29/10/2020

IJCLab
Irène Joliot-Curie
Laboratoire de Physique des 2 Infinis

How warm are non-thermal relics?

Marcos A. G. García
IFT-UAM

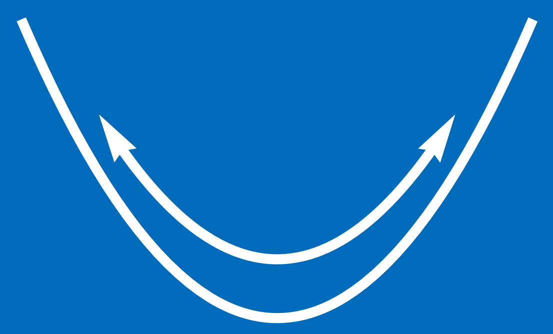
2011.XXXXX	with G. Ballesteros and M. Pierre
2006.03325	with Y. Mambrini, K. Olive and S. Verner
2004.08404	with Y. Mambrini, K. Olive and K. Kaneta
1806.01865	with M. Amin
1709.01549	with Y. Mambrini, K. Olive and M. Peloso

ift Instituto de Física Teórica UAM-CSIC

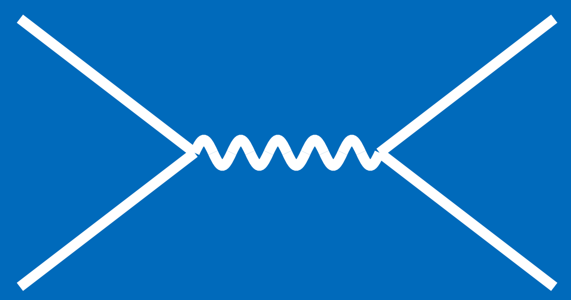
UAM Universidad Autónoma de Madrid

CSIC Consejo Superior de Investigaciones Científicas

1. Reheating



2. Freeze-in



3. Lyman-α

4. The end?

The Goals

Understand the (perturbative) repopulation of the Universe after the end of inflation (rates, energy densities, temperatures, distributions,..)

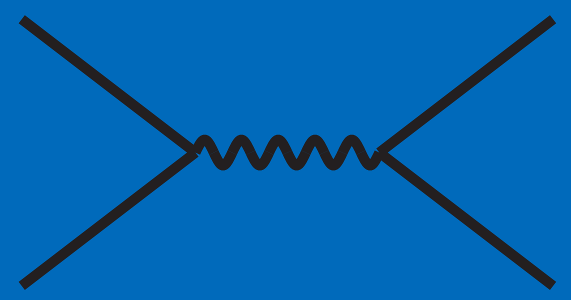
Characterize the out-of-equilibrium production of dark matter (from particle/condensate decays, scatterings, misalignment...)

In the absence of a direct signal, can we say more beyond Ω_{DM}?
Use the Ly-α measurement of the power spectrum to impose constraints

A short summary of what we (don't) understand

1. Reheating

2. Freeze-in



3. Lyman-α

4. The end?

Accelerated expansion can be driven by a slowly rolling scalar field

$$\mathcal{S} = \int d^4x \sqrt{-g}\left[\frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - V(\Phi)\right]$$

$$ds^2 = dt^2 - a(t)^2 d\boldsymbol{x}^2$$

$$G_{\mu\nu} = M_P^{-2} T_{\mu\nu}$$

$$\rho_\Phi = \frac{1}{2}\dot{\Phi} + V(\Phi)$$

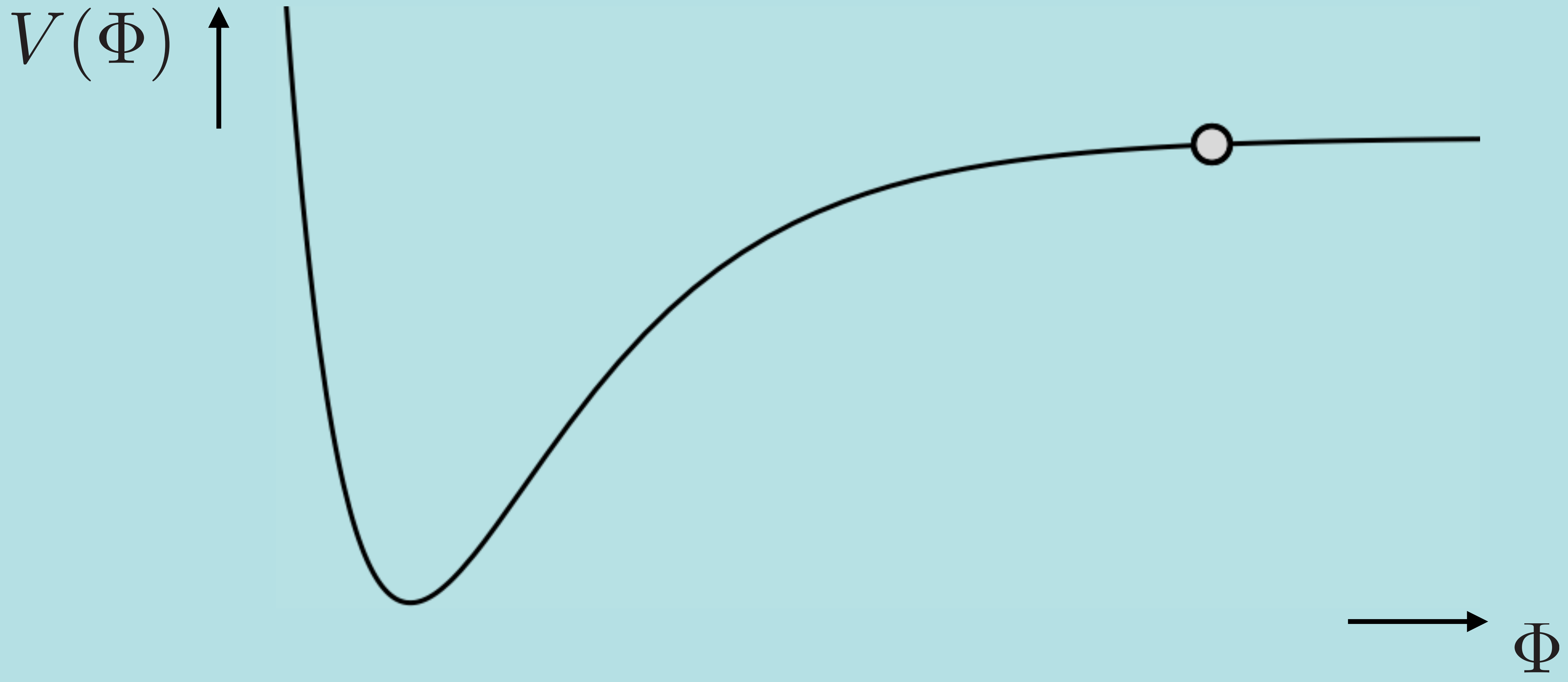
$$P_\Phi = \frac{1}{2}\dot{\Phi} - V(\Phi)$$

$$H \equiv \frac{\dot{a}}{a} = \left(\frac{\rho_\Phi}{3M_P^2}\right)^{1/2}$$

$$\dot{\rho}_\Phi + 3H(\rho_\Phi + P_\Phi) = 0$$

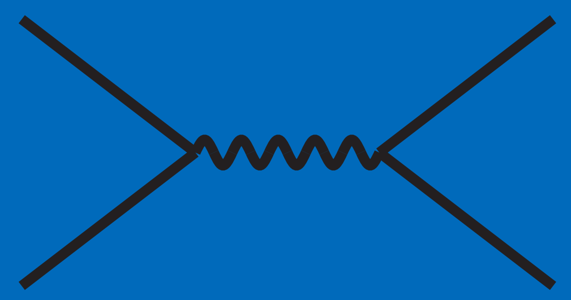
or

$$\ddot{\Phi} + 3H\dot{\Phi} + V'(\Phi) = 0$$



1. Reheating

2. Freeze-in



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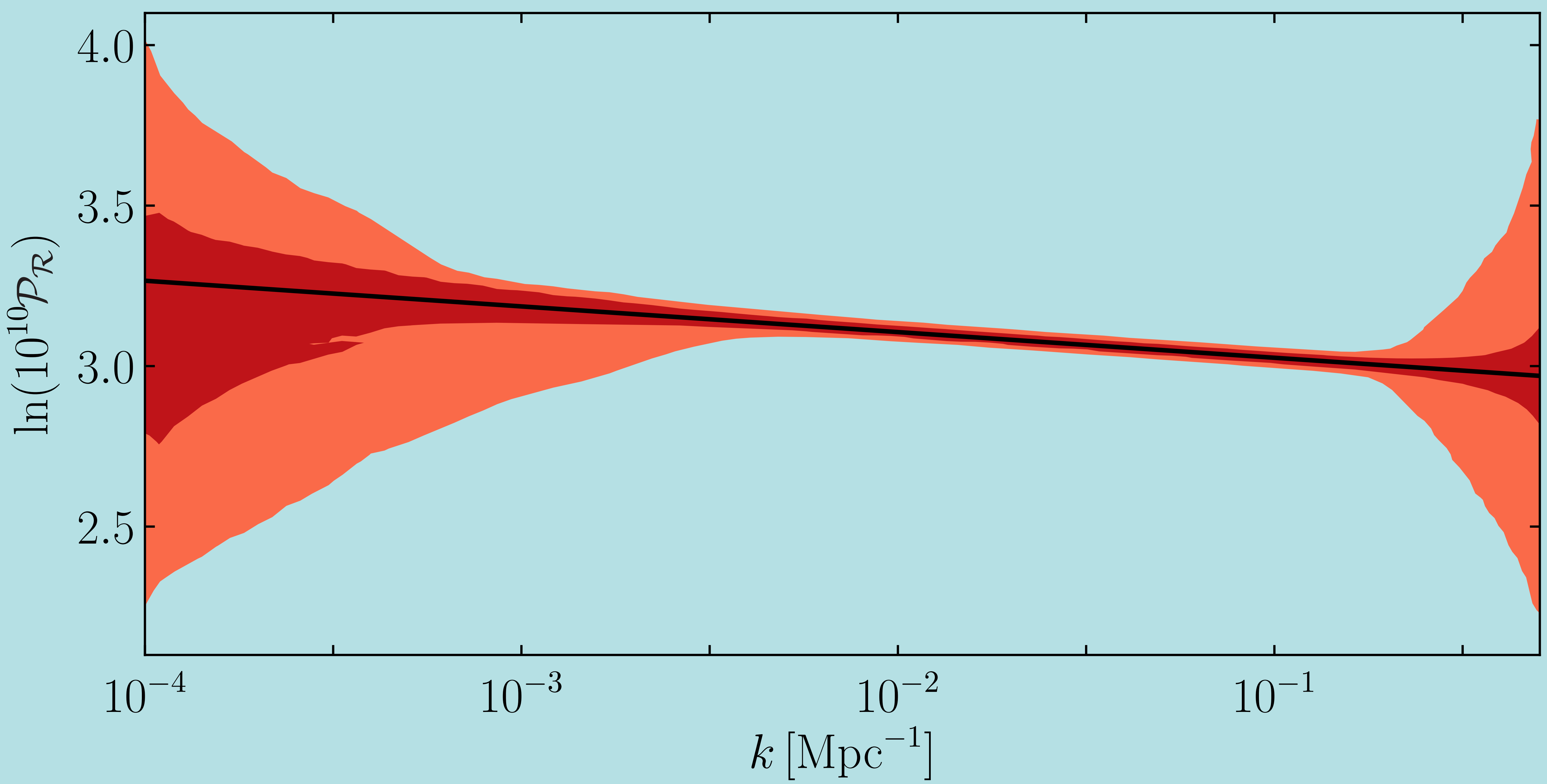
$$ds^2 = dt^2 - a(t)^2 d\boldsymbol{x}^2$$

$$\Phi \to \Phi + \delta\Phi$$

$$g \to g + \delta g$$

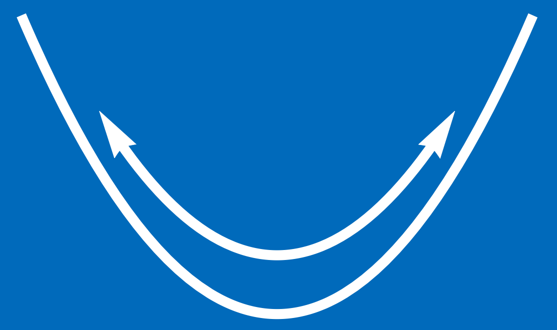
Planck TT, TE, EE + lowE + lensing + BK15 (1σ, 2σ)



$$\mathcal{P}_\mathcal{R} = \frac{H_*^4}{4\pi^2\dot{\phi}_*^2}\left(\frac{k}{aH}\right)^{n_s-1}$$

$$\mathcal{P}_\mathcal{T} = \frac{2}{\pi^2}H_*^2\left(\frac{k}{aH}\right)^{n_T}$$

1. Reheating



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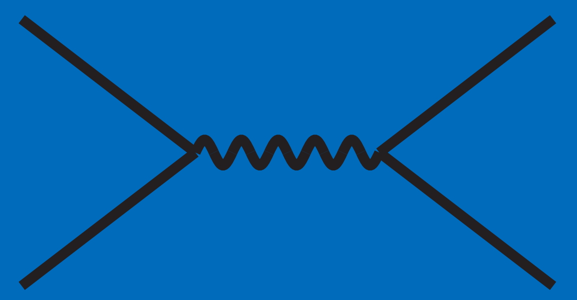

3. Lyman-α

4. The end?

After inflation ends, reheating begins

$$\mathcal{S} = \int d^4x \sqrt{-g}\left[\frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - V(\Phi)\right]$$

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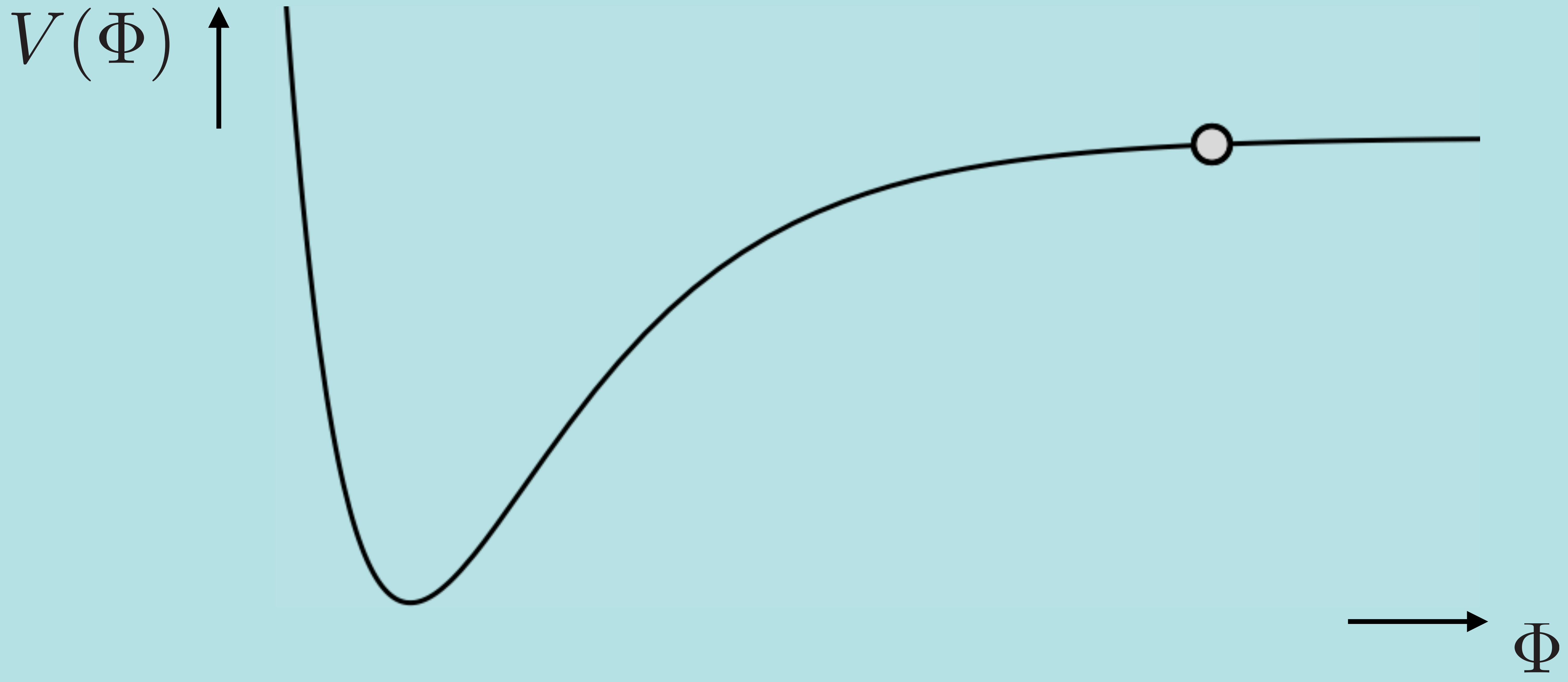
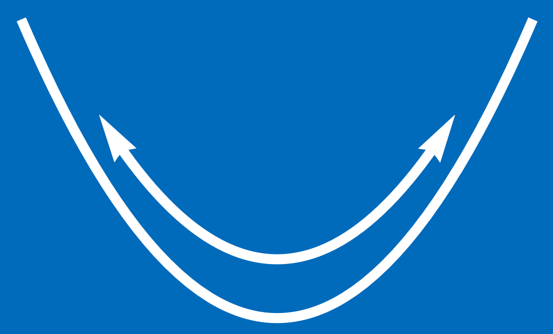
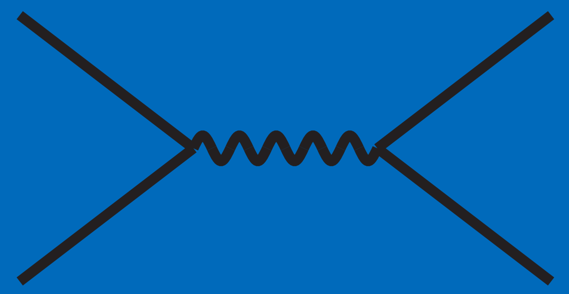
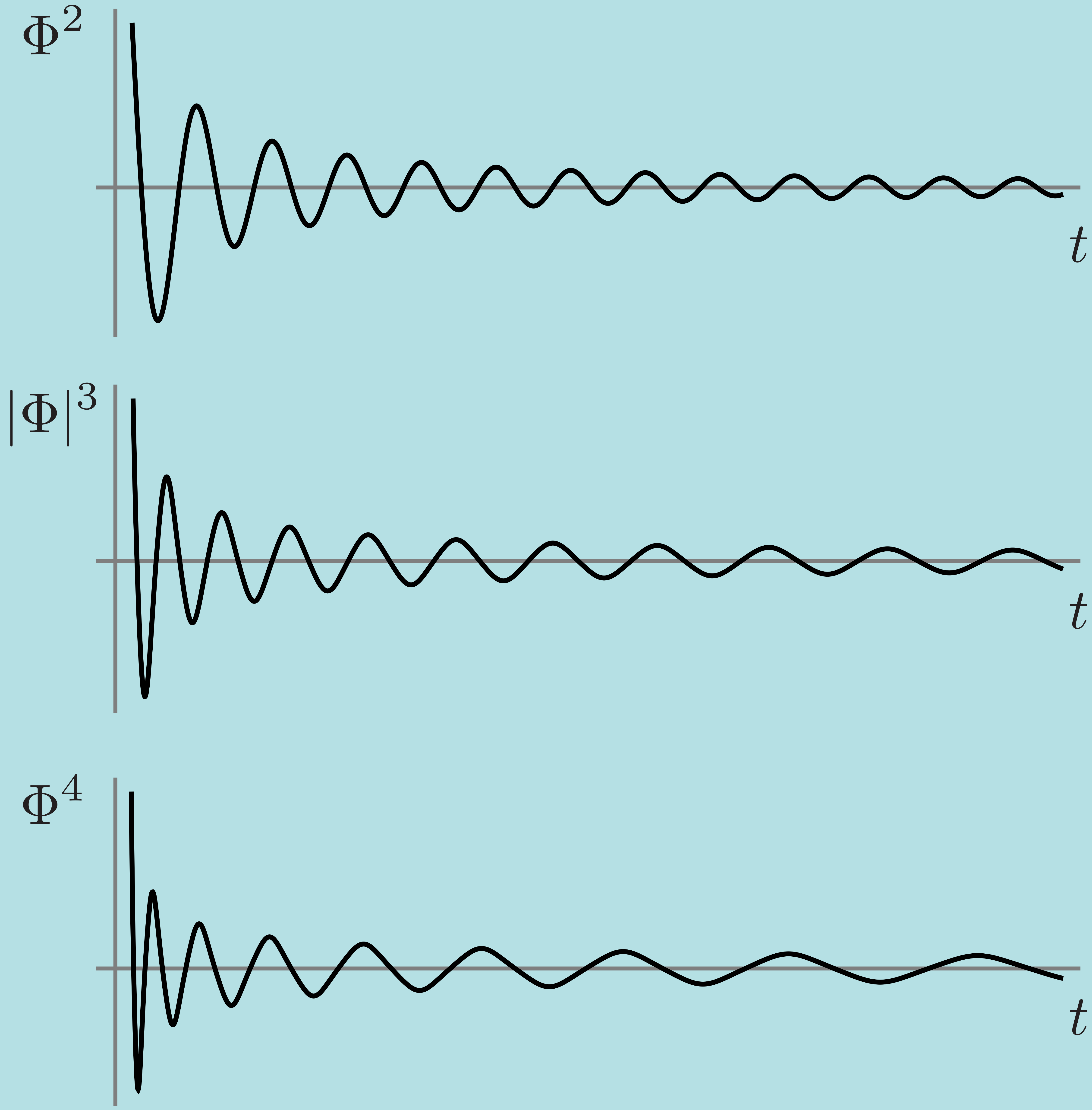






