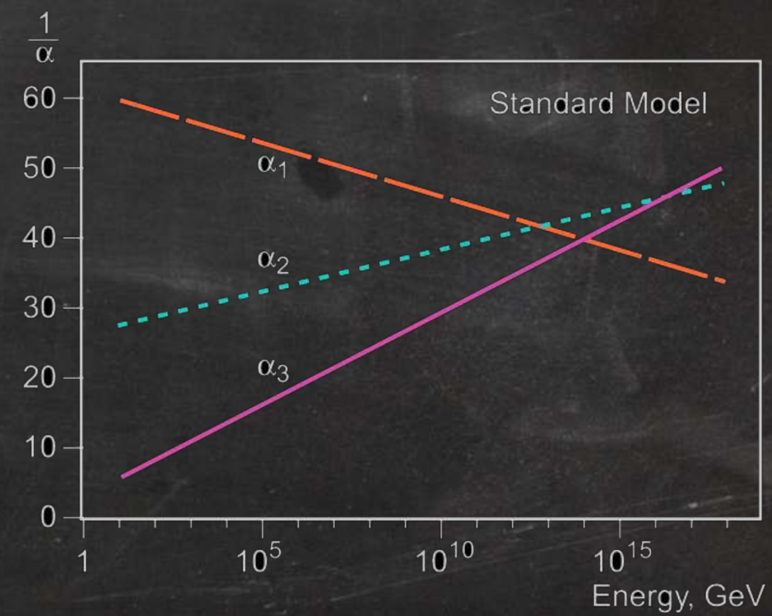
Grand Unification

The Standard Model provides a consistent picture of the dynamics and symmetries of the strong, weak and EM interactions. However…

- No understanding of discrete nature of electric charge
- Unification is partial, with seemingly unrelated interaction strengths

$$\text{Forces} \begin{cases} \text{Color} & SU(3) \\ \text{Weak} & SU(2) \\ \text{EM} & U(1) \end{cases}$$

$$\text{Matter} \begin{cases} \text{Quarks} \\ \text{Leptons} \end{cases}$$



- Mysterious technical coincidences (anomaly cancellation,…)

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$$\text{Forces}\left\{\begin{array}{ll}\text{Color} & SU(3)\\ \text{Weak} & SU(2)\\ \text{EM} & U(1)\end{array}\right\}$$

Extend the SM with a "bigger" interaction

$$SU(5)\xrightarrow{M_{\text{GUT}}} SU(3)\times SU(2)\times U(1)\xrightarrow{M_W} SU(3)\times U(1)_{\text{EM}}$$

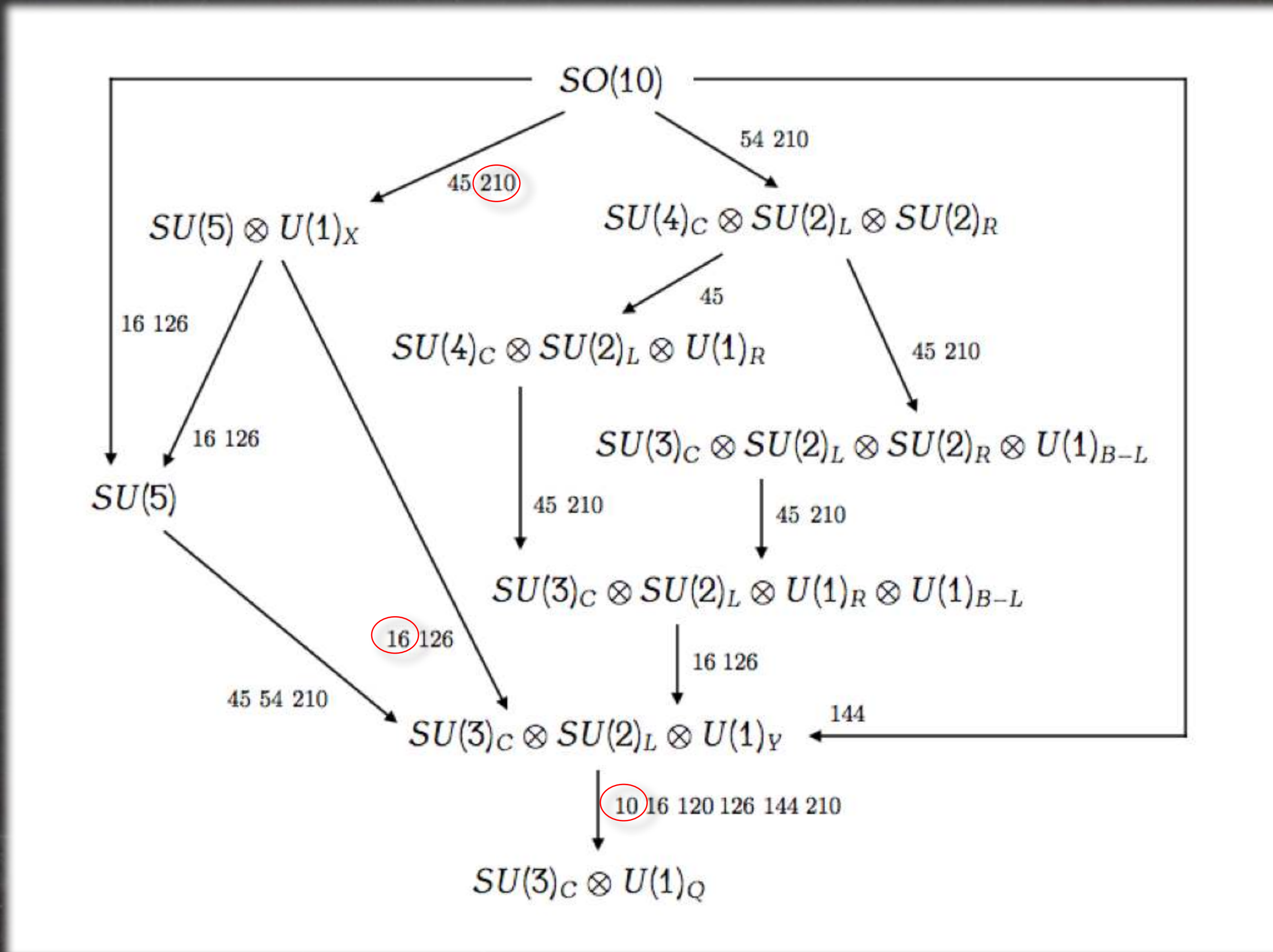
$$\text{Matter}\left\{\begin{array}{l}\text{Quarks}\\ \text{Leptons}\end{array}\right\}$$

Partial Q-L unification

$$d_R\leftrightarrow e_L,\nu_L$$
$$e_R\leftrightarrow u_R,u_L,d_L$$

- Mysterious technical coincidences (anomaly cancellation,…)

Gauge and Matter Unification: SO(10)



(from 1110.3210)

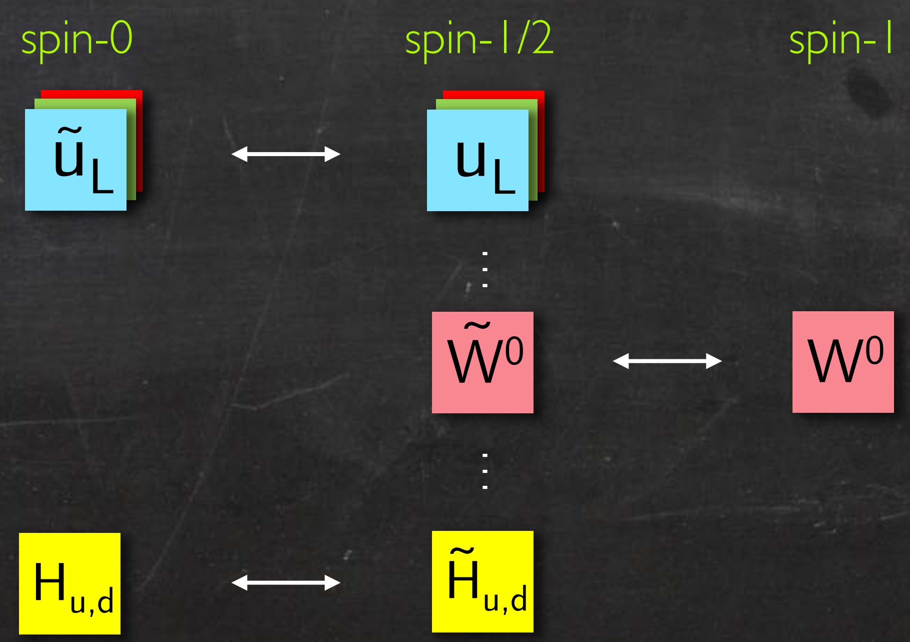
Spinor $\mathbf{16}$: $\psi \supset (Q, u^c, d^c, L, \nu^c, e^c)$ (right-handed neutrino)

Why Supersymmetry?

It is the spacetime symmetry that relates bosons and fermions. If it is realized in nature, it must be broken.

$$Q|\mathrm{Boson}\rangle = |\mathrm{Fermion}\rangle\,, \qquad Q|\mathrm{Fermion}\rangle = |\mathrm{Boson}\rangle$$

Lorentz invariance: particles $\longleftrightarrow$ antiparticles
Supersymmetry: particles $\longleftrightarrow$ sparticles



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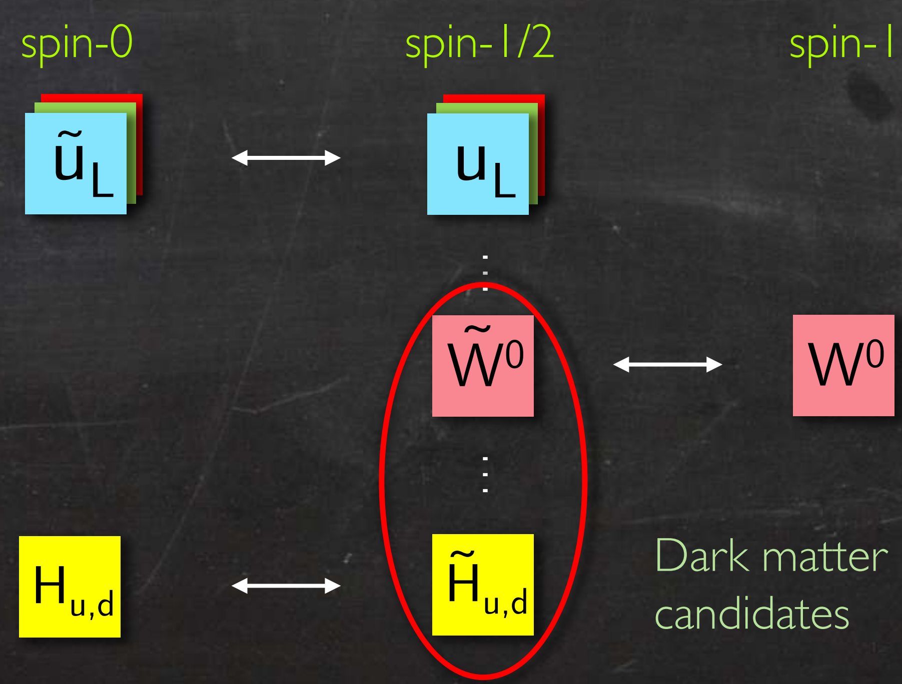
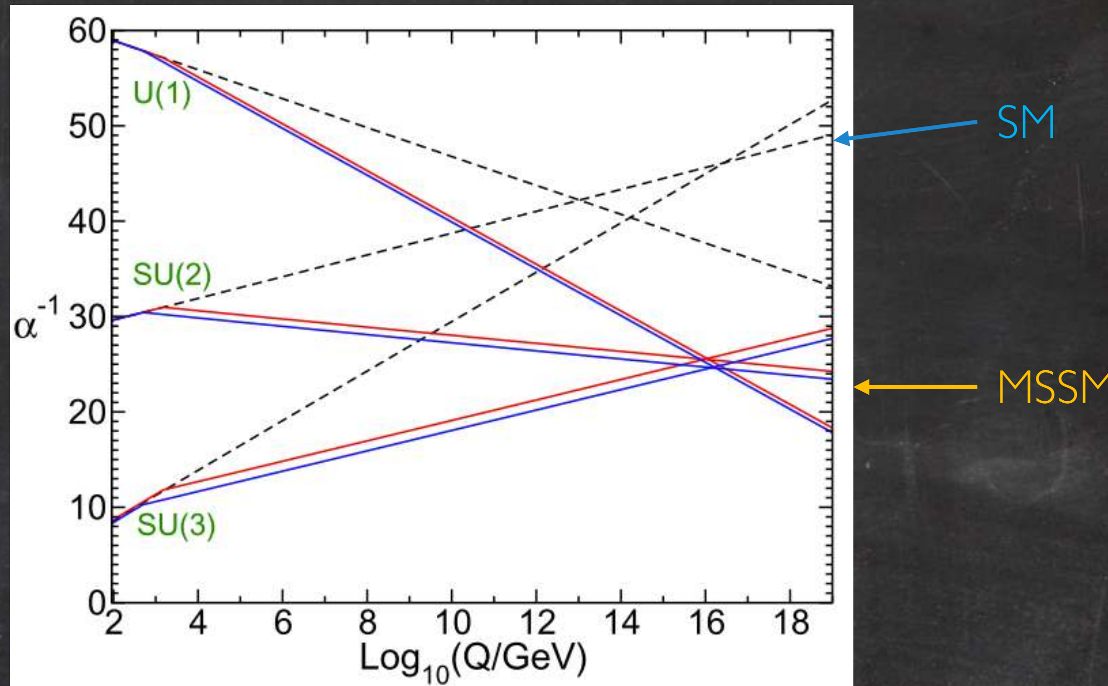