Reheating depends on the shape of the potential



$$V(\Phi) \propto |\Phi|^k$$

↓

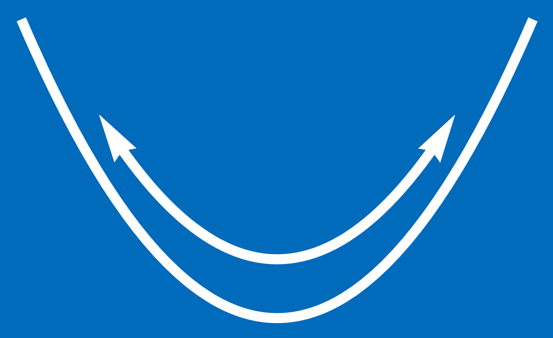
over one oscillation,

$$\langle \dot{\Phi}^2 \rangle \simeq \langle \Phi V'(\Phi) \rangle$$

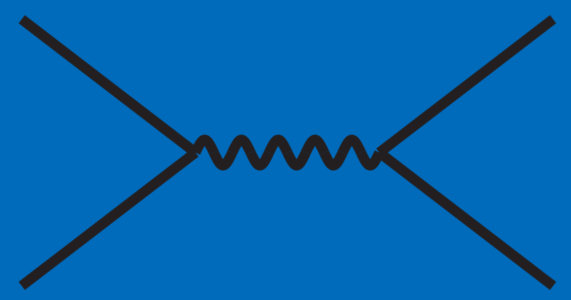
↓

$$\langle P_\Phi \rangle = \frac{k-2}{k+2} \langle \rho_\Phi \rangle$$

1. Reheating



2. Freeze-in



3. Lyman-α

4. The end?

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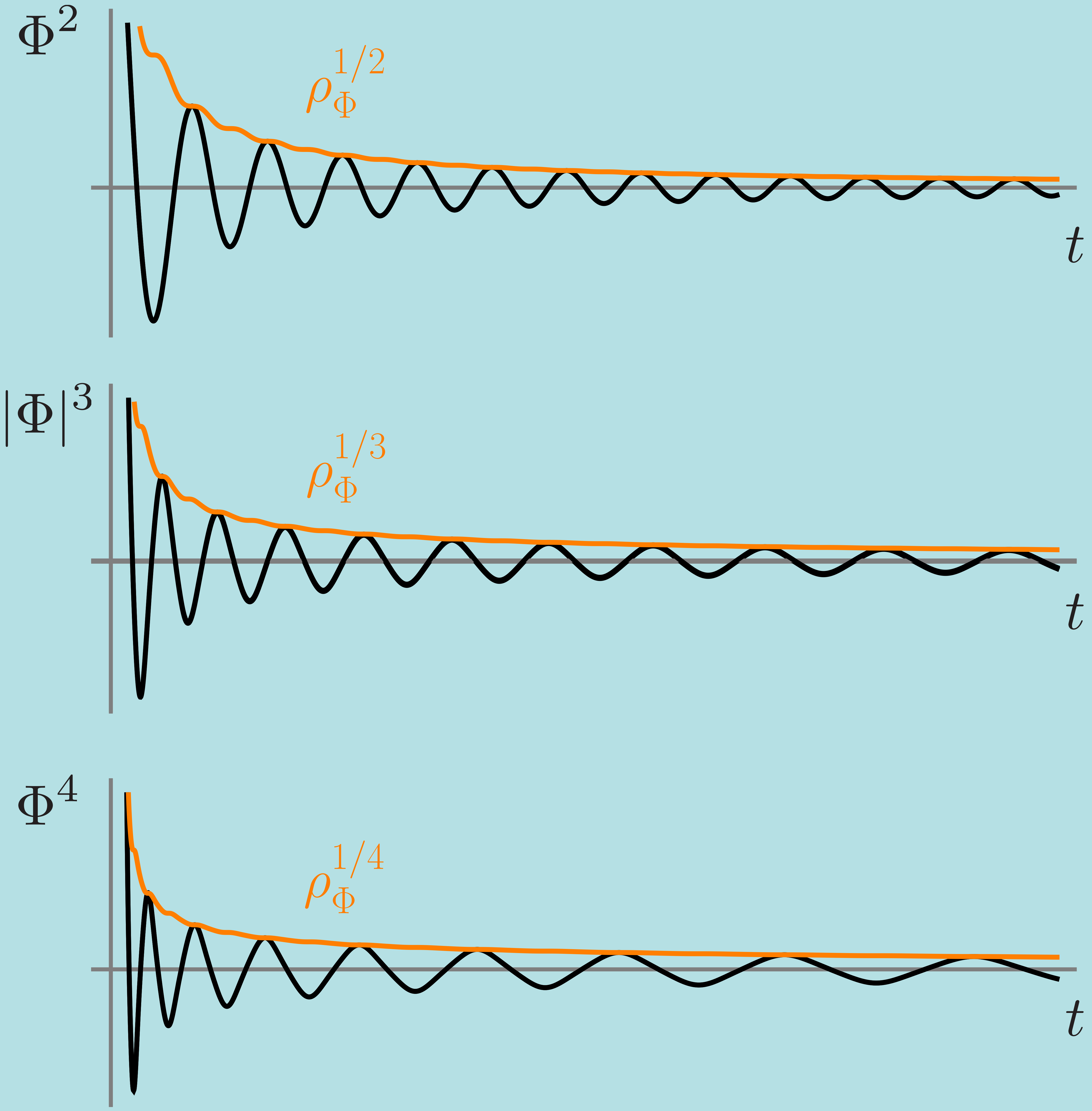


~ matter

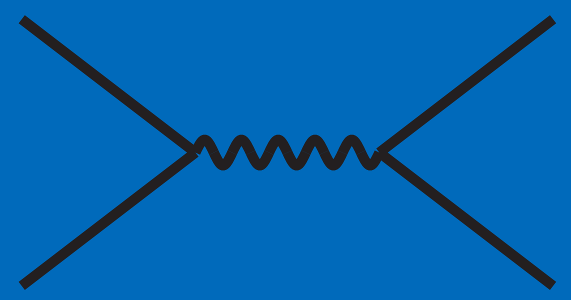
$$\rho_\Phi = \rho_{\text{end}} \left(\frac{a}{a_{\text{end}}} \right)^{-\frac{6k}{k+2}}$$

$$a \propto t^{\frac{k+2}{3k}}$$

~ radiation

1. Reheating

2. Freeze-in



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Particle production in the expanding Universe

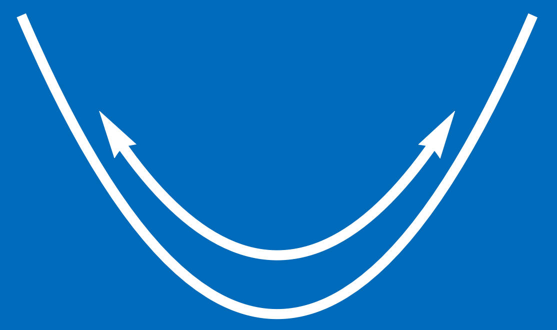
$$\chi + a + b + \cdots \longleftrightarrow i + j + \cdots$$

$$\frac{\partial f_\chi}{\partial t} - H|\boldsymbol{p}|\frac{\partial f_\chi}{\partial|\boldsymbol{p}|} = \mathcal{C}[f_\chi(|\boldsymbol{p}|, t)]$$

$$\begin{aligned}
\mathcal{C}[f_\chi] = -\frac{1}{2p_0}\int \frac{g_a d^3\boldsymbol{p}_a}{(2\pi)^3 2p_{a0}} \frac{g_b d^3\boldsymbol{p}_b}{(2\pi)^3 2p_{b0}} \cdots \frac{g_i d^3\boldsymbol{p}_i}{(2\pi)^3 2p_{i0}} \frac{g_j d^3\boldsymbol{p}_j}{(2\pi)^3 2p_{j0}} \cdots \\
\times (2\pi)^4 \, \delta^{(4)}(p_\chi + p_a + p_b + \cdots - p_i - p_j - \cdots) \\
\times \Big[|\mathcal{M}|^2_{\chi+a+b+\cdots \longrightarrow i+j+\cdots} \, f_a f_b \cdots f_\chi (1 \pm f_i)(1 \pm f_j) \cdots \\
- |\mathcal{M}|^2_{i+j+\cdots \longrightarrow \chi+a+b+\cdots} \, f_i f_j \cdots (1 \pm f_a)(1 \pm f_b) \cdots (1 \pm f_\chi) \Big]
\end{aligned}$$

$$n_\chi(t) = \frac{g_\chi}{(2\pi)^3}\int d^3\boldsymbol{p}\, f_\chi(p_0, t) \qquad \rho_\chi(t) = \frac{g_\chi}{(2\pi)^3}\int d^3\boldsymbol{p}\; p_0\, f_\chi(p_0, t)$$

1. Reheating



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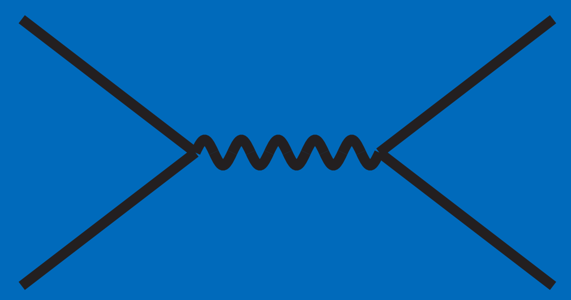


3. Lyman-α

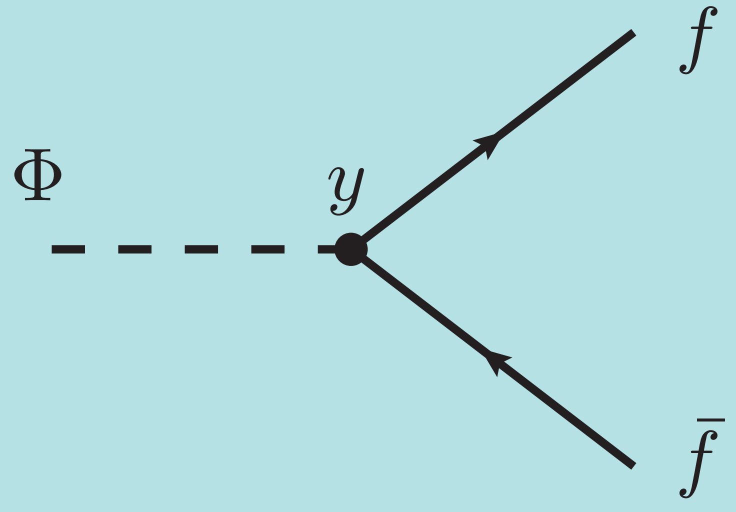
4. The end?

Integrated Boltzmann equations

$$\dot{\rho}_\Phi + 3\left(\frac{2k}{k+2}\right) H\rho_\Phi = -\Gamma_\Phi(t)\rho_\Phi$$

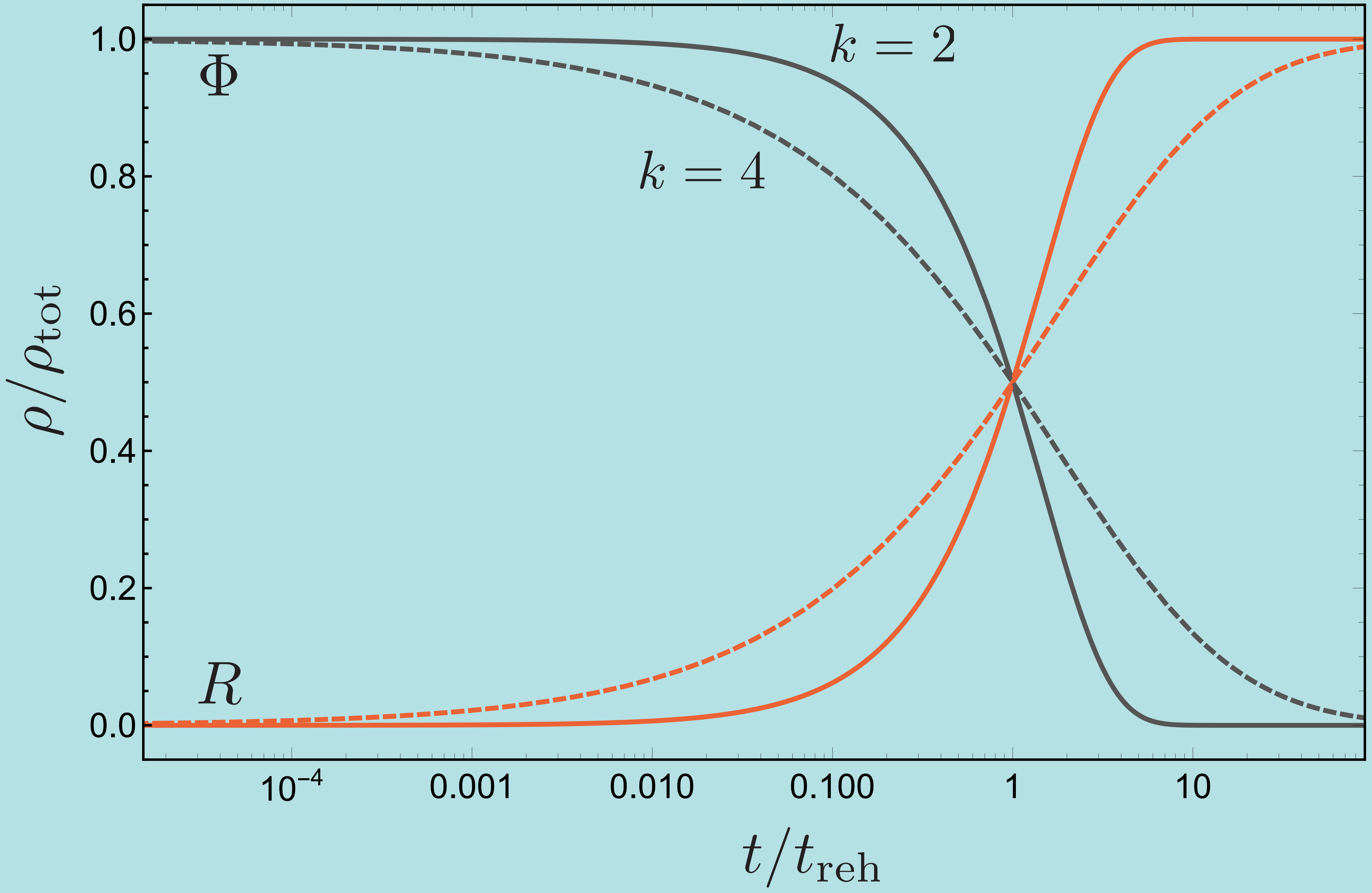
$$\dot{\rho}_R + 4H\rho_R = \Gamma_\Phi(t)\rho_\Phi$$

$$3M_P^2 H^2 = \rho_\Phi + \rho_R$$



$$\Gamma_\Phi = \frac{y^2}{8\pi} m_\Phi(t)\,,$$

$$m_\Phi^2 \equiv \partial_\Phi^2 V(\Phi) \propto \rho_\Phi^{\frac{k+2}{k}}$$



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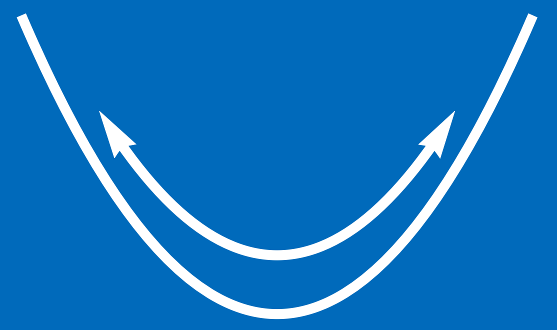

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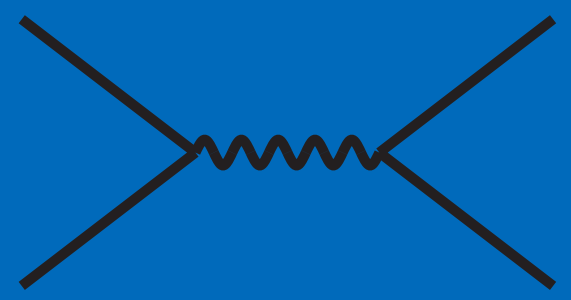

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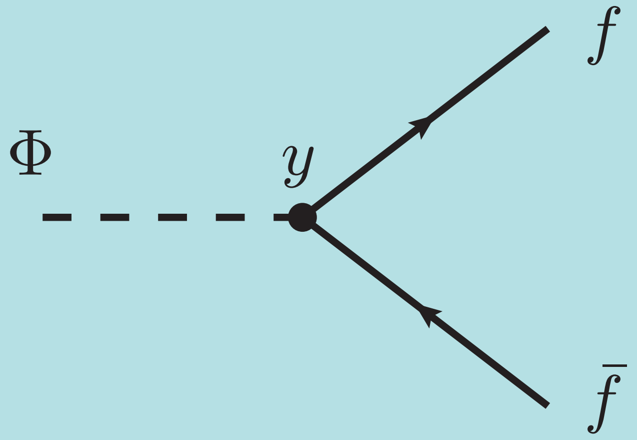