SUSY allows gauge coupling unification



Radiative corrections are under control

$$\Delta m_H^2 \sim \qquad -\Lambda^2 \qquad\qquad +\Lambda^2$$

When supersymmetry is promoted to a local symmetry, it automatically contains GR

$\Rightarrow$ SUPERGRAVITY

spin-2 graviton e_μ^a $\longleftrightarrow$ ψ_μ spin-3/2 gravitino

Blessing: all couplings determined by only two functions

$K(\Phi,\bar{\Phi})$ Kähler potential

$W(\Phi)$ superpotential

Curse: inflation is difficult to realize

$$V = -e^K\left[(K^{-1})_i^j(K^iW+W^i)(K_j\bar{W}+\bar{W}_j)-3|W|^2\right]$$

$$\sim \underbrace{e^{\phi^2}}_{\text{too steep!}}\underbrace{\left[\lambda_0+\lambda_1\phi+\cdots+\lambda_8\phi^8\right]}_{O(1)\text{ ``holes'' with } V_0\neq 0}$$

NO-SCALE SO(10) INFLATION

Model wish-list:

- ✓ Planck-compatible inflation
- ✓ The right amount of reheating
- ✓ GUT and SM breaking
- ✓ Neutrino masses
- ✓ Doublet-triplet splitting
- ✓ Baryogenesis
- ✓ Proton decay
- ✓ Dark matter

No-scale supergravity naturally contains R²-inflation

- A very symmetric theory; the EFT of the compactified string

$$K = -3\ln\underbrace{\left(T + \bar{T} - |\phi|^2/3 + \cdots\right)}_{SU(N,1)\,/\,SU(N)\times U(1)} + \cdots$$

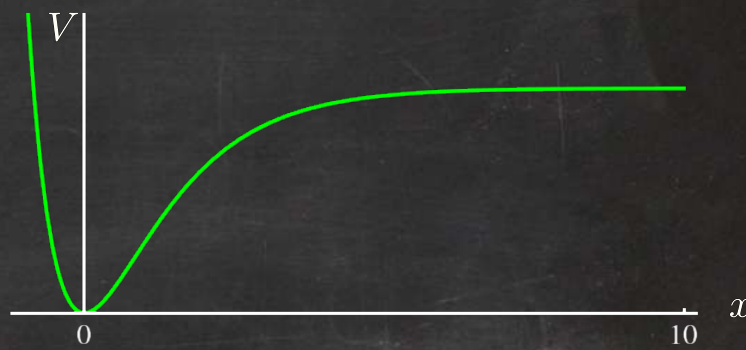
- The scalar potential is naturally flat; flatness can be lifted by mass parameters that break the underlying symmetry

protected by supersymmetry

- Supersymmetry is broken through radiative corrections

- Cecotti (1987):

$$K = -3\ln\left(T + \bar{T} - \frac{|\phi|^2}{3}\right)$$
$$W = \sqrt{3}m\phi(T - 1/2)$$
$$\mathrm{Re}\, T = \frac{1}{2}e^{\sqrt{2/3}x}$$

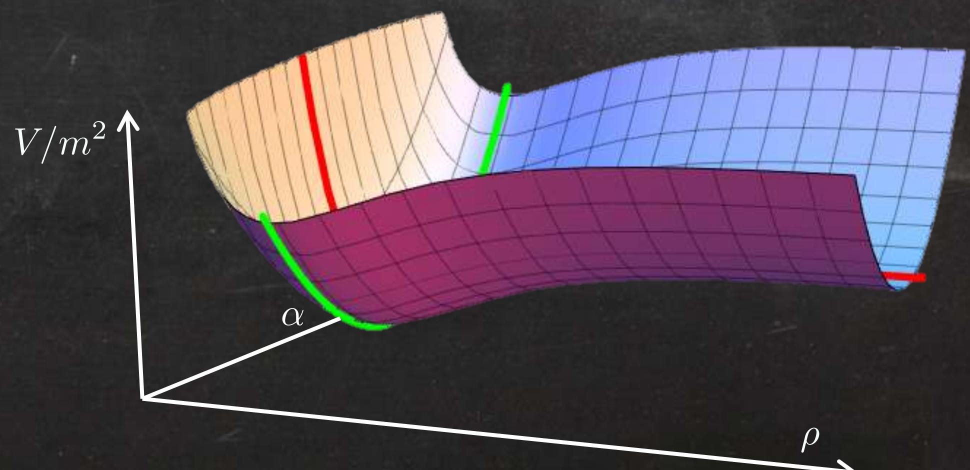


- Ellis, Nanopoulos, Olive (2013):

$$K = -3\ln\left(T + \bar{T} - \frac{|\phi|^2}{3}\right)$$
$$W = m\left(\frac{\phi^2}{2} - \frac{\phi^3}{3\sqrt{3}}\right)$$
$$\phi = \sqrt{3}\tanh(x/\sqrt{6})$$

- Ellis, MG, Nanopoulos, Olive (2014):

$$K = -3\ln(T + \bar{T}) + \frac{|\varphi|^2}{(T + \bar{T})^3}$$
$$W = \sqrt{3}m\varphi(T - 1/2)$$
$$T = \frac{1}{2}e^{-\sqrt{2/3}\rho} + \frac{i}{\sqrt{6}}\alpha$$



Inflaton: a SO(10) singlet S (×3)

16 matter: $\psi \supset (Q, u^c, d^c, L, \nu^c, e^c)$

210 Higgs: $\Sigma \supset (p, a, \omega, \ldots)$ $\qquad SO(10) \longrightarrow SU(5) \times U(1)$

16 $\overline{\mathbf{16}}$ Higgs: $\Phi \supset (\phi_R, \ldots)$ $\qquad SU(5) \times U(1) \longrightarrow SU(3) \times SU(2) \times U(1)$

10 Higgs: $H \supset (H_u, H_d, \ldots)$ $\qquad SU(3) \times SU(2) \times U(1) \longrightarrow SU(3) \times U(1)_{\mathrm{EM}}$

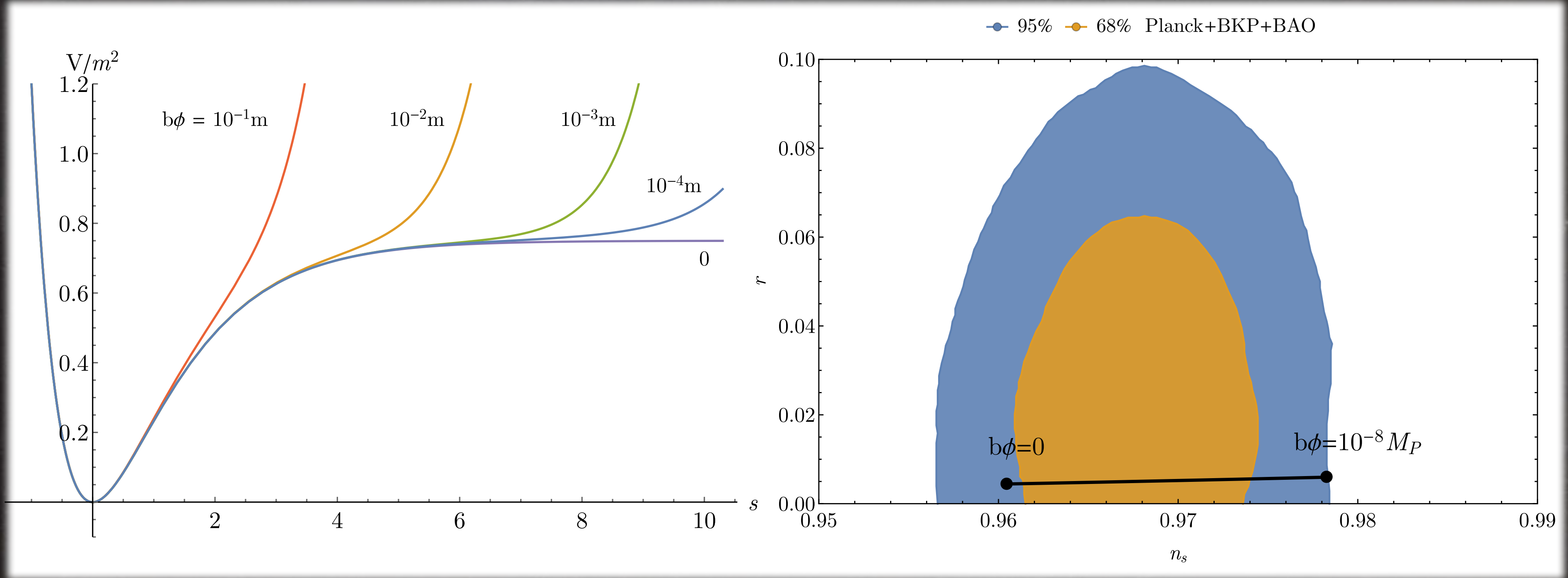
1 'Higgs': T $\qquad$ break SUSY

$$K = -3 \ln \left[T + T^* - \frac{1}{3} \left(S^* S + H^\dagger H + \psi^\dagger \psi + \Phi^\dagger \Phi + \bar{\Phi}^\dagger \bar{\Phi} + \frac{1}{4!} \Sigma^\dagger \Sigma \right) \right]$$

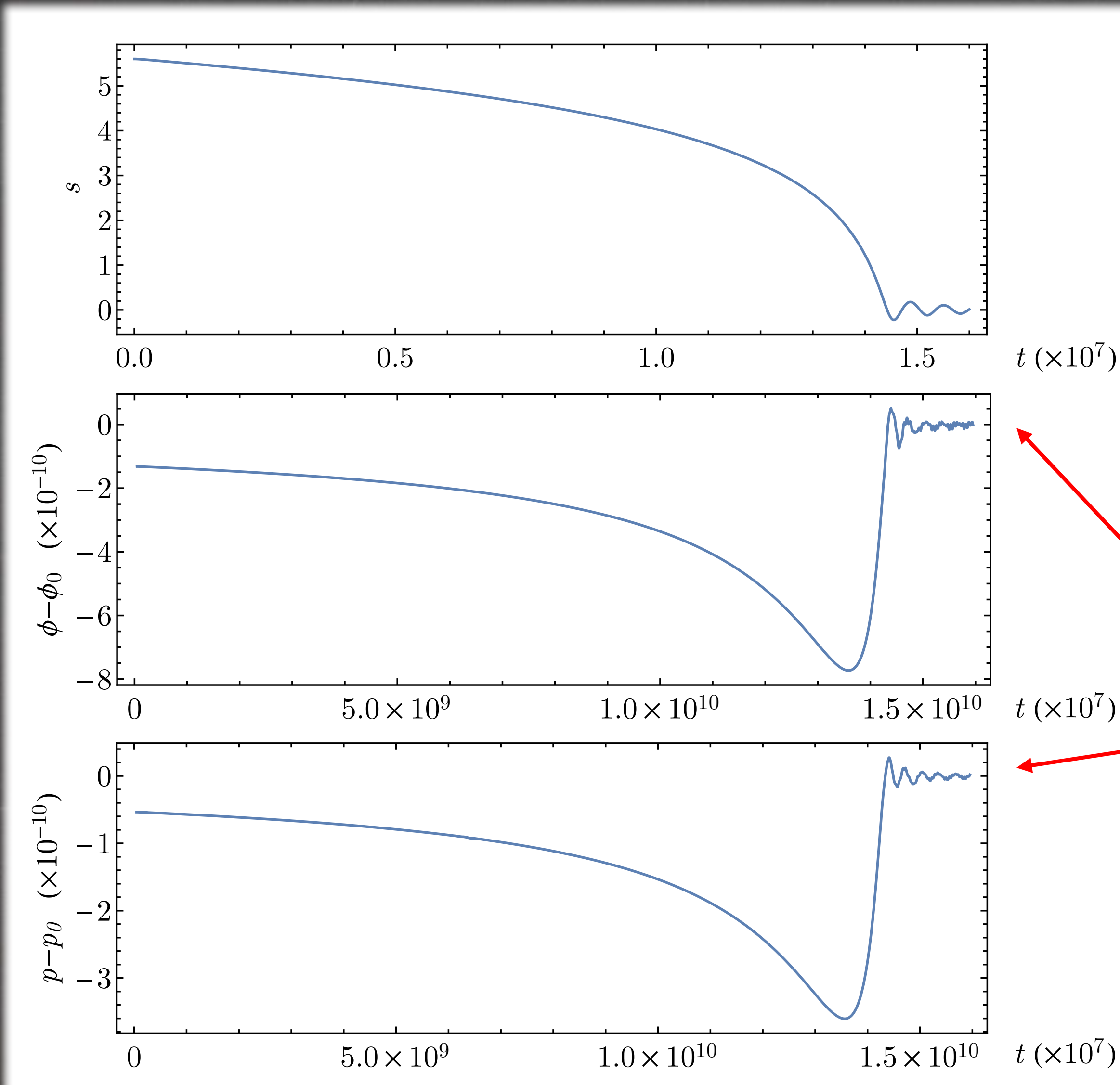
$$W = \frac{m}{2} S^2 - \frac{\lambda}{3} S^3 + bS\bar{\Phi}\psi + m_\Phi \bar{\Phi}\Phi + \frac{\eta}{4!} \bar{\Phi}\Phi\Sigma + \frac{m_\Sigma}{4!} \Sigma^2 + \frac{\Lambda}{4!} \Sigma^3$$
$$+ yH\psi\psi + m_H H^2 + H(\alpha\Phi\Phi + \bar{\alpha}\bar{\Phi}\bar{\Phi}) + \text{constrained}$$