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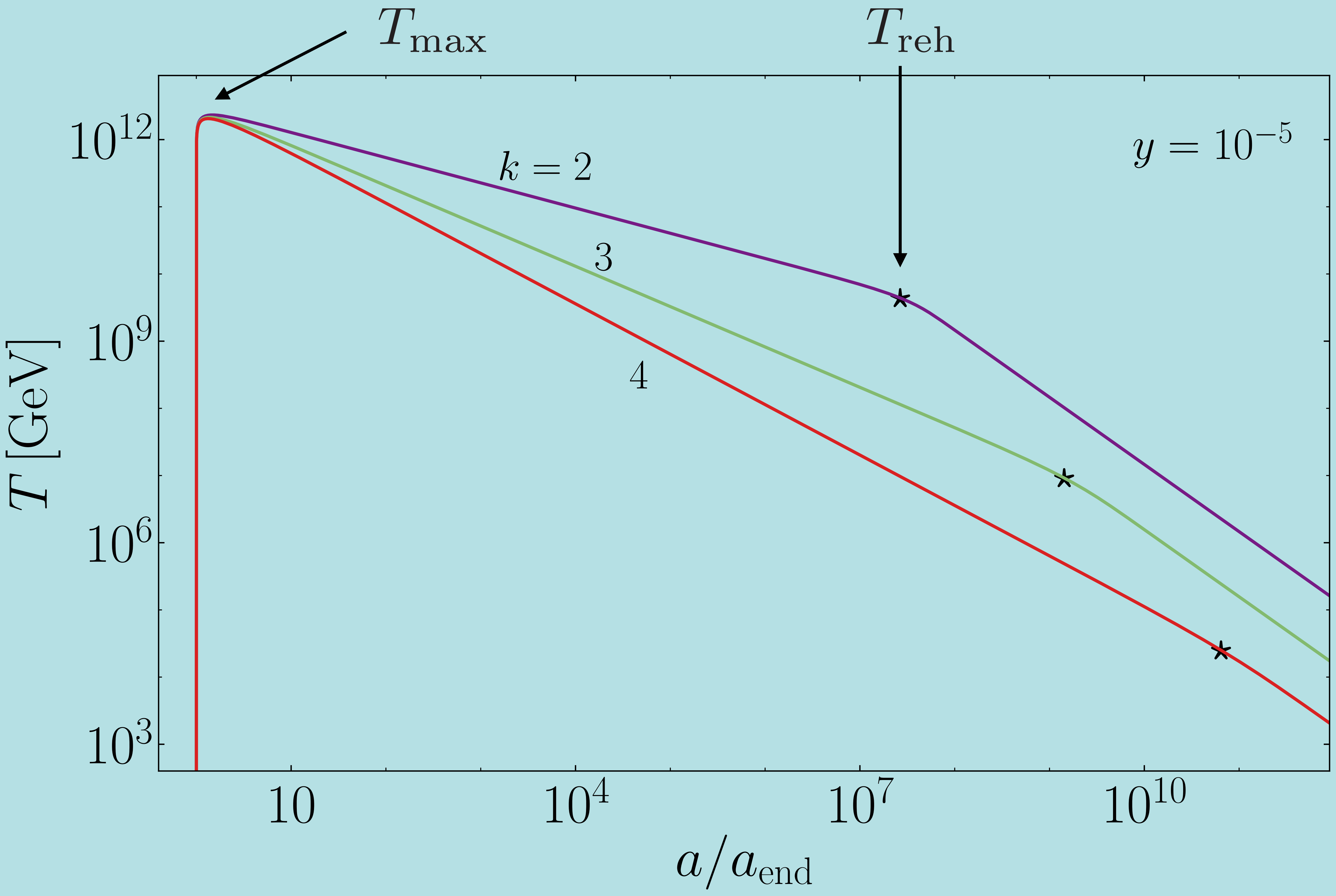
$$T = \left(\frac{30\rho_R}{\pi^2 g_*}\right)^{1/4}$$

$$\propto a^{-\frac{3k-3}{2k+4}}$$



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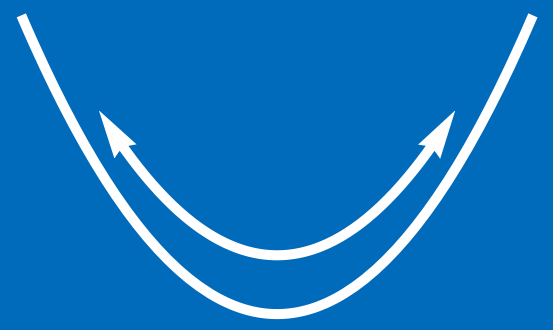

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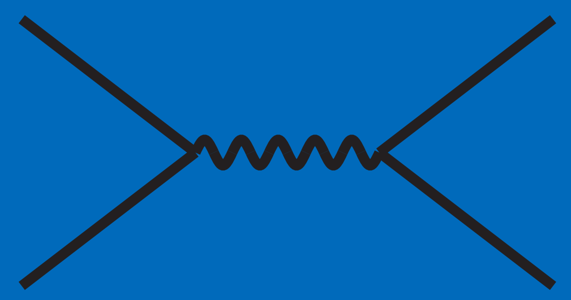

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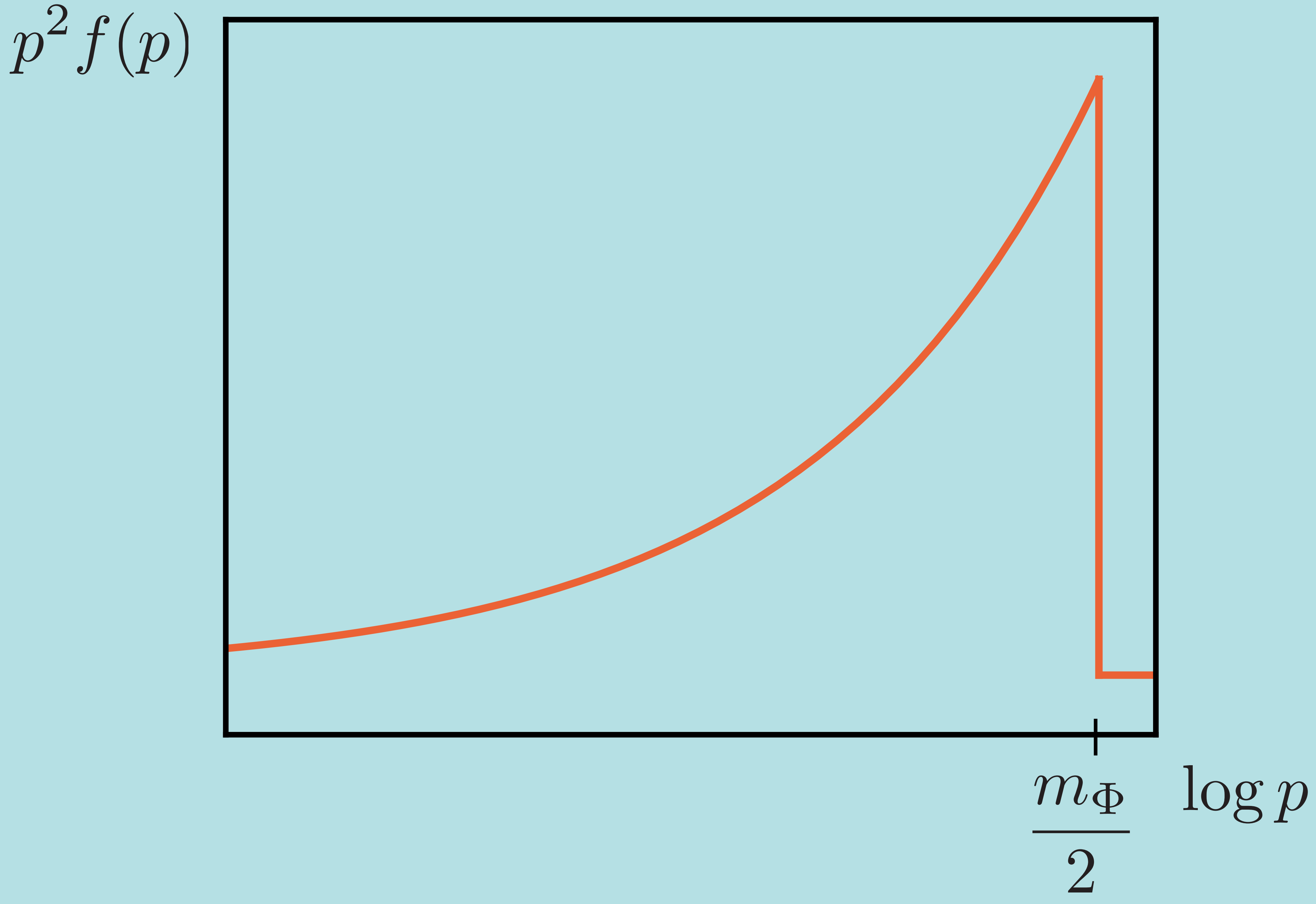
A closer look: thermalization is not instantaneous! (quadratic only)

$$\mathcal{C}[f_\chi(p,t)] = \frac{8\pi^2}{g_\chi m_\Phi^2} n_\Phi \Gamma_{\Phi\to\chi\psi}\delta(p - m_\Phi/2)$$

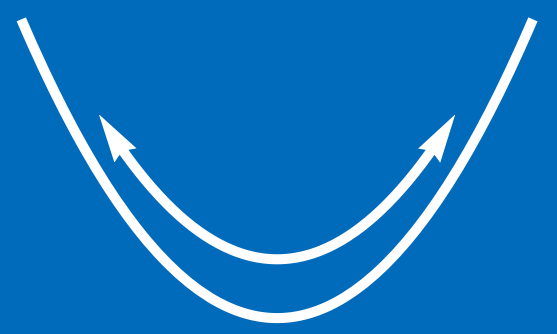
$$f_\chi(p,t) = \frac{16\pi^2\Gamma_{\Phi\to\chi\psi}n_\Phi(\hat{t})}{g_\chi m_\Phi^3 H(\hat{t})}\theta(t-\hat{t})\,, \qquad \frac{a(t)}{a(\hat{t})} = \frac{m_\Phi}{2p}$$



$$f_\chi(p,t) \simeq \frac{24\pi^2 n_\chi(t)}{g_\chi m_\Phi^3}\left(\frac{m_\Phi}{2p}\right)^{3/2}\theta(m_\Phi/2 - p)$$

$$(t \ll t_{\rm reh})$$

1. Reheating



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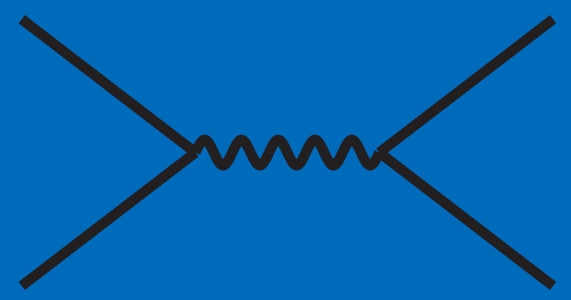

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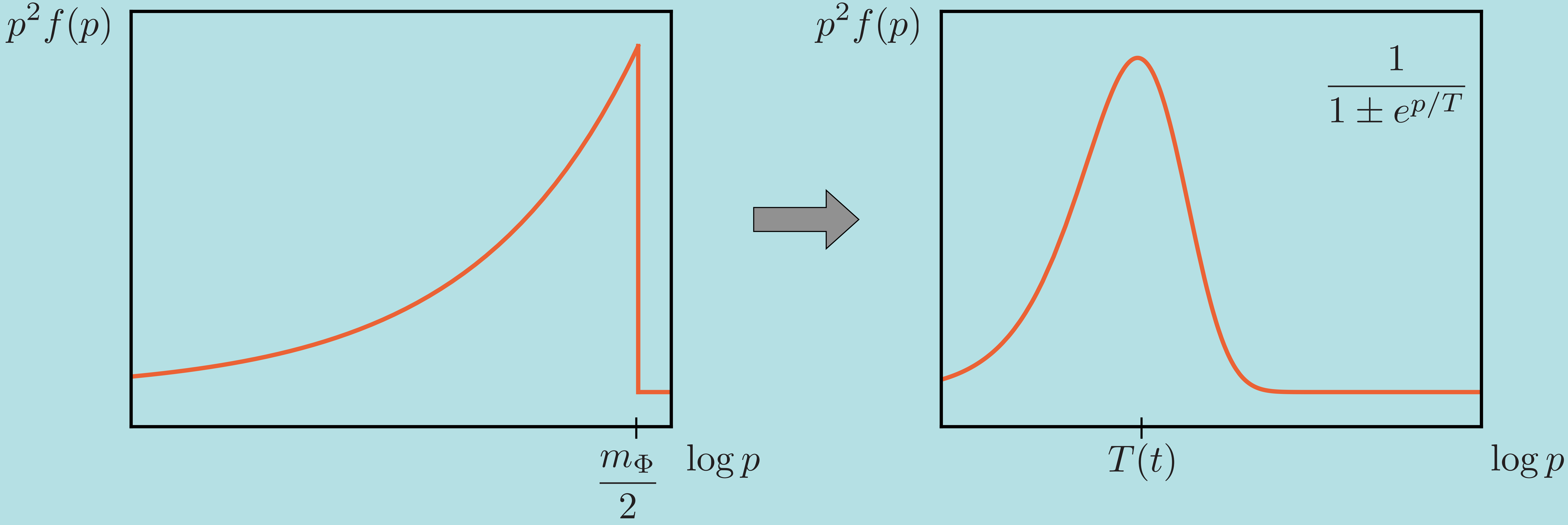

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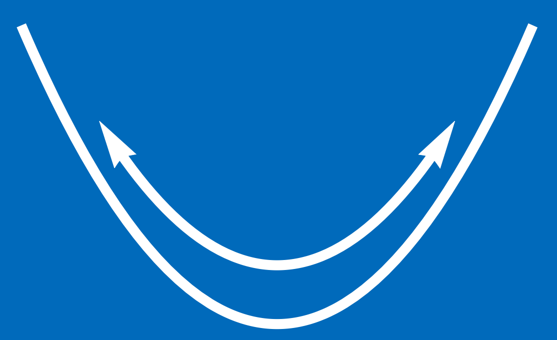

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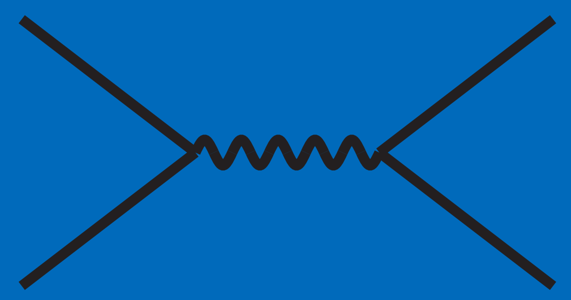

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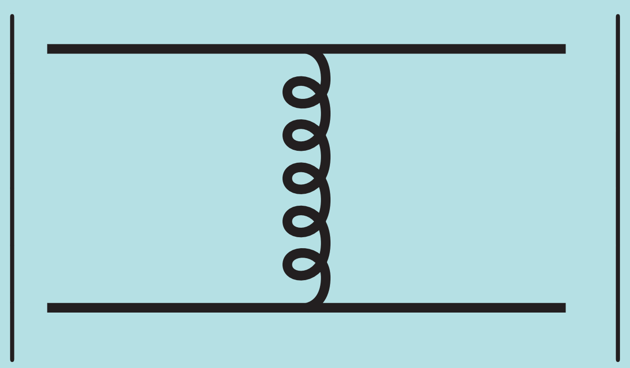
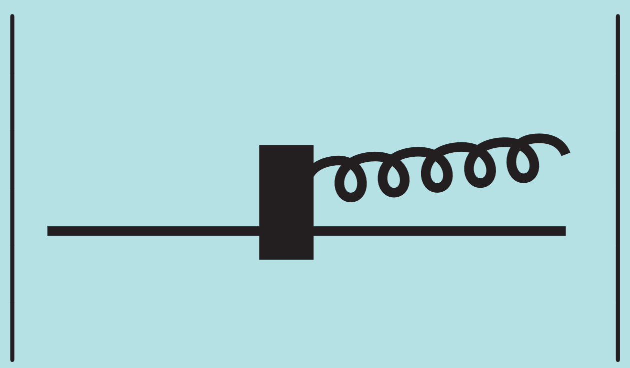
$$\frac{\partial f_\chi}{\partial t} - Hp\frac{\partial f_\chi}{\partial p} = \left| \text{diagram 1} \right|^2 + \left| \text{diagram 2} \right|^2 + \cdots$$



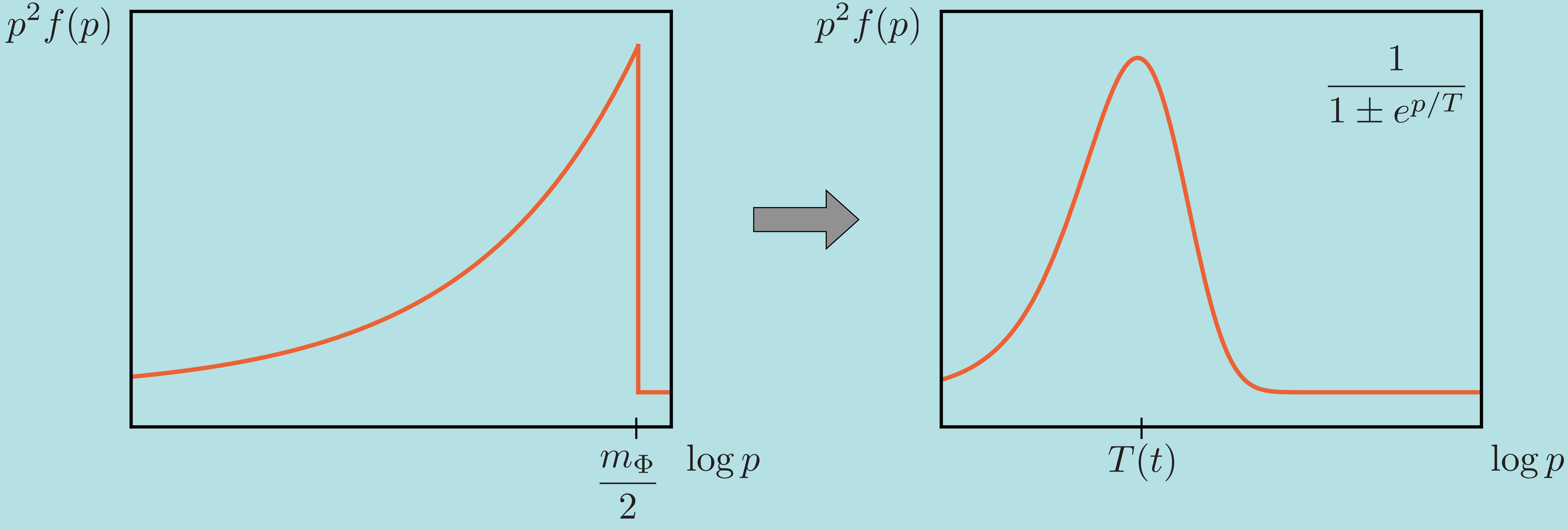


$$\equiv -\mathcal{C}^{2\leftrightarrow 2}[f_\chi] - \mathcal{C}^{\text{“}1\leftrightarrow 2\text{”}}[f_\chi] + \cdots ,$$

K. Harigaya, K. Mukaida, JHEP 05 (2014) 006

K. Mukaida, M. Yamada, JCAP 02 (2016) 003

$$\Gamma_\Phi t_{\rm th} \simeq \alpha_{\rm SM}^{-16/5} \left(\frac{\Gamma_\Phi m_\Phi^2}{M_P^3} \right)^{2/5}$$



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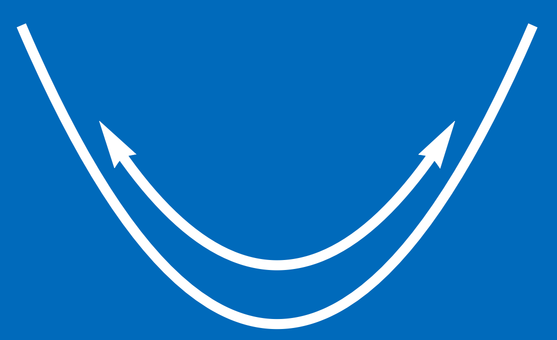

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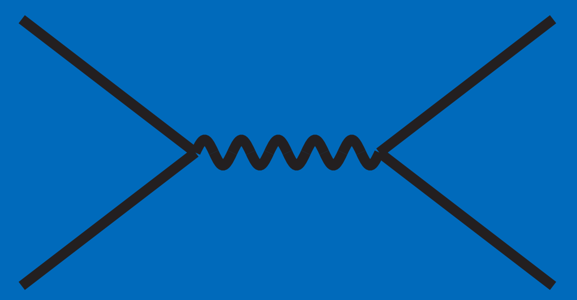