Deformation of the inflationary potential

$$V \simeq \frac{3}{4} m^2 \left(1 - e^{-\sqrt{2/3}\, s}\right)^2 + \frac{3}{4} |b\phi|^2 \sinh^2(\sqrt{2/3}\, s)$$



Time evolution of scalar fields:



GUT-breaking minimum

Neutrino masses

$$\mathcal{L}_{\rm mass} = -\begin{pmatrix}\overline{\nu}_L & \overline{\nu}_R^c & \overline{\tilde{S}}\end{pmatrix}\begin{pmatrix}0 & -m_u & 0\\ -m_u & 0 & -b\phi\\ 0 & -b\phi & m\end{pmatrix}\begin{pmatrix}\nu_L\\ \nu_R^c\\ \tilde{S}\end{pmatrix}$$

$$m_u \ll b\phi \ll m$$

$$\Rightarrow \qquad \mathcal{M}_{\rm diag} \simeq \begin{pmatrix} m\left(\frac{m_u}{b\phi}\right)^2 & 0 & 0\\ 0 & -\frac{(b\phi)^2}{m} & 0\\ 0 & 0 & m\end{pmatrix}$$

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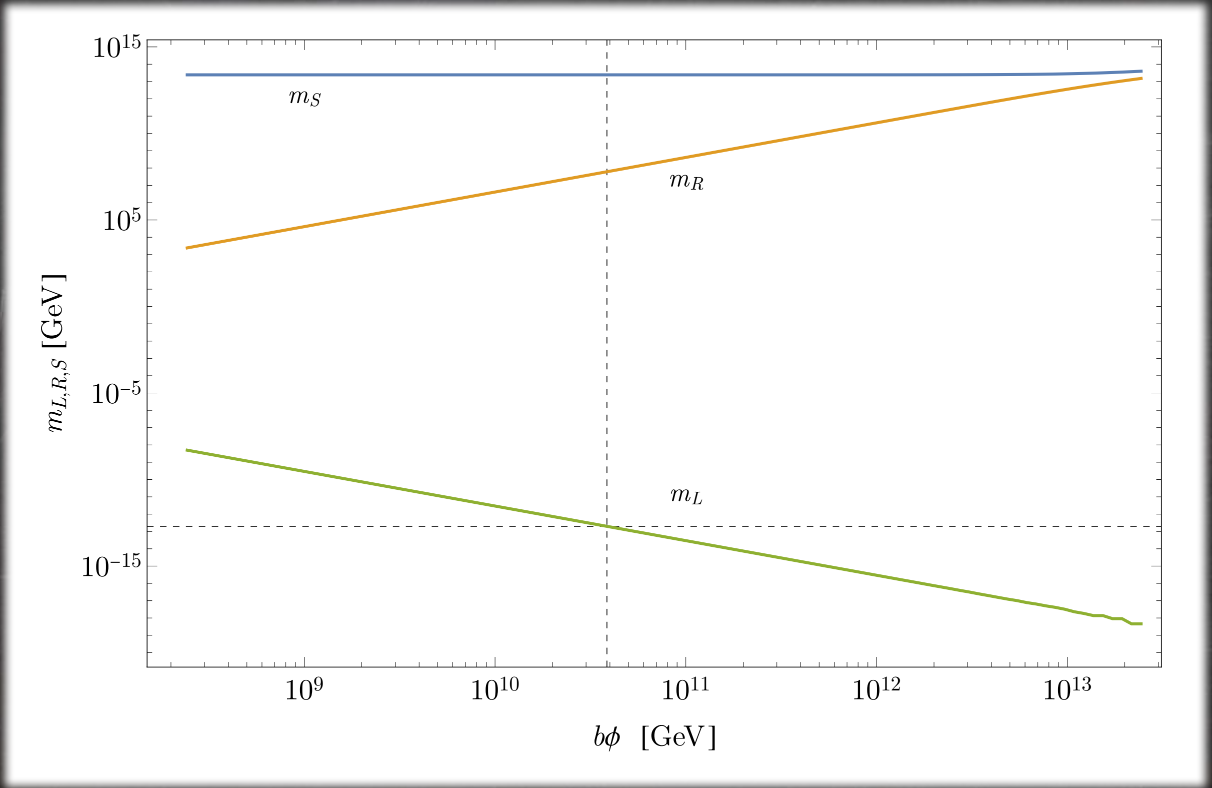

$$\sum m_\nu < 0.12\,\mathrm{eV}$$

Neutrino masses

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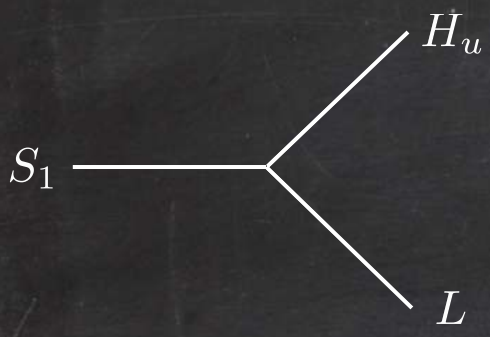
$$n_s \simeq -\frac{2}{N_*} + \frac{8}{3}\left(\frac{m}{m_\nu}\right)\left(\frac{m_u}{m}\right)^2 N_*^2 + \cdots$$

$$r \simeq \frac{12}{N_*^2} + \frac{32}{3}\left(\frac{m}{m_\nu}\right)\left(\frac{m_u}{m}\right)^2 N_* + \cdots$$

Neutrino physics can probe the unification and inflationary scales!