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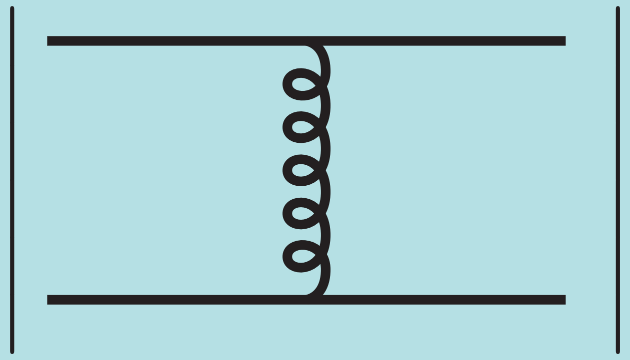
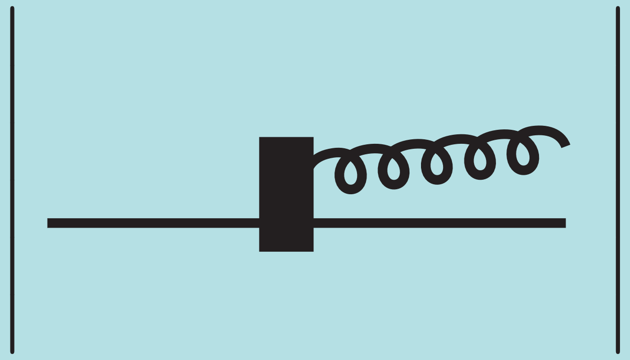
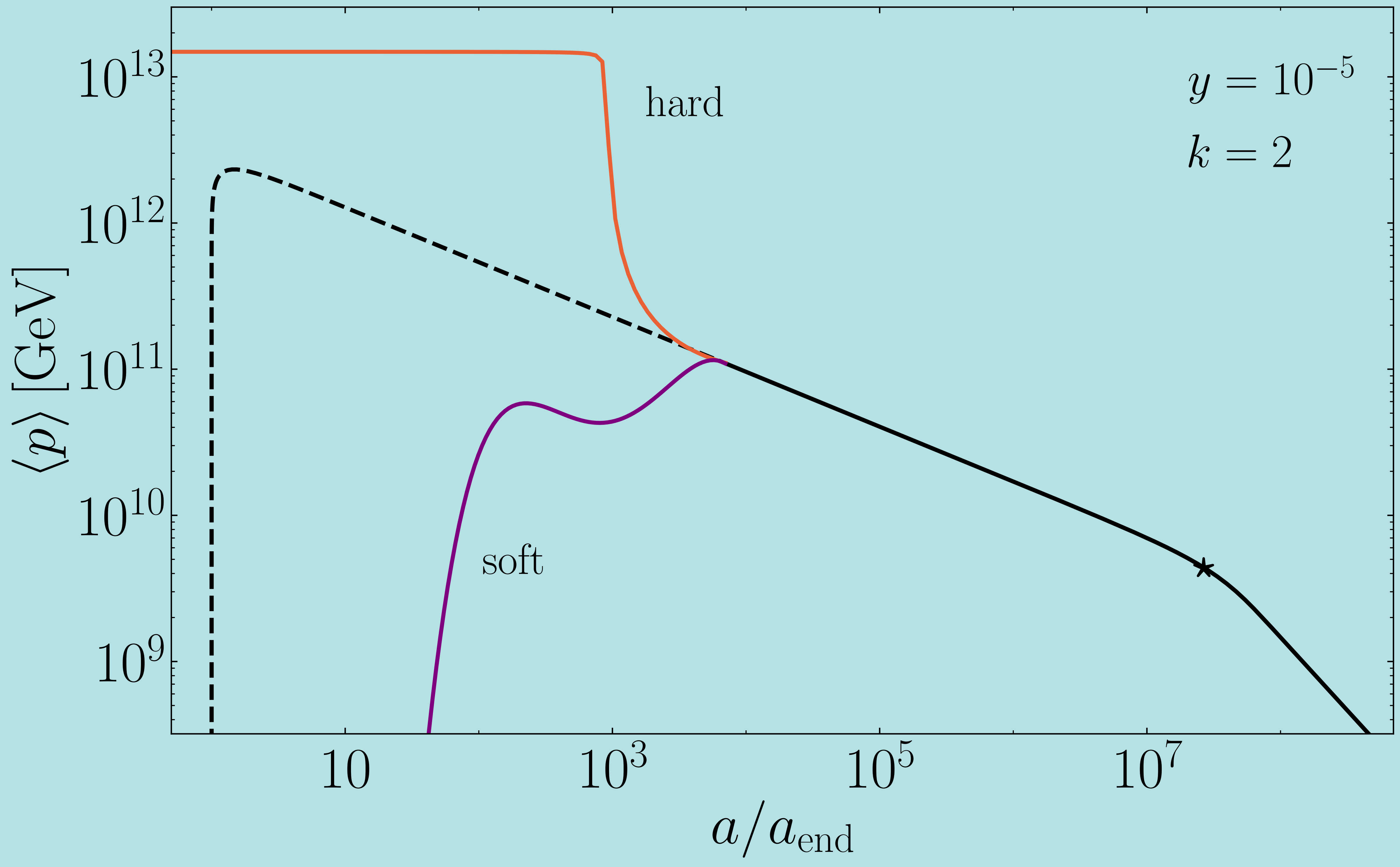




$$\equiv -\mathcal{C}^{2\leftrightarrow 2}[f_\chi] - \mathcal{C}^{\text{“}1\leftrightarrow 2\text{”}}[f_\chi] + \cdots ,$$

$$\Gamma_\Phi t_{\rm th} \simeq \alpha_{\rm SM}^{-16/5}\left(\frac{\Gamma_\Phi m_\Phi^2}{M_P^3}\right)^{2/5}$$



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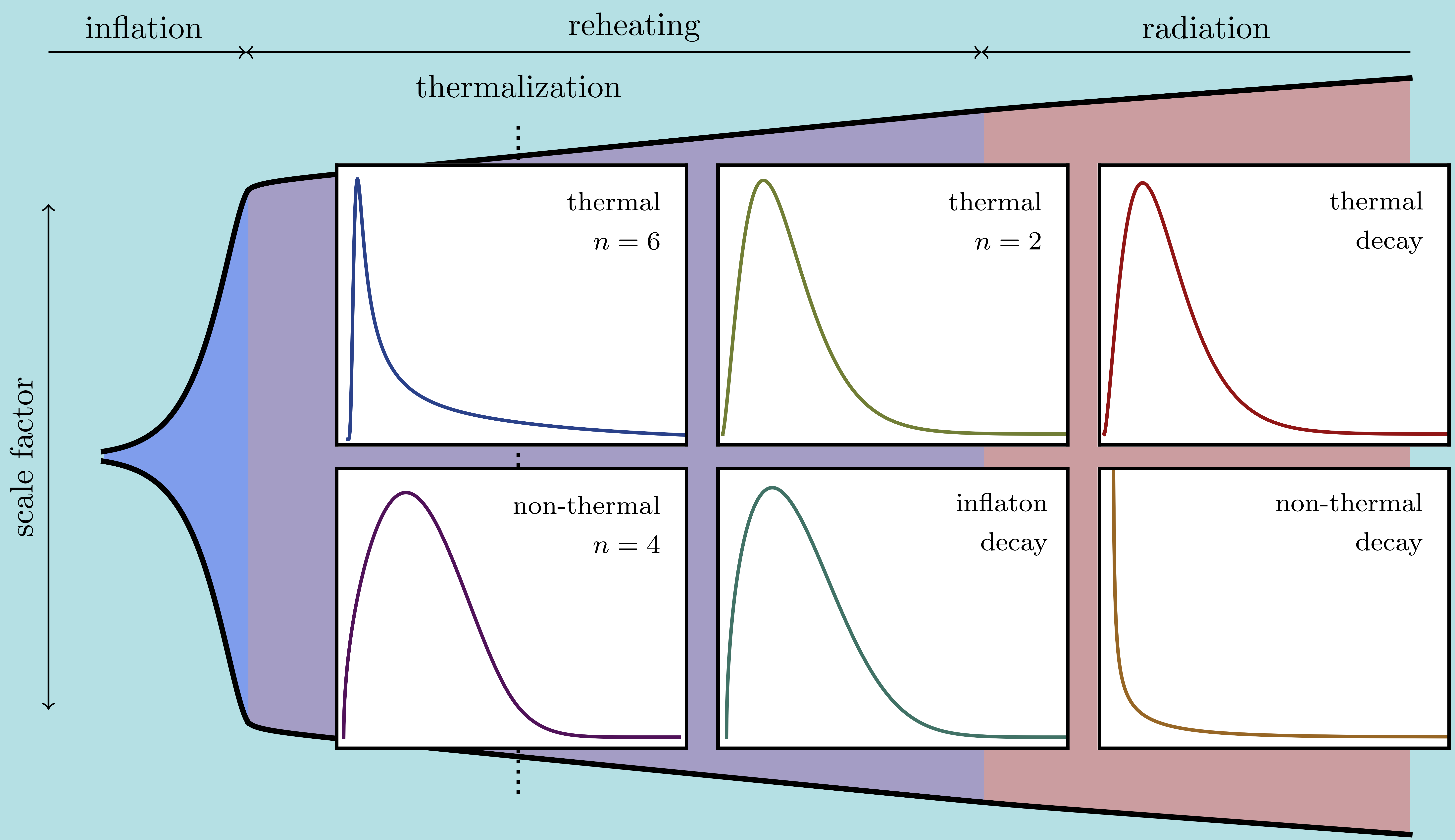
Freeze-in: producing dark matter out-of-equilibrium



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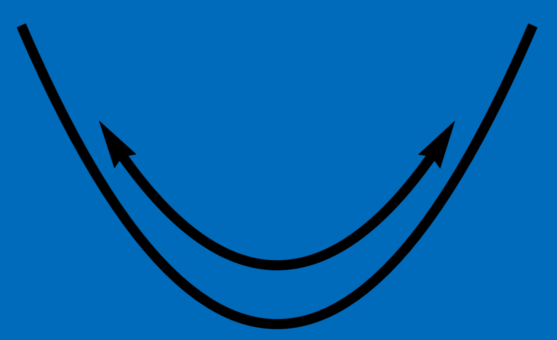

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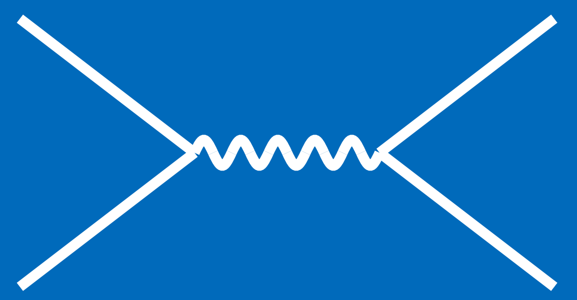

3. Lyman-α

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Decay of an oscillating condensate (inflaton, non-stabilized moduli, ...)

Notation. If $f_\chi(p) \equiv f_\chi(p/p_0)$ at decoupling $(t = t_*)$, then for $t > t_*$

$$f_\chi\left(\frac{p}{p_0}\frac{a(t)}{a_*}\right) = f_\chi\left(\frac{p\,a(t)/a_0}{p_0\,a_*/a_0}\right) = f_\chi\left(\frac{p_{\rm co}}{\underbrace{p_0\,a_*/a_0}_{T_*}}\right) \equiv f_\chi(q)$$

T_* (or $T_{\rm NCDM}$)

Standard in literature and cosmology codes (CLASS)

C. Ma, E. Bertschinger, Astrophys. J. 455 (1995) 7

D. Blas, J. Lesgourgues, T. Tram, JCAP 07 (2011) 034

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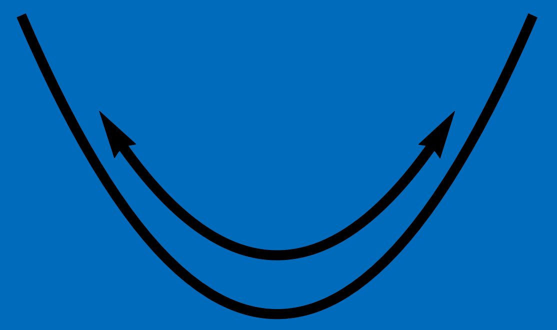

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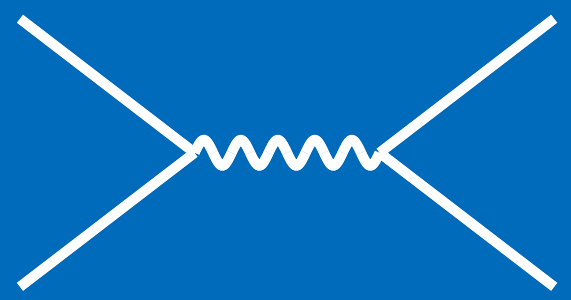

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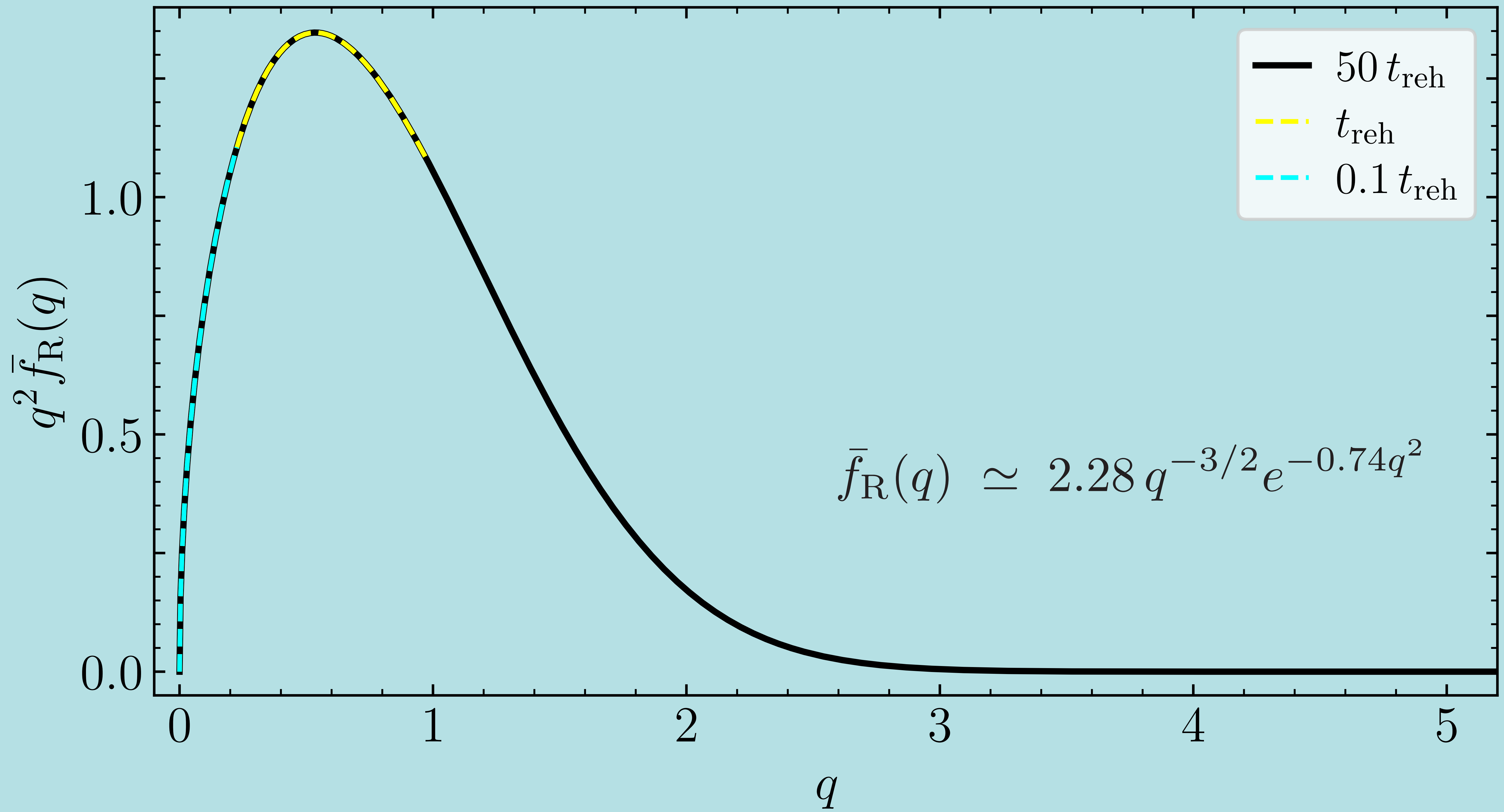
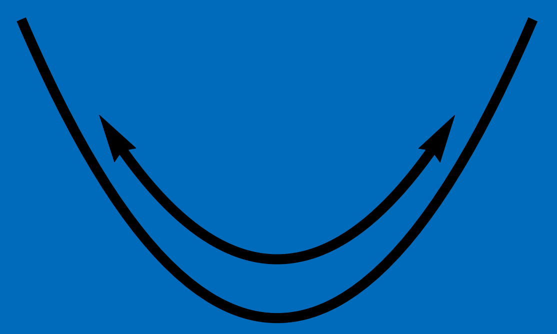
4. The end?

Decay of an oscillating condensate (inflaton, non-stabilized moduli, ...)

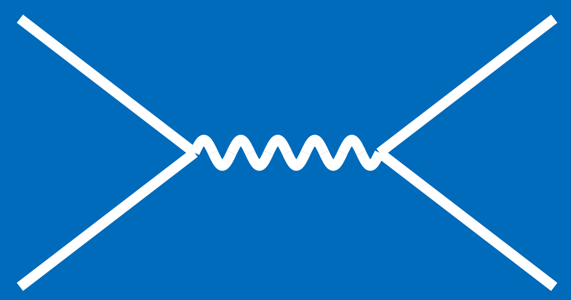
$$f_\chi(p,t)\, d^3\boldsymbol{p} \;=\; \frac{4\pi^4 \mathrm{Br}_\chi g_{*s}^{\mathrm{reh}}}{5 g_\chi} \left(\frac{T_{\mathrm{reh}}}{m_\Phi}\right)^4 \left(\frac{a_0}{a(t)}\right)^3 T_*^3\, \bar{f}_{\mathrm{R}}(q)\, d^3\boldsymbol{q}\,, \qquad T_* \;=\; \left(\frac{g_{*s}^0}{g_{*s}^{\mathrm{reh}}}\right)^{1/3} \frac{m_\Phi}{2T_{\mathrm{reh}}}\, T_0$$



1. Reheating



2. Freeze-in



3. Lyman-α

4. The end?

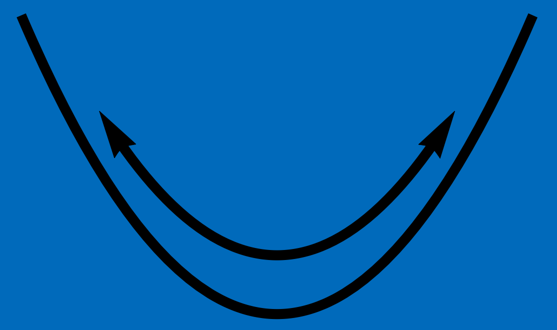
Decay of an oscillating condensate (inflaton, non-stabilized moduli, ...)

$$f_\chi(p,t)\,d^3\boldsymbol{p} \;=\; \frac{4\pi^4 \mathrm{Br}_\chi g_{*s}^{\mathrm{reh}}}{5 g_\chi}\left(\frac{T_{\mathrm{reh}}}{m_\Phi}\right)^4\left(\frac{a_0}{a(t)}\right)^3 T_*^3\,\bar{f}_{\mathrm{R}}(q)\,d^3\boldsymbol{q}\,, \qquad T_* \;=\; \left(\frac{g_{*s}^0}{g_{*s}^{\mathrm{reh}}}\right)^{1/3}\frac{m_\Phi}{2T_{\mathrm{reh}}}\,T_0$$

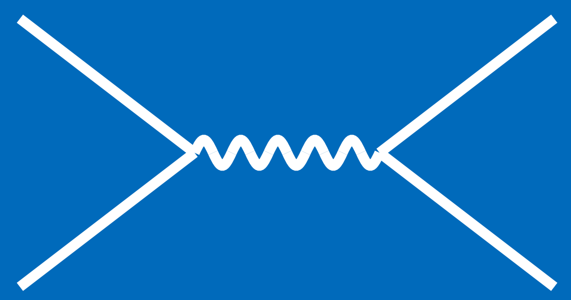
$$n_\chi(t) \;\simeq\; 0.70\pi^2 \mathrm{Br}_\chi g_{*s}^{\mathrm{reh}}\left(\frac{T_{\mathrm{reh}}}{m_\Phi}\right)^4\left(\frac{a_0}{a(t)}\right)^3 T_*^3$$

$$\Omega_\chi h^2 \;\simeq\; 0.1\left(\frac{\mathrm{Br}_\chi}{5.5\times10^{-4}}\right)\left(\frac{m_{\mathrm{DM}}}{1\,\mathrm{MeV}}\right)\left(\frac{T_{\mathrm{reh}}}{10^{10}\,\mathrm{GeV}}\right)\left(\frac{3\times10^{13}\,\mathrm{GeV}}{m_\Phi}\right)$$

1. Reheating



2. Freeze-in



3. Lyman-α

4. The end?

Out-of-equilibrium decay chain $\Phi \to A \to \chi$

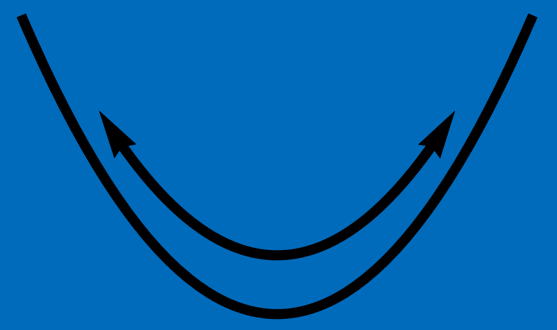
$$\Omega_\chi h^2 \;\simeq\; 0.1 \left(\frac{\mathrm{Br}_\chi}{5.5\times 10^{-4}}\right)\left(\frac{m_{\rm DM}}{1\,\mathrm{MeV}}\right)\left(\frac{T_{\rm reh}}{10^{10}\,\mathrm{GeV}}\right)\left(\frac{3\times 10^{13}\,\mathrm{GeV}}{m_\Phi}\right)$$

$$f_\chi(p,t)\,d^3\boldsymbol{p} \;=\; \frac{24\pi^3\sqrt{10 g_{*s}^{\rm reh}}\,\mathrm{Br}_\chi \mathrm{Br}_A \Gamma_A M_P}{5 g_A m_A^2}\left(\frac{T_{\rm reh}}{m_\Phi}\right)^2 \mathcal{F}(q, Y_{\rm dec})\left(\frac{a_0}{a(t)}\right)^3 T_*^3\, d^3\boldsymbol{q}$$

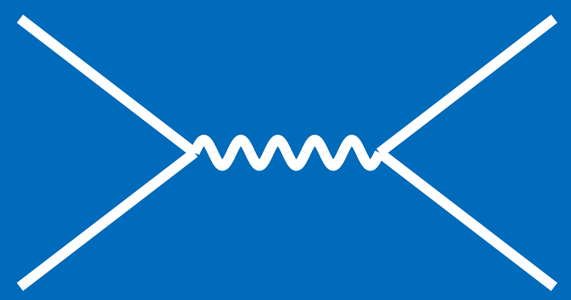
$$\mathcal{F}(q,Y_{\rm dec}) \;=\; q^{-2}\int_0^{Y_{\rm dec}} dy\, y^2 \int_{\left|q-\frac{y^2}{q}\right|}^{\infty} \frac{z\,dz}{\sqrt{q^2+4y^2}}\,\bar{f}_R(z) \;\simeq\; \begin{cases} \bar{f}_{\rm D,NR}(q)\,, & Y_{\rm dec}\gg 1\,,\\ \dfrac{Y_{\rm dec}^3}{3}\,\bar{f}_{\rm D,R}(q)\,, & Y_{\rm dec}\ll 1 \end{cases}$$

$$Y_{\rm dec} \;=\; \left(\frac{g_{*s}^{\rm reh}}{g_{*s}^{\rm dec}}\right)^{1/3}\frac{m_A T_{\rm reh}}{m_\phi T_{\rm dec}}$$

1. Reheating



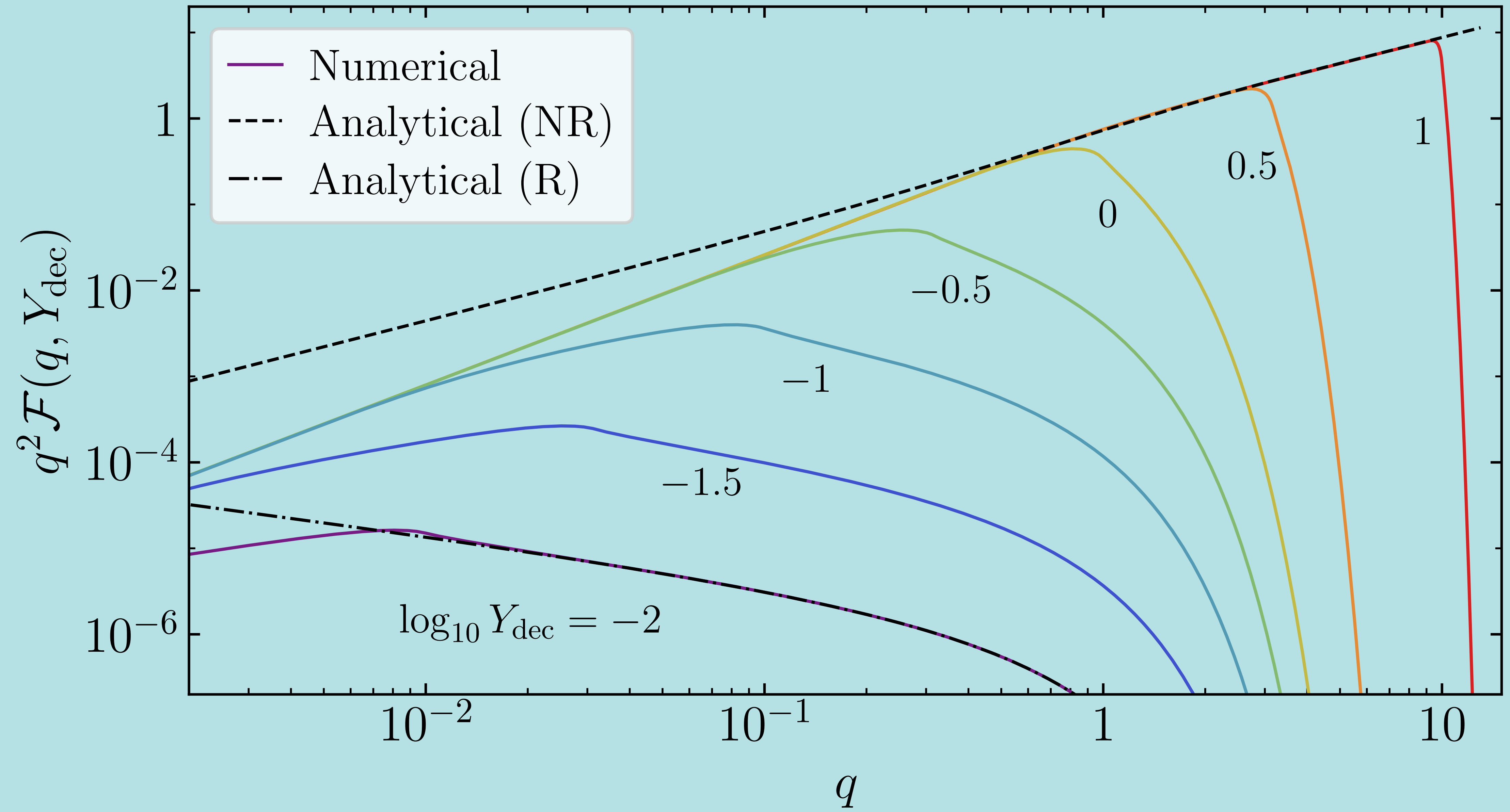
2. Freeze-in



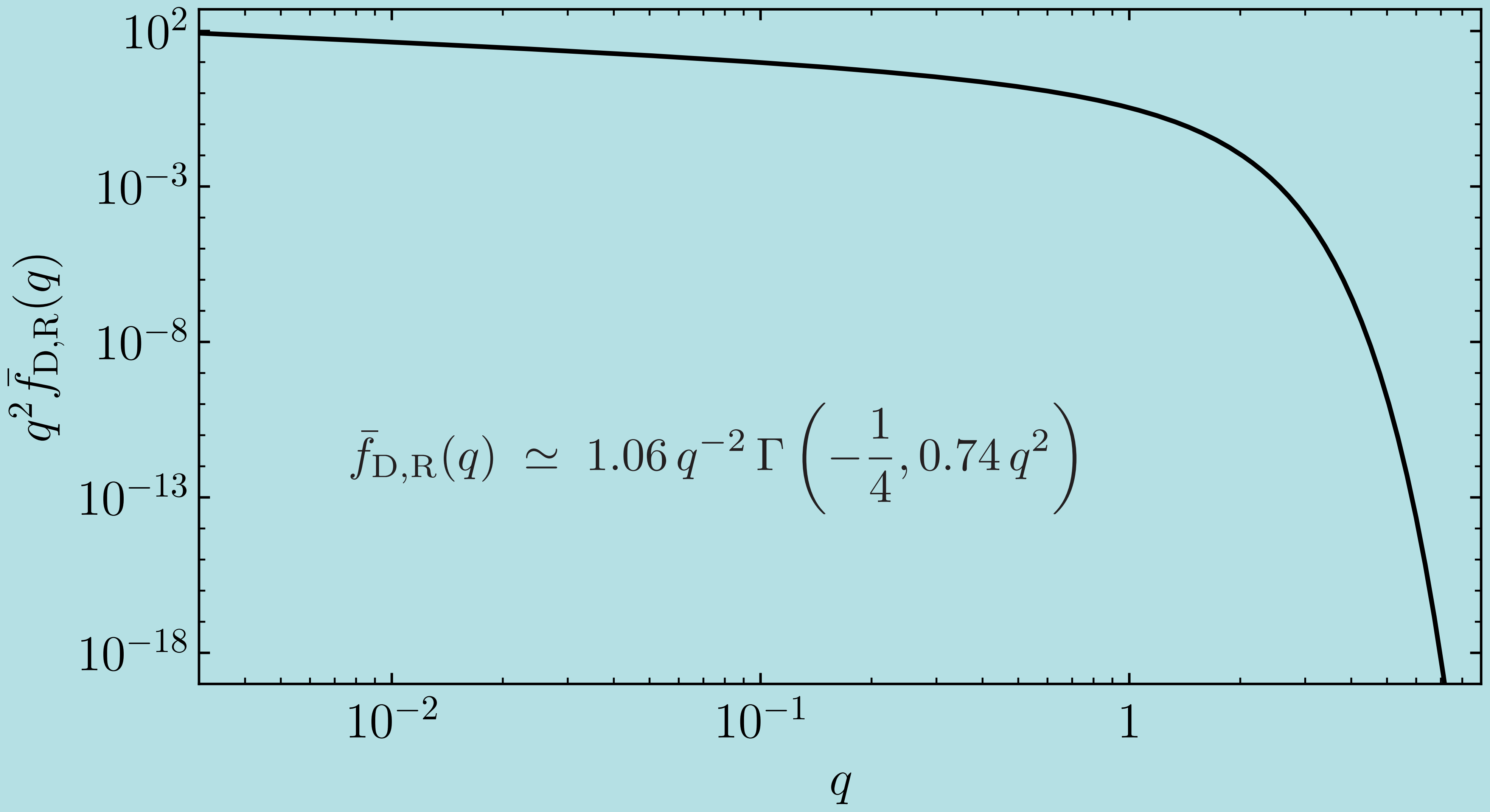
3. Lyman-α

4. The end?

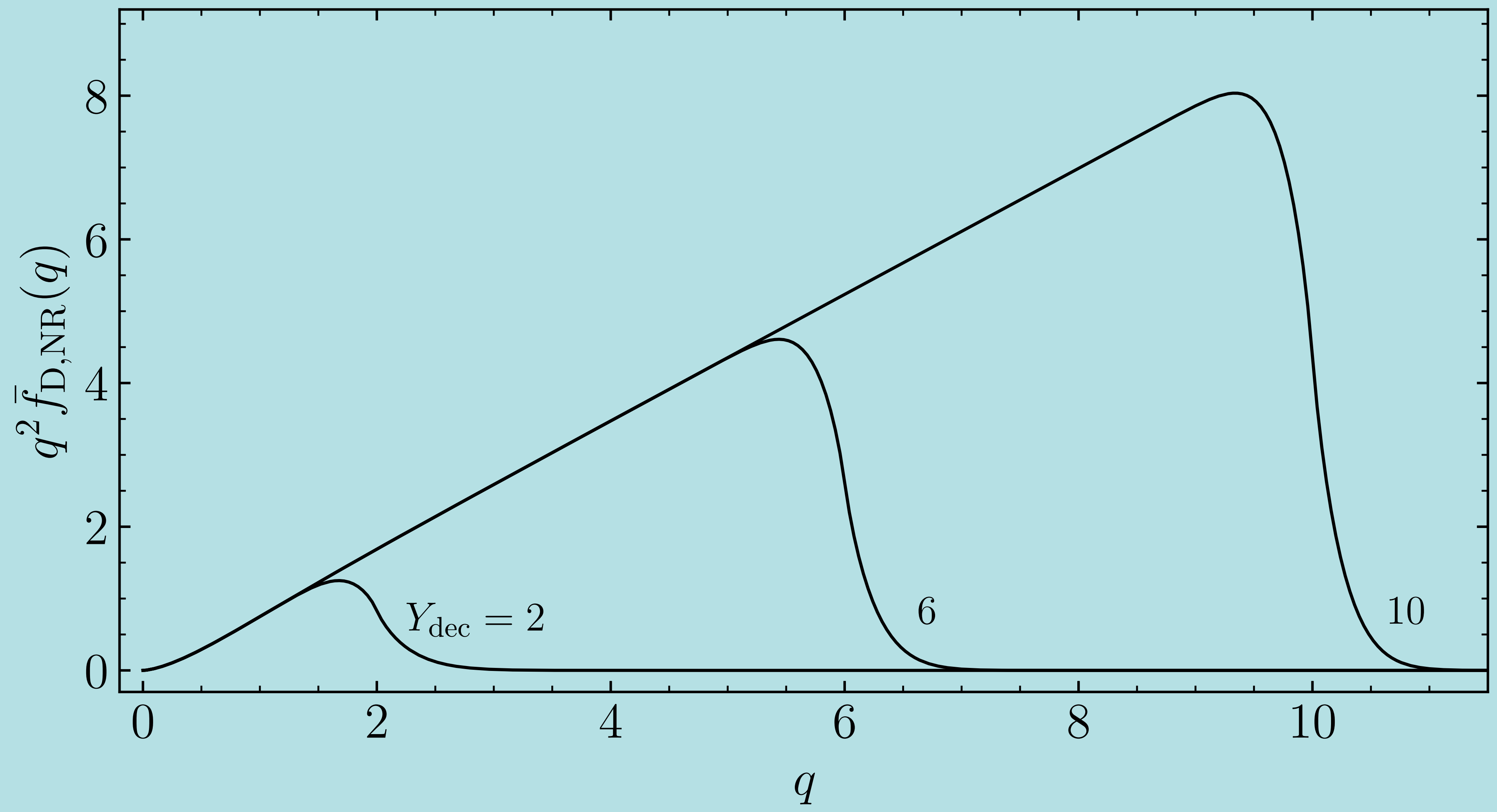
Out-of-equilibrium decay chain $\Phi \to A \to \chi$