Reheating, Dark Matter and Baryogenesis

When inflation ends, the Universe must be re-populated ($\rho \sim e^{-3Ht}$)



$$T_R \simeq 10^{15}\ \mathrm{GeV} \times \left| b\left(\mathcal{U}_{21} - \frac{y\phi}{m}\right)\right| \left(\frac{g_*}{915/4}\right)^{-1/4} \left(\frac{m}{10^{-5} M_P}\right)^{1/2}$$

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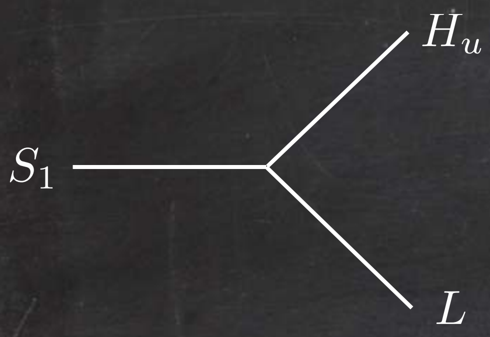
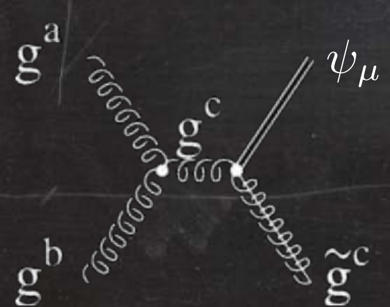


$$T_R \simeq 10^{15}\ \mathrm{GeV} \times \underbrace{\left| b\left(\mathcal{U}_{21} - \frac{y\phi}{m}\right)\right|}_{\lesssim 10^{-5}} \left(\frac{g_*}{915/4}\right)^{-1/4} \left(\frac{m}{10^{-5} M_P}\right)^{1/2}$$

Dark matter production:



$$\Rightarrow \qquad \psi_\mu \longrightarrow G + \tilde{G}$$

(neutralino)

$$\Omega_{\tilde{G}} h^2 \lesssim 0.12$$

Reheating, Dark Matter and Baryogenesis

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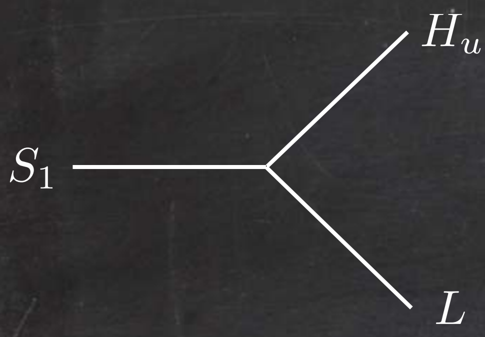
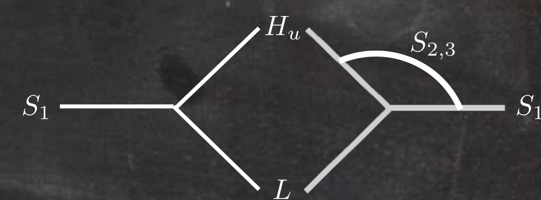
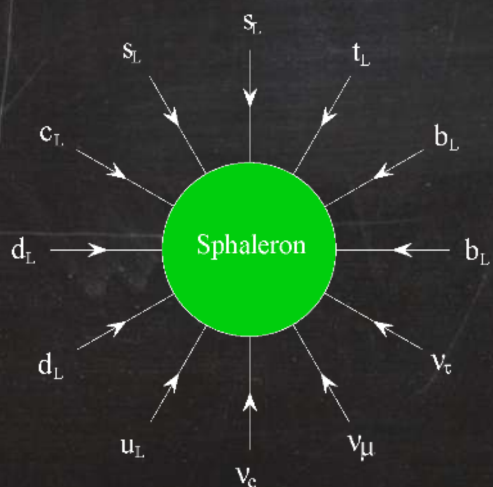


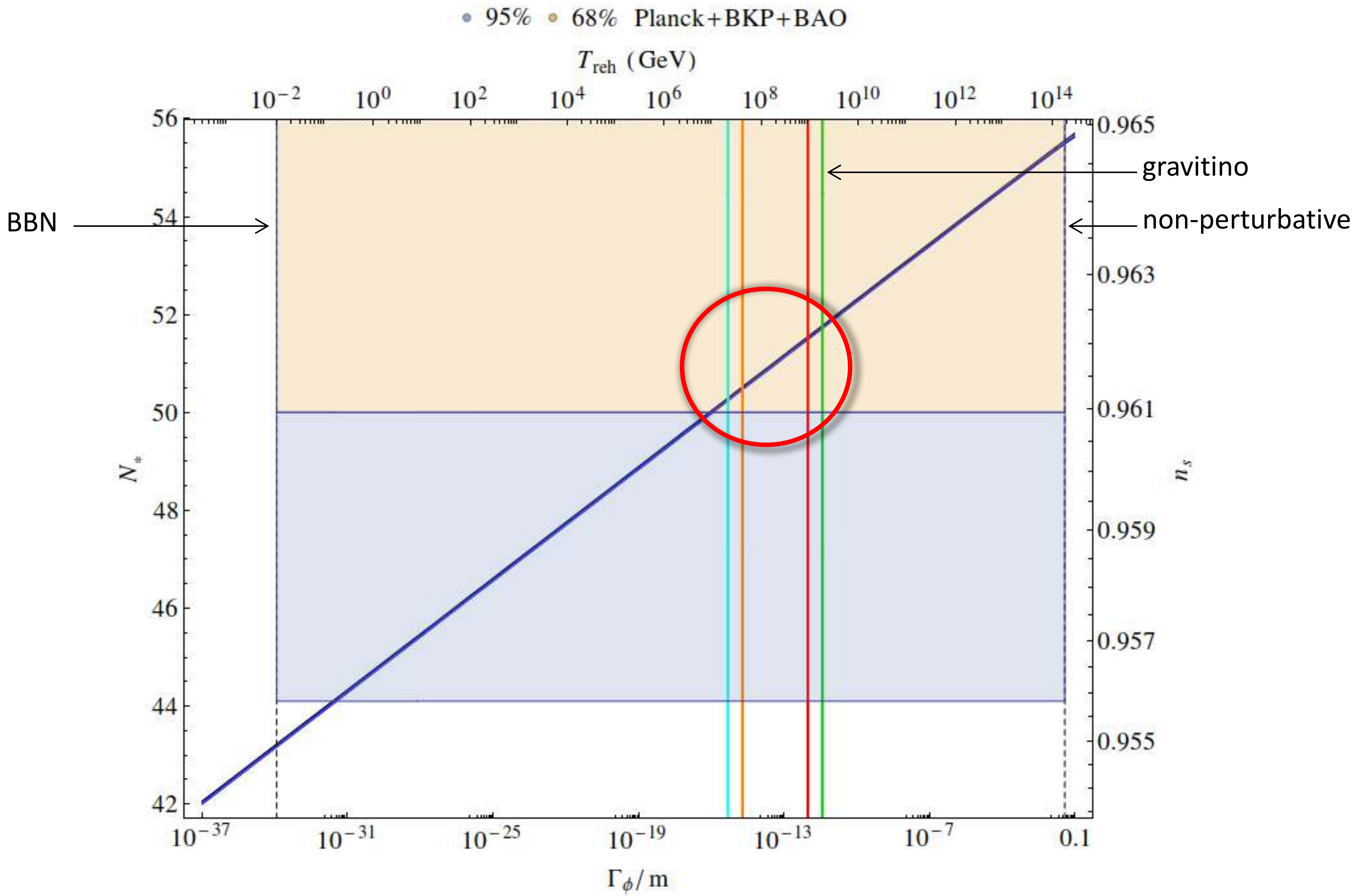
lepton-number asymmetric! $\Rightarrow$ $\dfrac{n_L}{s} \sim \epsilon \dfrac{T_R}{m}$