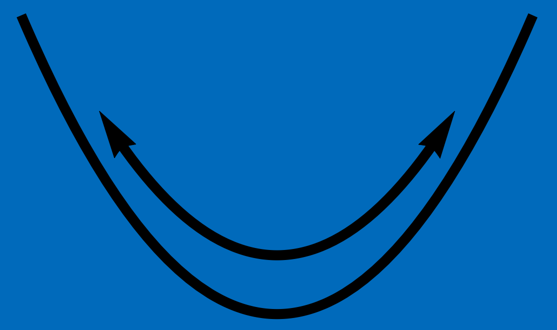
R



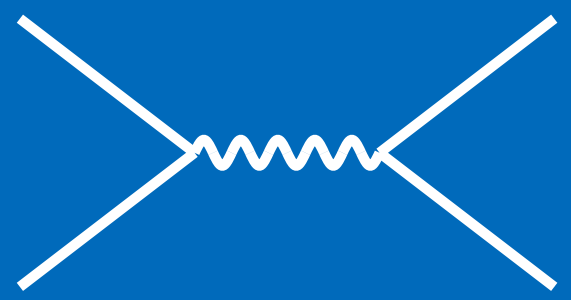
NR



1. Reheating



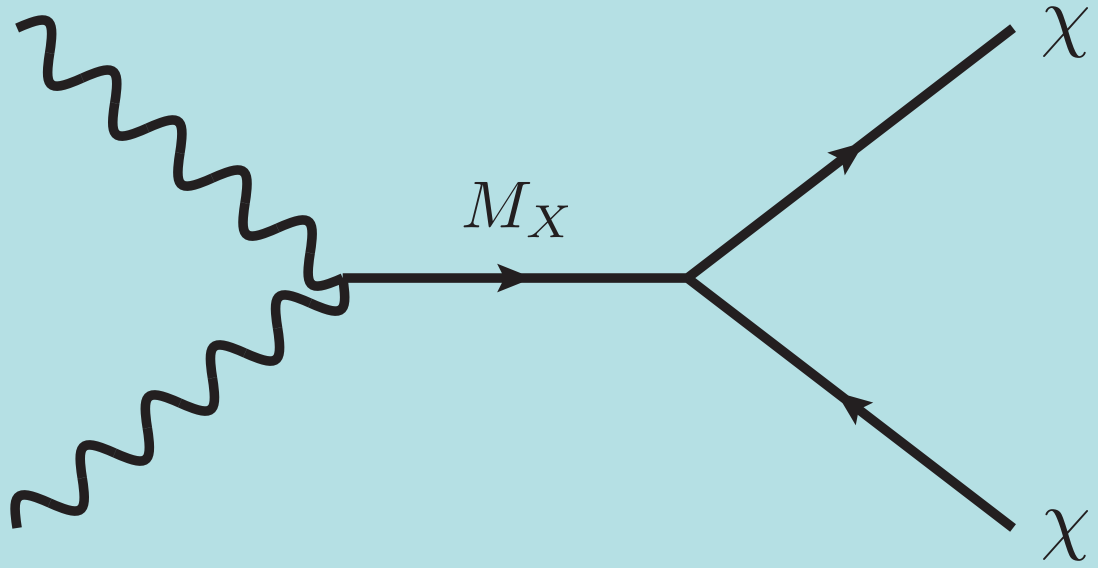
2. Freeze-in



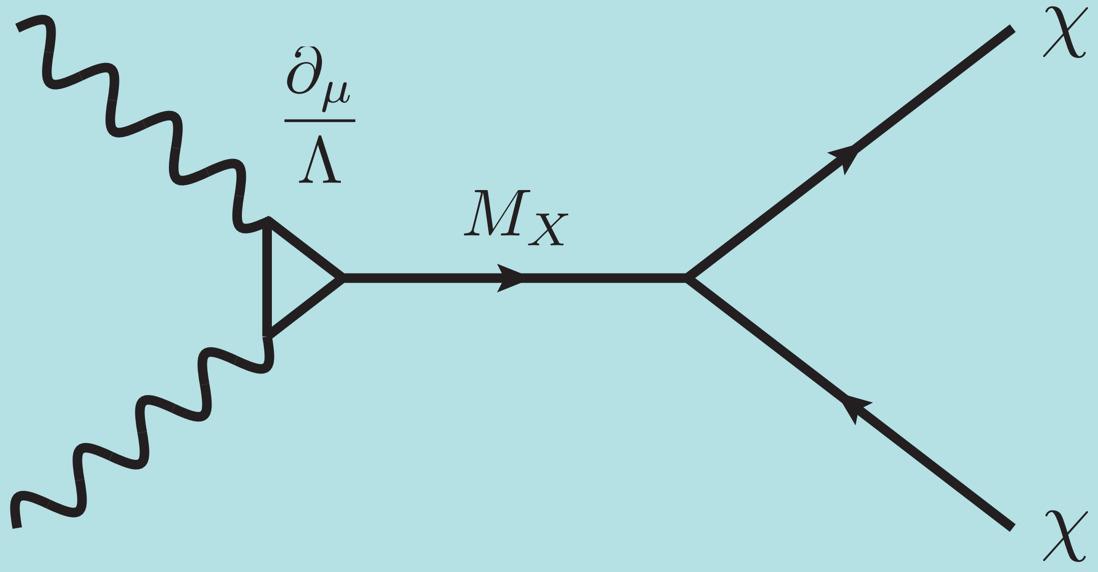
3. Lyman-α

4. The end?

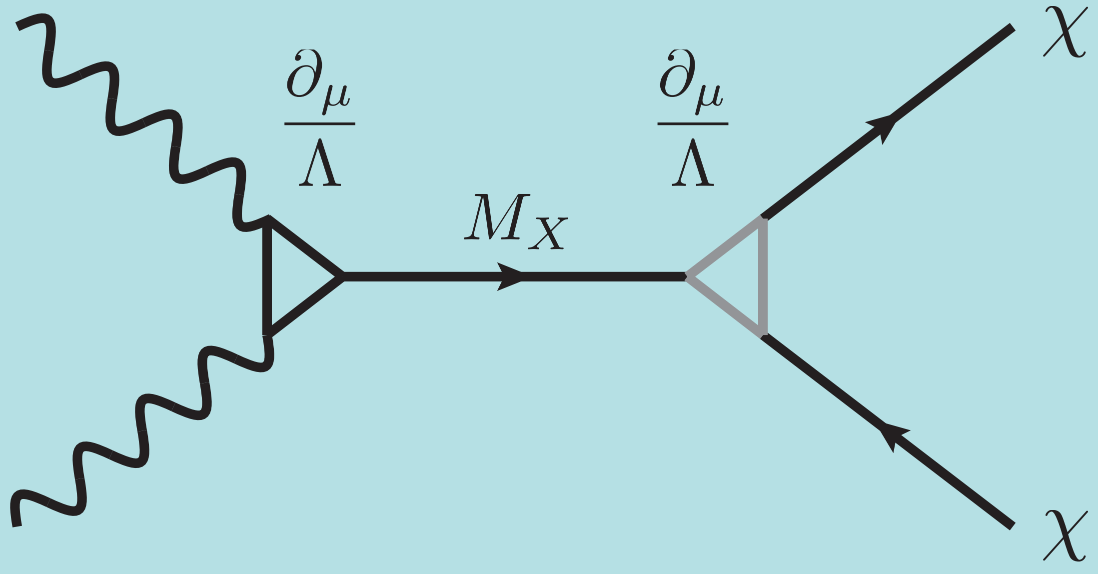
Freezing-in through scatterings



$$\sigma(s) = \frac{s}{M_X^4}$$



$$\sigma(s) = \frac{s^2}{\Lambda^2 M_X^4}$$



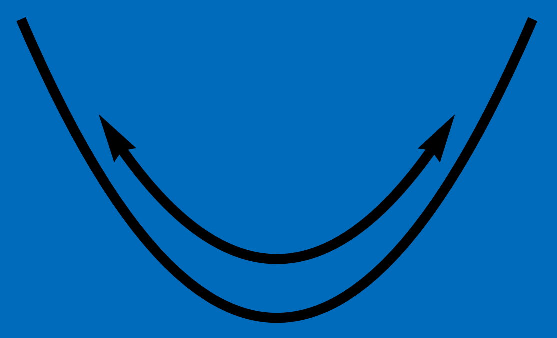
$$\sigma(s) = \frac{s^3}{\Lambda^4 M_X^4}$$

Production at $t \leq t_{\mathrm{reh}}$ if

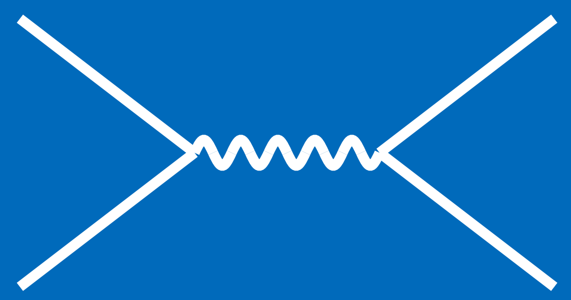
$$\sigma(s) = \frac{s^{n/2}}{\Lambda^{n+2}}$$

with $n > -1$

1. Reheating



2. Freeze-in



3. Lyman-α

4. The end?

Freezing-in through scatterings (examples)

- $n = 0:$ Low scale susy gravitino, $\sigma \propto M_P^{-2}$. Axino, $\sigma \propto f_a^{-2}$

  V. Rychkov, A. Strumia, Phys. Rev. D 75 (2007) 075011 ; A. Strumia, JHEP 06 (2010) 036

- $n = 2:$ $SO(10) \rightarrow SU(4) \times SU(2)_L \times U(1)_R \rightarrow \mathrm{SM}$ (heavy DM), $\sigma \propto s/M_{\mathrm{int}}^4$
  Light spin-2 mediator (light DM), $\sigma \propto s/M_P^4$

  Y. Mambrini et al., Phys. Rev. Lett. 110 (2013) 241306; N. Bernal et al., Phys. Rev. D 97 (2018) 115020

- $n = 4:$ Non-susy spin-3/2 DM + sterile neutrino, $\sigma(s) \propto (s/m_{3/2} m_R M_P)^2$

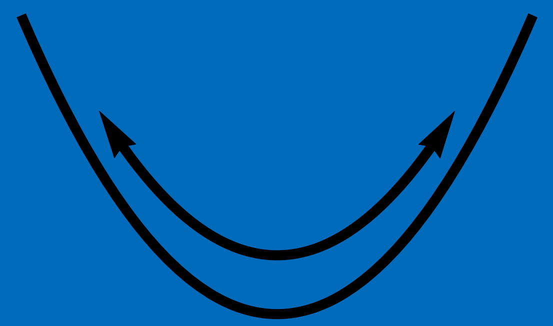
- $n = 6:$ High scale susy gravitino (heavy DM), $\sigma \propto s^3/(m_{3/2} M_P)^4$
  Heavy spin-2 mediator (light DM), $\sigma \propto s^3/(m_{\tilde{h}} M_P)^4$

  K. Benakli et al., Phys. Rev. D 95 (2017) 095002; N. Bernal et al., Phys. Rev. D 97 (2018) 115020

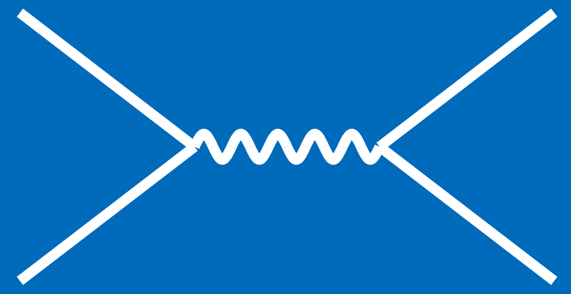
- $n > 6:$ Vector non-Abelian DM with heavy Z'

  G. Bhattacharyya, M. Dutra, Y. Mambrini, M. Pierre, Phys. Rev. D 98 (2018) 035038

1. Reheating



2. Freeze-in



3. Lyman-α

4. The end?

Thermal freeze-in

$n < 6:$

$$\Omega_\chi^{(n)} h^2 \simeq \frac{g_A g_B g_\psi g_\chi \sqrt{c}\, 2^{n+3} \Gamma(\frac{n}{2}+3)^2 \zeta(\frac{n}{2}+3)^2 \mathcal{S}(n)}{(6-n)(n+4)} \left(\frac{106.75}{g_{*s}^{\rm reh}}\right)^{3/2} \left(\frac{T_{\rm reh}}{\Lambda}\right)^{n+1} \left(\frac{10^{16}\,\mathrm{GeV}}{\Lambda}\right) \left(\frac{m_{\rm DM}}{1\,\mathrm{keV}}\right)$$

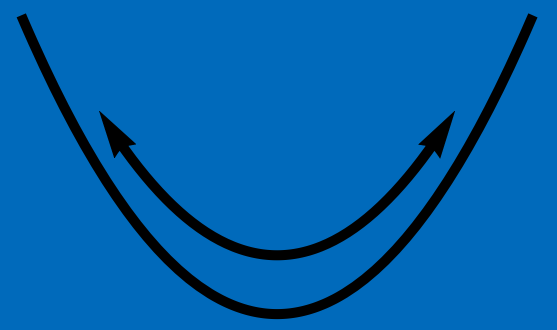
$n = 6:$

$$\Omega_\chi^{(6)} h^2 = g_A g_B g_\psi g_\chi \sqrt{c}\, \mathcal{S}(6) \left(\frac{106.75}{g_{*s}^{\rm reh}}\right)^{3/2} \left(\frac{m_{\rm DM}}{1.2\,\mathrm{keV}}\right) \left(\frac{T_{\rm reh}}{10^6\,\mathrm{GeV}}\right)^{7} \left(\frac{10^8\,\mathrm{GeV}}{\Lambda}\right)^{8} \ln\left(\frac{T_{\max}}{T_{\rm reh}}\right)$$

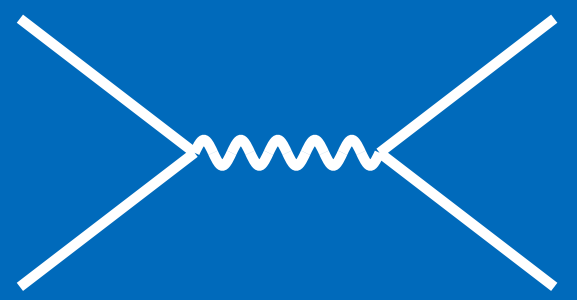
$n > 6:$

$$\Omega_\chi^{(n)} h^2 \simeq \frac{g_A g_B g_\psi g_\chi \sqrt{c}\, 2^{n+3} \Gamma(\frac{n+4}{2}) \Gamma(\frac{n+6}{2})}{n-6} \left(\frac{106.75}{g_{*s}^{\rm reh}}\right)^{3/2}$$
$$\times \left(\frac{T_{\max}}{\Lambda}\right)^{n+1} \left(\frac{T_{\rm reh}}{T_{\max}}\right)^{7} \left(\frac{10^{16}\,\mathrm{GeV}}{\Lambda}\right) \left(\frac{m_{\rm DM}}{1.8\,\mathrm{keV}}\right)$$

1. Reheating



2. Freeze-in



3. Lyman-α

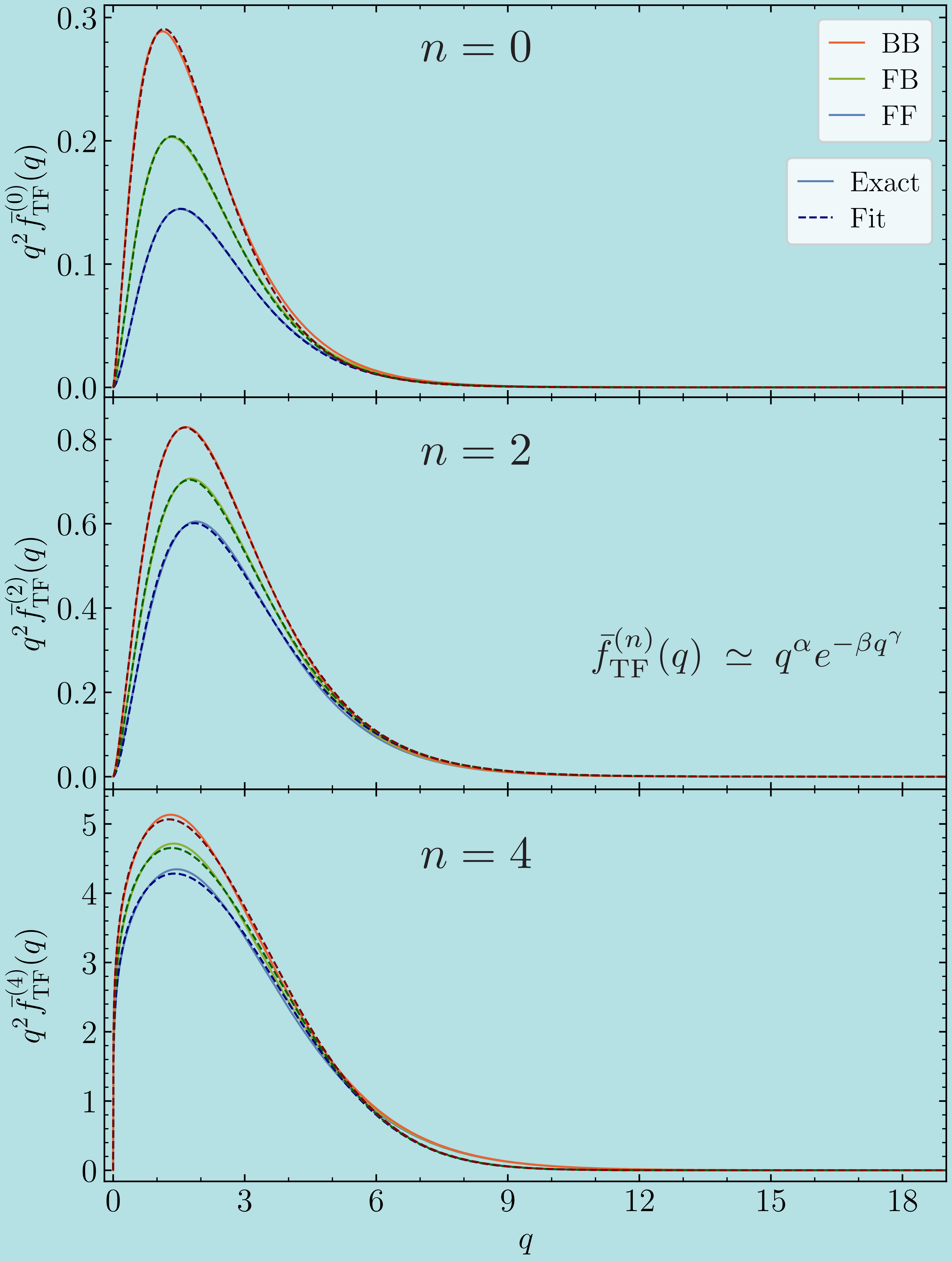
4. The end?

Thermal freeze-in

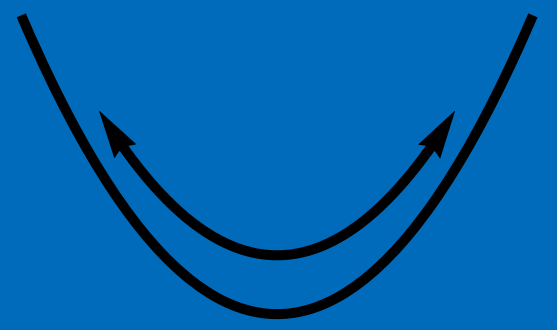
$n < 6:$

$$f_\chi(p,t)\,d^3\boldsymbol{p} \;\simeq\; \frac{3\cdot 2^{n+6}\,\Gamma\!\left(\frac{n+4}{2}\right) g_A g_B g_\psi M_P T_{\rm reh}^{n+1}}{5(2\pi)^3\Lambda^{n+2}} \times \left(\frac{6c}{g_{*s}^{\rm reh}}\right)^{1/2}\left(\frac{a_0}{a(t)}\right)^3 T_*^3\,\bar{f}_{\rm TF}^{(n)}(q)\,d^3\boldsymbol{q}$$

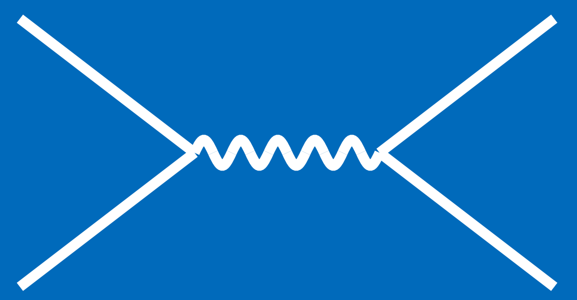
$$T_* \;=\; \left(\frac{g_{*s}^0}{g_{*s}^{\rm reh}}\right)^{1/3} T_0$$



1. Reheating



2. Freeze-in



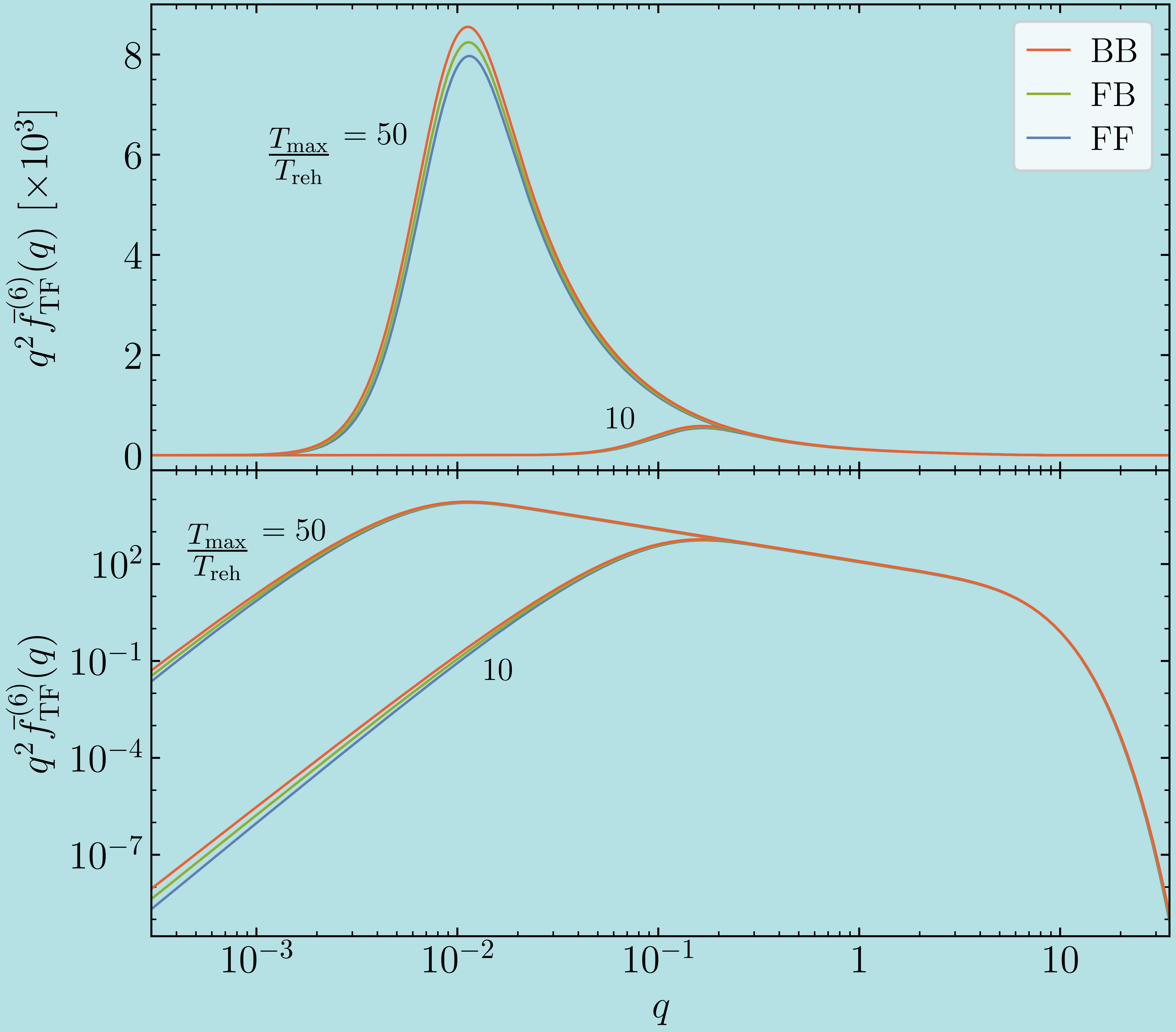
3. Lyman-α

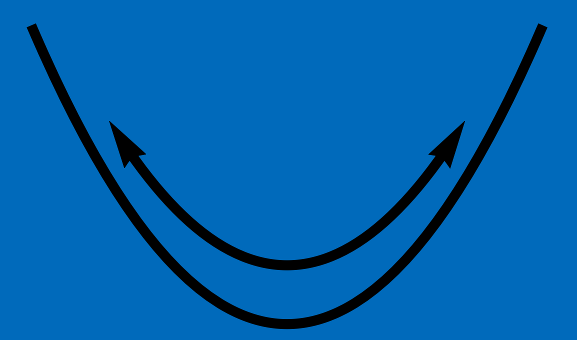
4. The end?

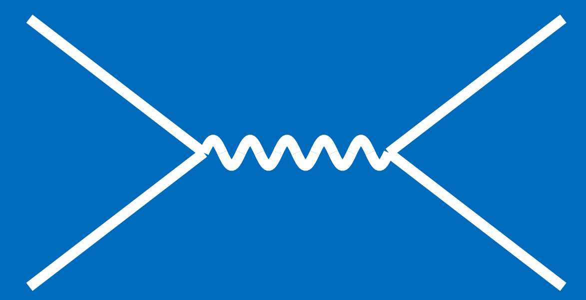
Thermal freeze-in

$n \geq 6:$

$$\bar{f}^{(n)}_{\rm TF}(q) \simeq q^{\frac{3}{5}(1-n)}\left[\Gamma\left(\frac{11}{10}n - \frac{3}{5}, q\right) - \Gamma\left(\frac{11}{10}n - \frac{3}{5}, q\left(\frac{T_{\rm max}}{T_{\rm reh}}\right)^{5/3}\right)\right]$$

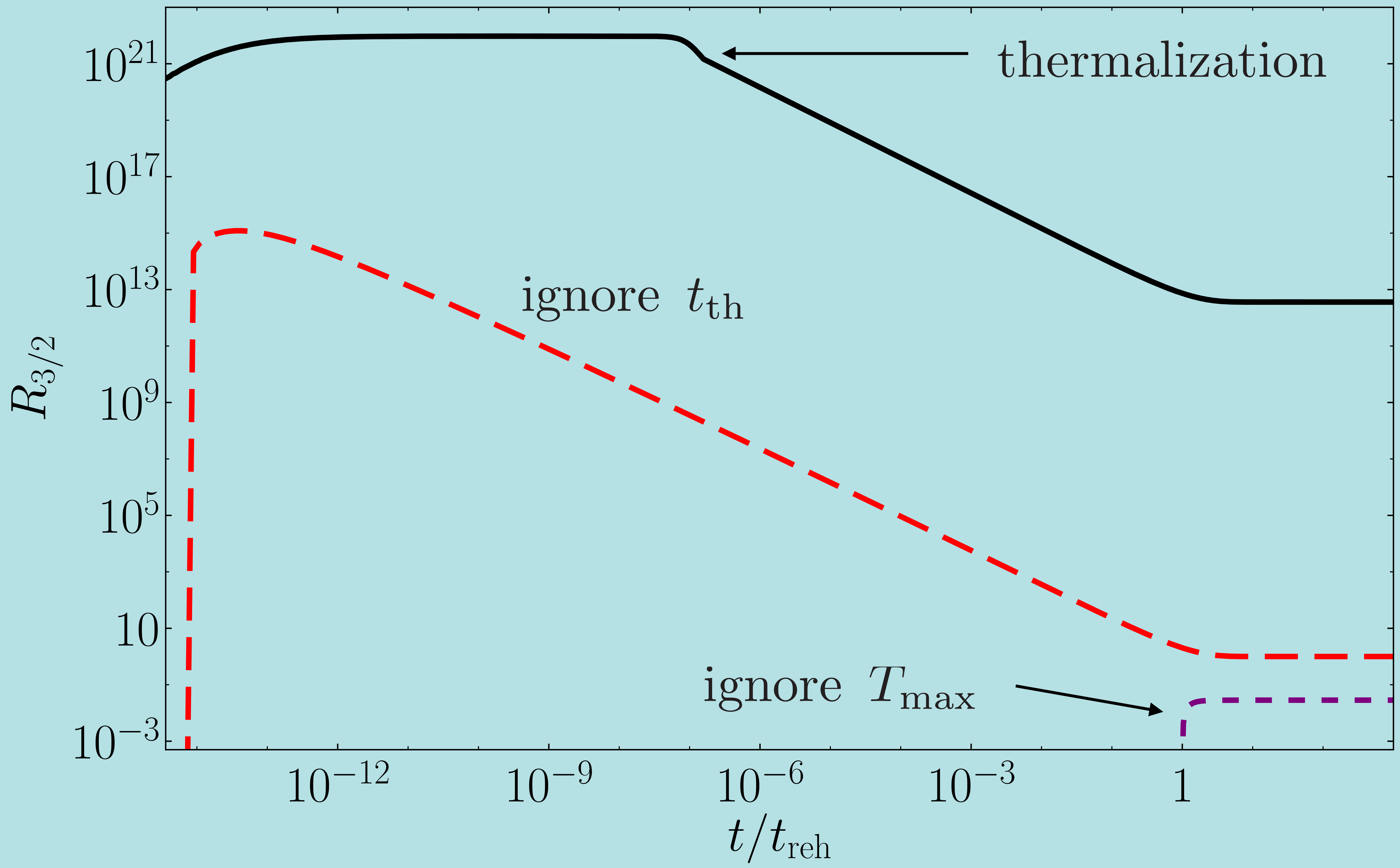






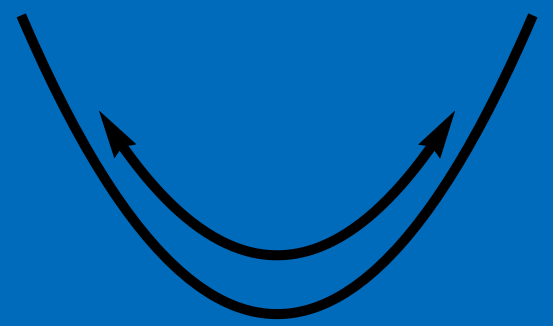
Non-thermal freeze-in

High scale supersymmetry breaking: $m_{\mathrm{susy}} \gg m_\Phi \gg m_{3/2} \qquad (n = 6)$

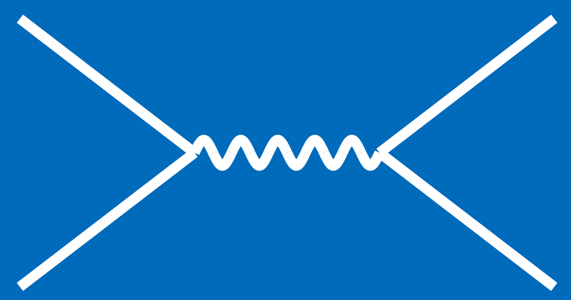


$$R_{3/2} \equiv \frac{n_{3/2}^{\mathrm{NT}}}{n_{3/2}^{\mathrm{T}}}$$

1. Reheating



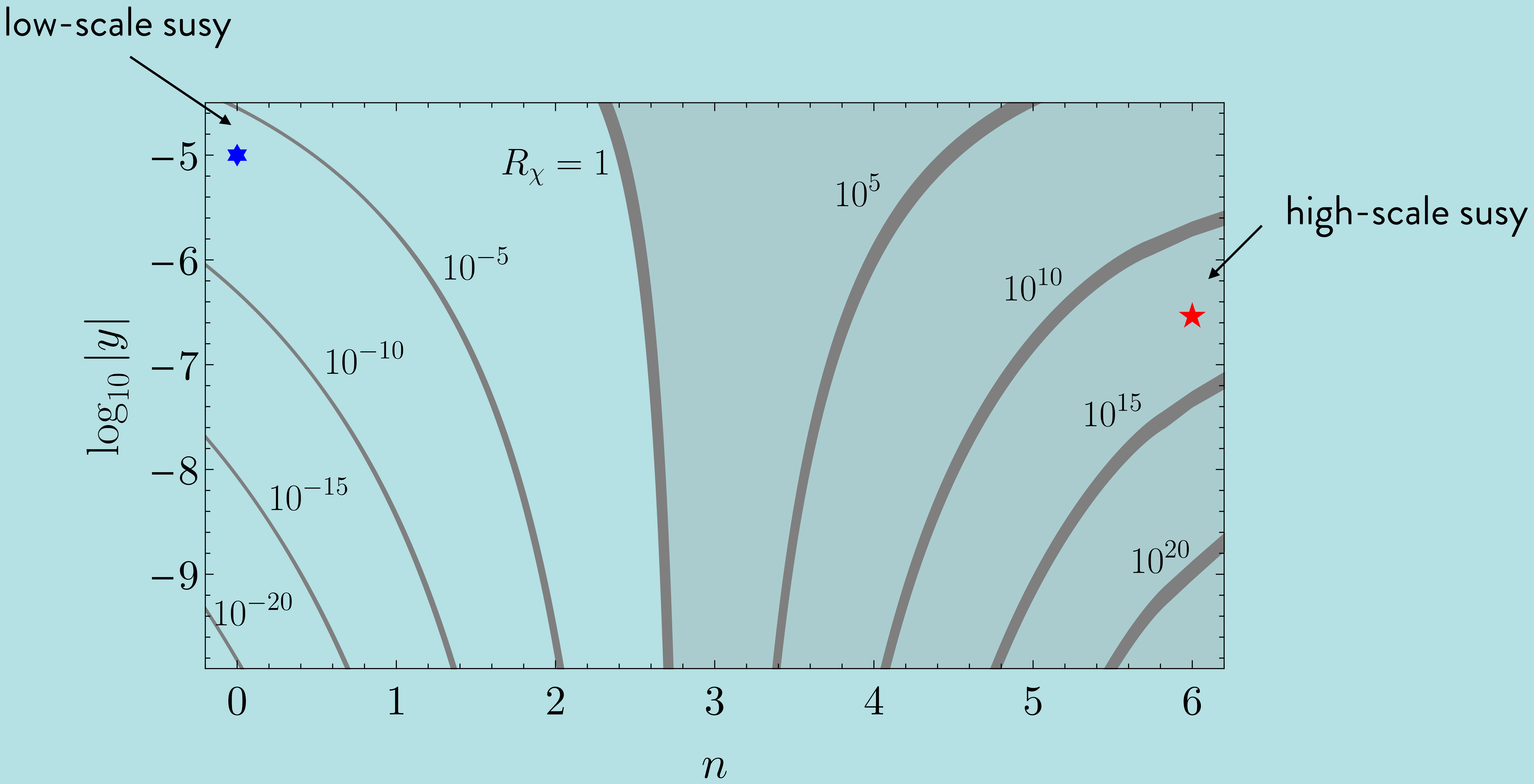
2. Freeze-in



3. Lyman-α

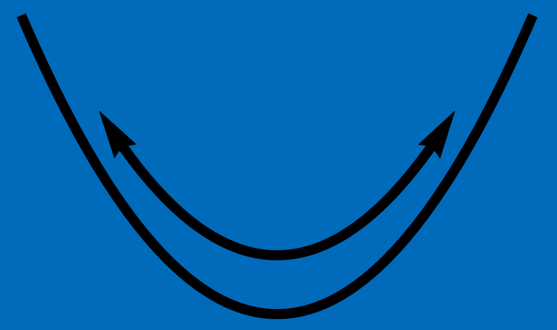
4. The end?

Non-thermal freeze-in

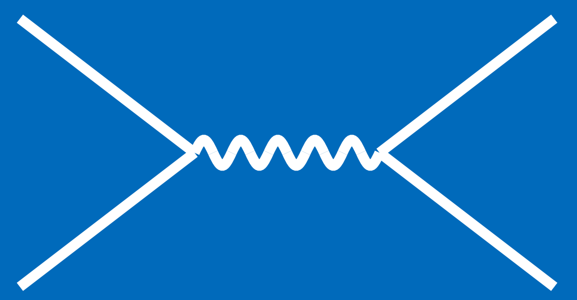


$$\Omega_\chi \propto \frac{m_\Phi^{n-2} M_P T_{\rm reh}^3}{M^{n+2}} \left(\Gamma_\Phi t_{\rm th}\right)$$

1. Reheating



2. Freeze-in



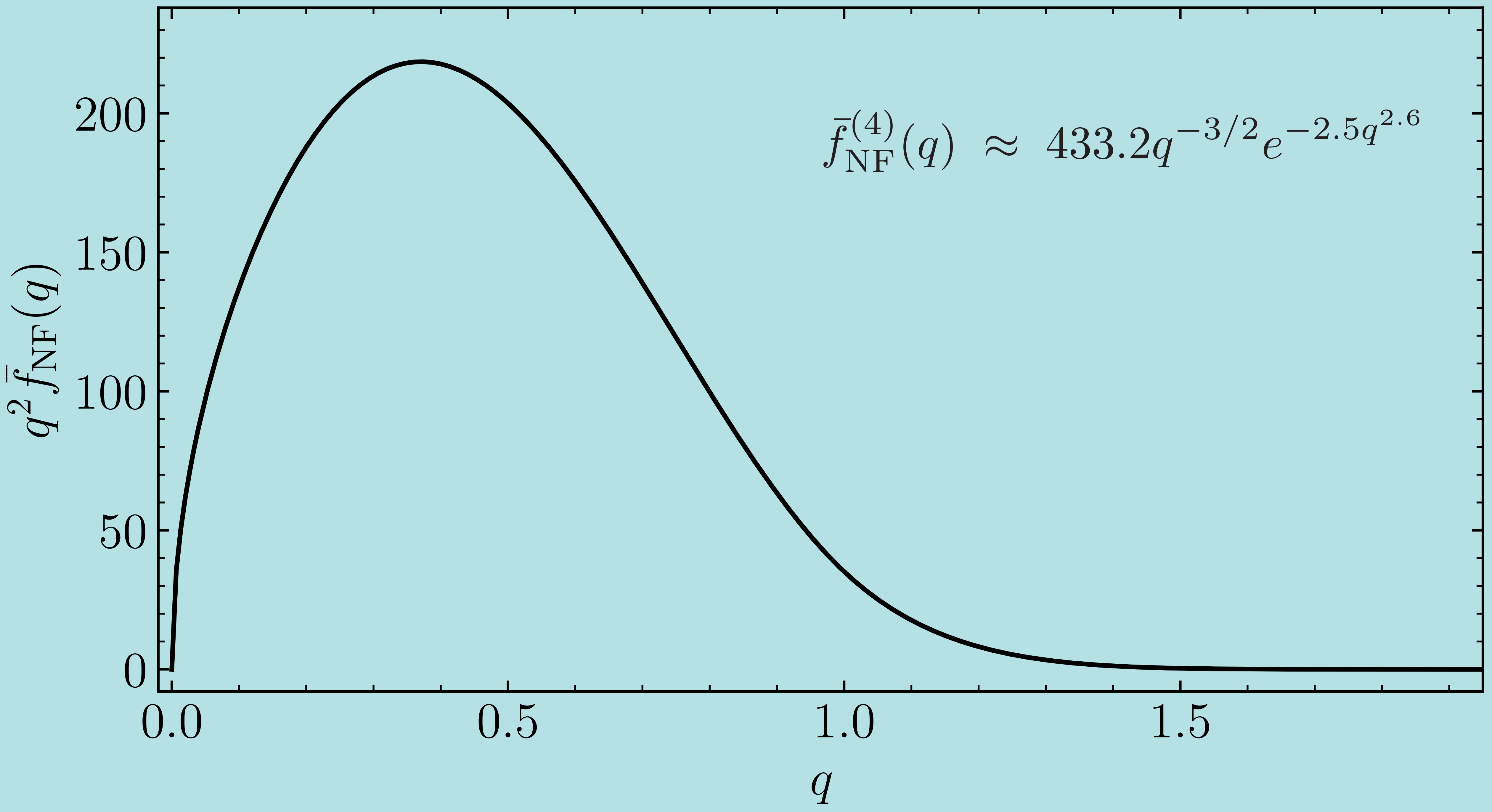
3. Lyman-α

4. The end?

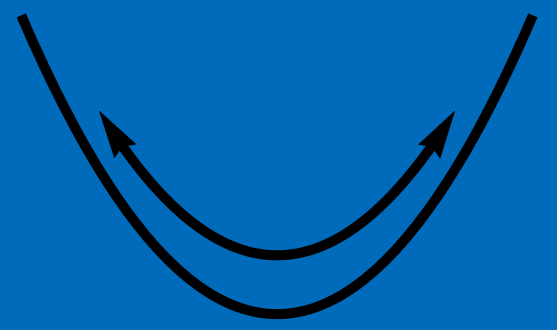
Non-thermal freeze-in $\quad (n=4)$

$$f_\chi(p,t)\,d^3\boldsymbol{p} \;\simeq\; \frac{256\pi^2 g_\psi}{15015\Lambda^6}\left(\frac{\pi^2 c g_{*s}^{\rm reh}}{24}\right)^{13/10}\left(\frac{\alpha_{\rm SM}^{16}T_{\rm reh}^{26}M_P^{13}}{m_\Phi^9}\right)^{1/5}\left(\frac{a_0}{a(t)}\right)^3 T_*^3\,\bar{f}_{\rm NF}^{(4)}(q)\,d^3\boldsymbol{q}$$

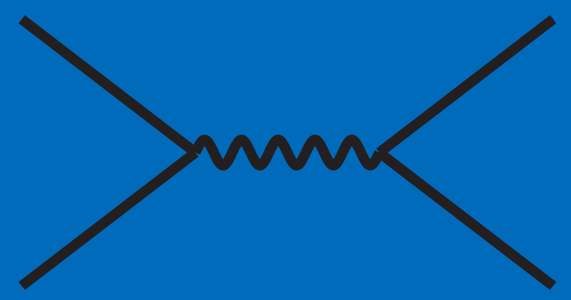
$$T_* \;=\; \frac{\alpha_{\rm SM}^{-32/15}}{2}\left(\frac{g_{*s}^0}{g_{*s}^{\rm reh}}\right)^{1/3}\left(\frac{\pi^2 c g_{*s}^{\rm reh}}{24}\right)^{2/15}\left(\frac{m_\Phi}{T_{\rm reh}}\right)^{7/15}\left(\frac{m_\Phi}{M_P}\right)^{16/15} T_0$$