$\epsilon \sim$





L-asymmetry converted to B-asymmetry through non-perturbative processes



95% 68% Planck+BKP+BAO
$T_{\rm reh}$ (GeV)
10^{-2} 10^{0} 10^{2} 10^{4} 10^{6} 10^{8} 10^{10} 10^{12} 10^{14}
56 54 52 50 48 46 44 42
N_*
0.965 0.963 0.961 0.959 0.957 0.955
n_s
10^{-37} 10^{-31} 10^{-25} 10^{-19} 10^{-13} 10^{-7} 0.1
Γ_ϕ/m
gravitino
BBN
non-perturbative

CONCLUSION

- Embedding of R^2 inflation in a supersymmetry GUT model
- GUT symmetry broken automatically during/after inflation
- Natural (?) initial conditions
- Fermion masses: unification of b and τ Yukawa couplings; double neutrino see-saw
- Gravitino (over)production bound consistent with first generation coupling
- Broken R-parity, but LSP is long-lived
- Successful leptogenesis
- High SUSY breaking scale needed to satisfy proton decay bounds
- Rapid inflaton decay favored by data; no-scale reheating lies in the favored range.

The future

- Flipped $SU(5) \times U(1)$ (EGNNO)
- A statistical description of the reheating epoch

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H. Xie
(Wisconsin)

J. Shen

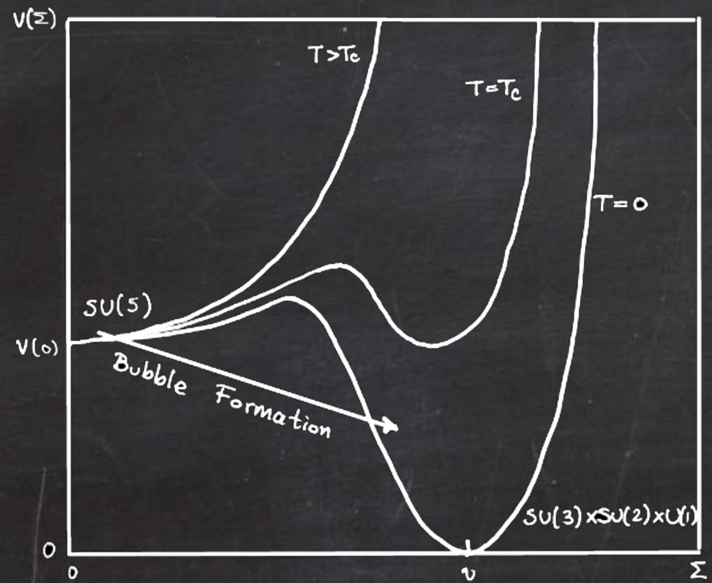
O. Wen

S. Carlsten

Thank you!

GUT + Inflation?

- Inflation is possible in GUT theories (*old inflation*)



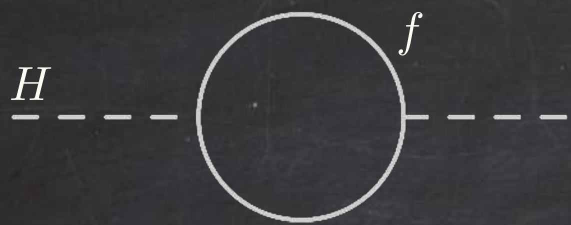
But lacks "graceful exit",

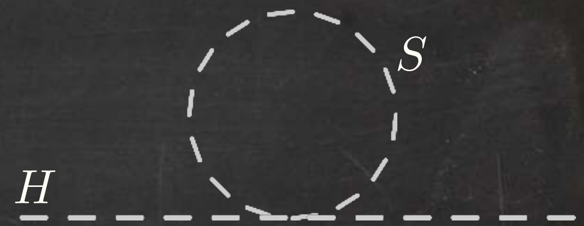
- Nucleation of bubbles doesn't generate radiation
- Collisions can, but for the amount of inflation needed, no collisions happen.

- Not impossible (*Higgs inflation*) (Bezrukov, Shaposhnikov)

$$S = \int d^4x\,\sqrt{g}\left(-\frac{R}{2} + \xi H^\dagger H\,R + \mathcal{L}_{\rm SM}\right) \qquad \Rightarrow \qquad V(\chi) = \frac{\lambda}{4\xi^2}\left(1+e^{-2\chi/\sqrt{6}}\right)^{-2}$$

- Radiative corrections (hierarchy problem)



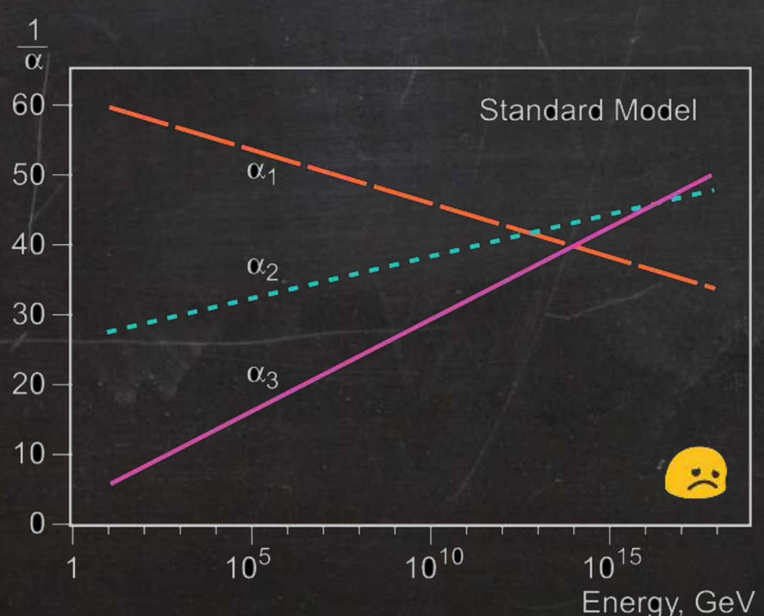


$$\Delta m_H^2 = \frac{\lambda_f^2}{8\pi^2}\left[-\Lambda^2 + 6m_f^2 \ln\left(\frac{\Lambda}{m_f}\right)\right]$$

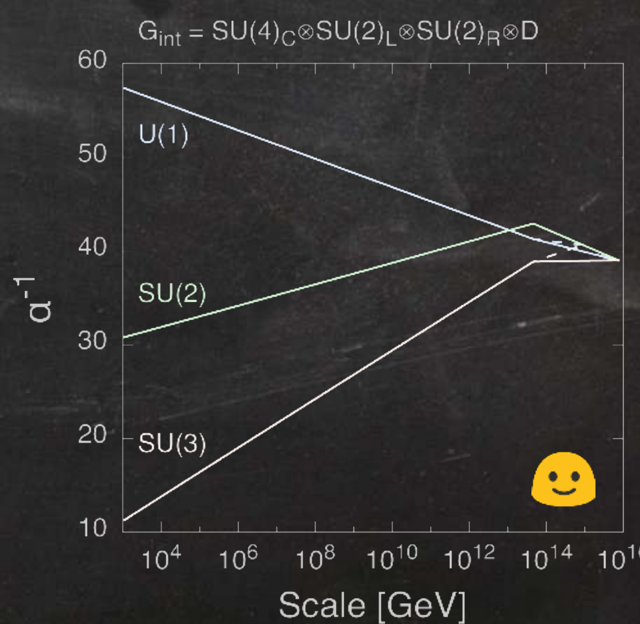
$$\Delta m_H^2 = \frac{\lambda_S}{16\pi^2}\left[\Lambda^2 - 2m_S^2 \ln\left(\frac{\Lambda}{m_S}\right)\right]$$

- Matching of unification scales is not automatic

SU(5):



SO(10):



(from 1502.06929)

The other goodies

3) Doublet-triplet splitting

$$W_\mu = (\bar{H}_L, \phi_L) \begin{pmatrix} m_H & -\bar{\alpha}\phi \\ -\alpha\phi & m_\Phi + \eta(p - 3a) \end{pmatrix} \begin{pmatrix} H_L \\ \bar{\phi}_L \end{pmatrix}$$

$$\Rightarrow \quad \mu_{1,2} = \frac{1}{2}\left[m_H + 2\eta(p+3\omega) \mp \sqrt{[m_H + 2\eta(p+3\omega)]^2 - 4\Delta} \right] ,$$

$\Delta = 2\eta m_H(p+3\omega) - \alpha\bar{\alpha}\phi^2$ No significant fine tuning unless $x \simeq 0, 1/3, \pm i$

4) Proton decay

Color-triplet mediated $p \to K^+ \bar{\nu} \quad \sim \begin{cases} \tan\beta \simeq m_t/m_b \\ m_{H-\mathrm{triplet}}^{-2} \end{cases}$ $M_{\rm SUSY} \gtrsim 10\,\mathrm{TeV}$

Gauge mediated

$$\tau(p \to e^+\pi^0) \simeq 5 \times 10^{34} \times \left(\frac{1/25}{\alpha_{\rm GUT}}\right)^4 \left(\frac{M_X}{10^{16}\ \mathrm{GeV}}\right)^4 \left(\frac{3}{A_R}\right)^2 \ \mathrm{years}$$

How to realize WZ ($\phi^2 + \phi^3$) no-scale inflation within SO(10) ?

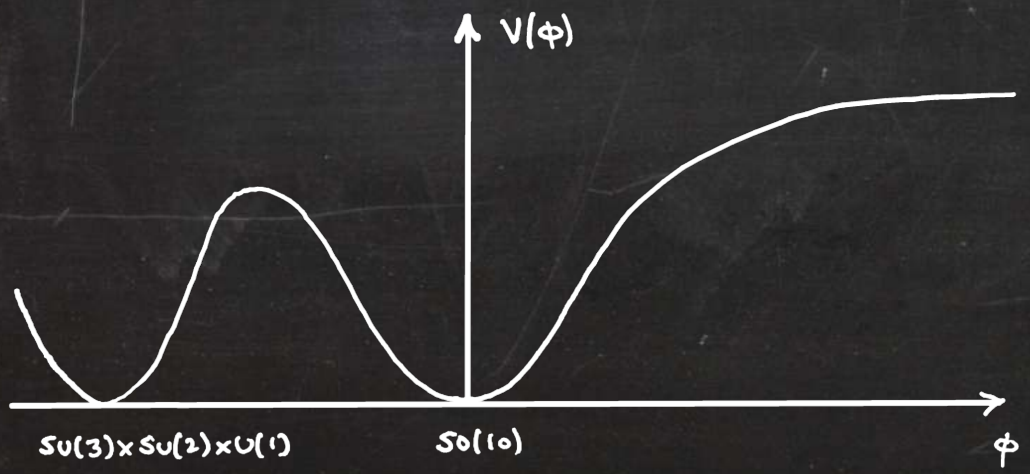
1) Right-handed sneutrino.

Spinor $\mathbf{16}$: $\psi \supset (Q, u^c, d^c, L, \nu^c, e^c)$

Can't write $\mathbf{16}^2$, $\mathbf{16}^3$ gauge-invariant interactions ✘

2) Singlet components of Higgs fields.

Quadratic and cubic terms possible for **210** , **126** SO(10)-breaking Higgs (Garg, Mohanty 2015). But…



- $M_{\rm GUT} \gg M_{\rm inf}$
- Too much (or too little) reheating ✘

Why Inflation?

Cosmological inflation is the early period of accelerated expansion, $a \sim e^{Ht}$
It provides a solution to the problem of initial conditions in standard cosmology:

- The Flatness Problem
- The Horizon Problem
- Small Scale Inhomogeneities
- The Relics Problem

Heavy and stable relics can be produced in the early universe following phase transitions (monopoles, strings, domain walls,…)

1 monopole per horizon $\longrightarrow$ $\Omega_M \sim 10^{13}$

Inflation $\Rightarrow$ $\rho \sim e^{-3Ht}$ ✔