1. Reheating



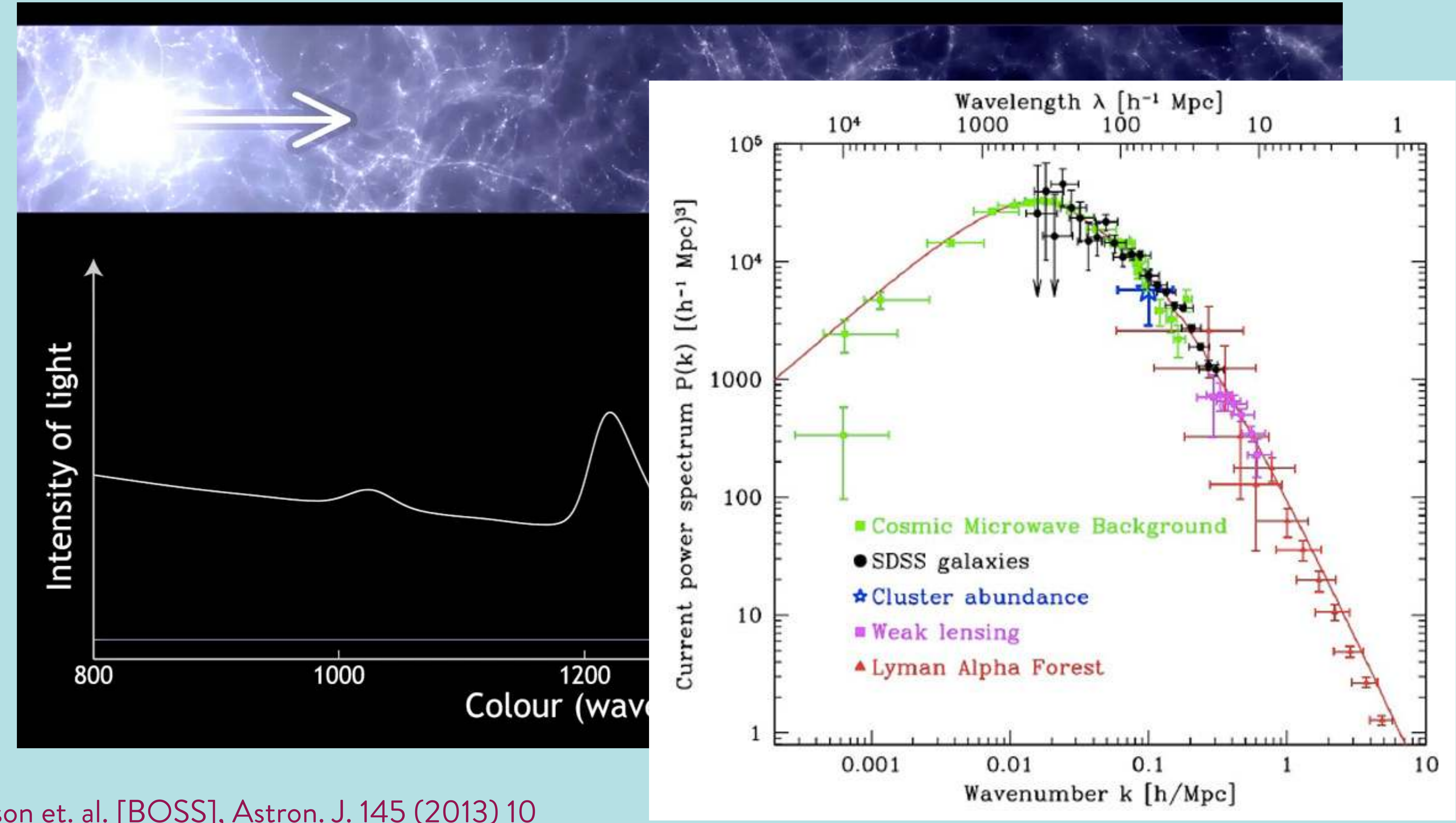
2. Freeze-in



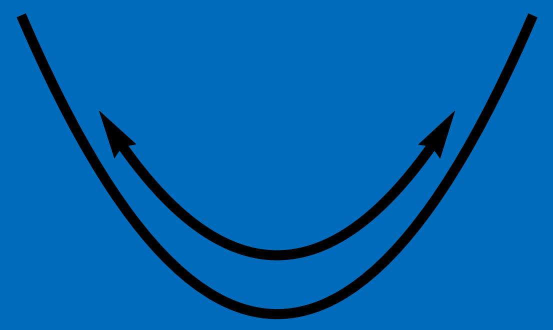
3. Lyman-α

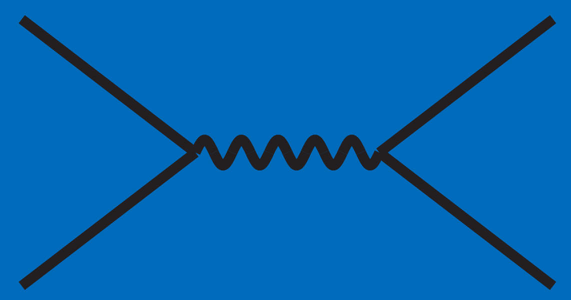
4. The end?

The Lyman-α constraint on Warm Dark Matter



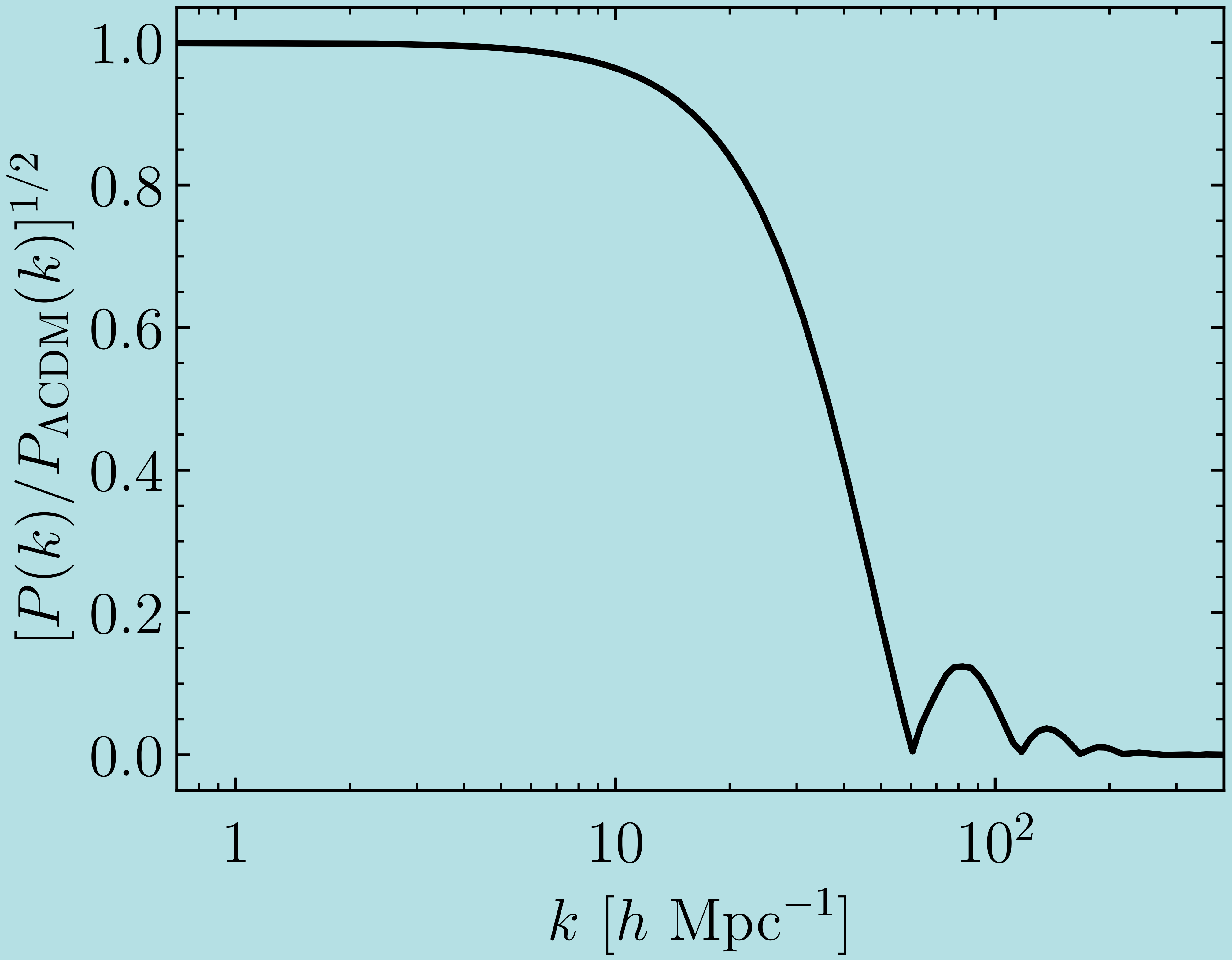
K. Dawson et. al. [BOSS], Astron. J. 145 (2013) 10





The perturbed Boltzmann equation

$$f(\boldsymbol{x},\boldsymbol{p},\tau) = f_0(|\boldsymbol{p}|,\tau)[1+\Psi(\boldsymbol{x},\boldsymbol{p},\tau)] \longrightarrow \ddot{\delta} + \mathcal{H}\dot{\delta} - \frac{3}{2}\mathcal{H}^2\left(1 - w\frac{10}{9}\frac{k^2}{\mathcal{H}^2}\right)\delta = 0$$



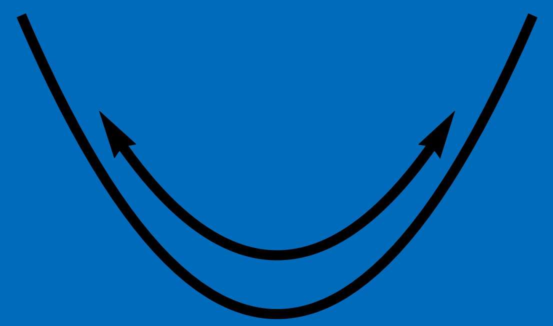
$$k_{\mathrm{FS}}(a) = \frac{9}{10}\frac{\mathcal{H}^2}{w}$$

$$k_H(a) \equiv \left[\int_0^a \frac{1}{k_{\mathrm{FS}}(\tilde{a})}\frac{d\tilde{a}}{\tilde{a}}\right]^{-1}$$

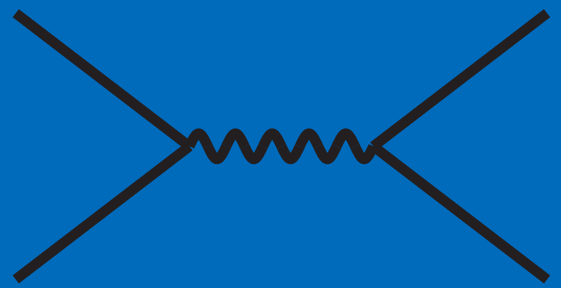
$$w \simeq \frac{\delta P}{\delta\rho} = \frac{T_*^2}{3m_{\mathrm{DM}}^2}\frac{\langle q^2\rangle}{a^2}$$

J. Lesgourgues, T. Tram, JCAP 09 (2011) 032

1. Reheating



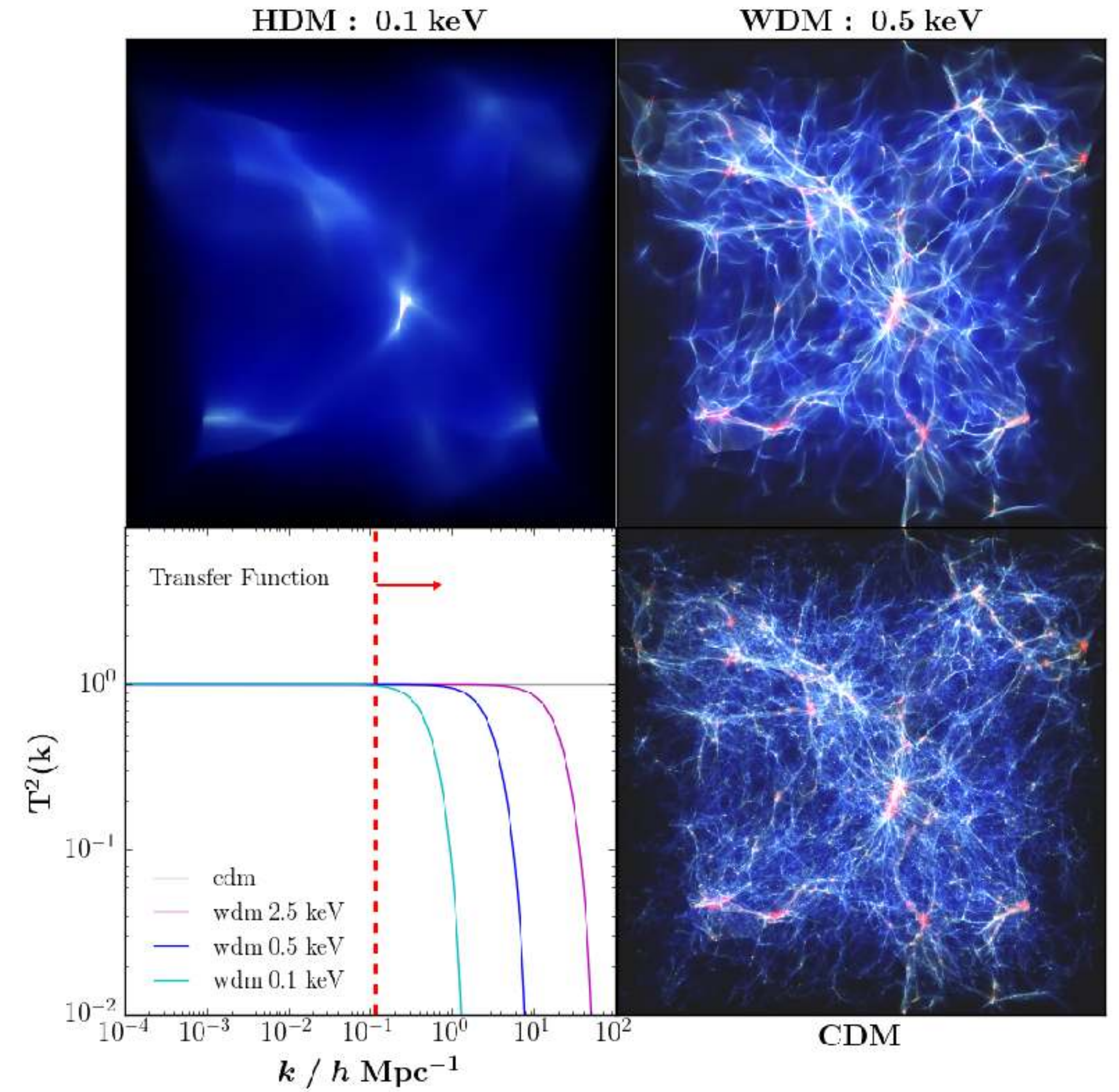
2. Freeze-in



3. Lyman-α

4. The end?

The perturbed Boltzmann equation



$$\ddot{\delta} + \mathcal{H}\dot{\delta} - \frac{3}{2}\mathcal{H}^2\left(1 - w\frac{10}{9}\frac{k^2}{\mathcal{H}^2}\right)\delta = 0$$

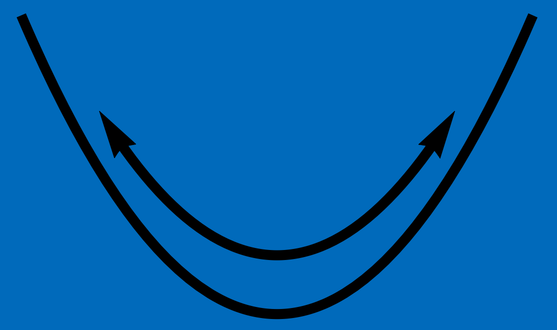
$$k_{\rm FS}(a) = \frac{9}{10}\frac{\mathcal{H}^2}{w}$$

$$k_H(a) \equiv \left[\int_0^a \frac{1}{k_{\rm FS}(\tilde{a})}\frac{d\tilde{a}}{\tilde{a}}\right]^{-1}$$

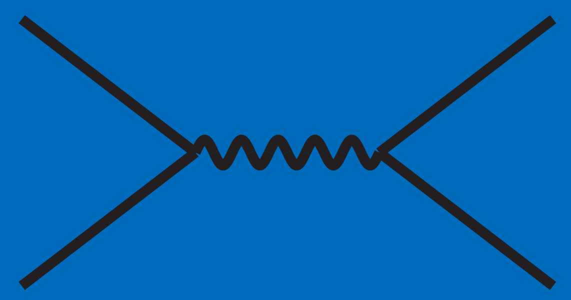
J. Baur et al., JCAP 08 (2016) 012

R. Murguia et al., JCAP 11 (2017) 046

1. Reheating



2. Freeze-in

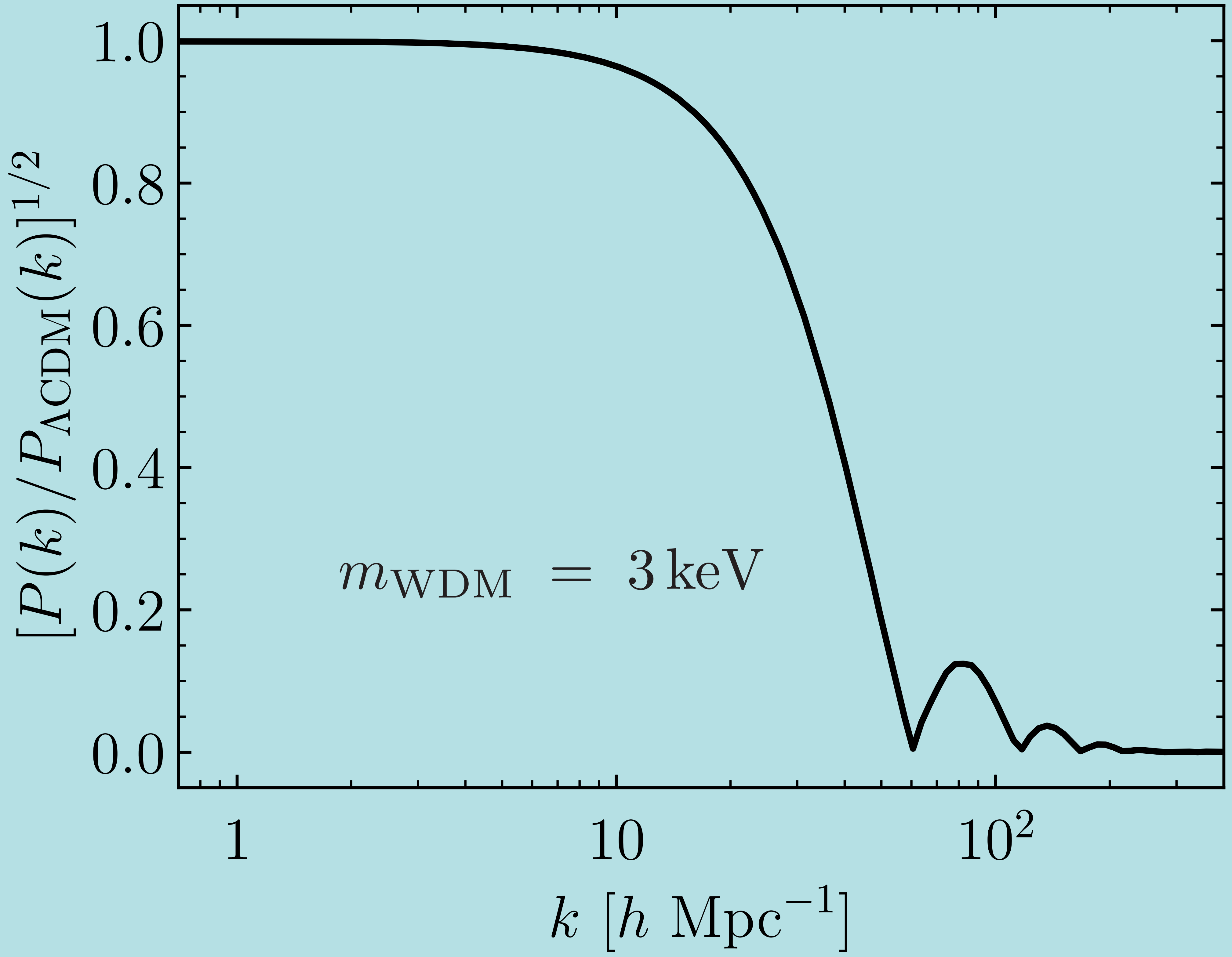


3. Lyman-α

4. The end?

The perturbed Boltzmann equation

$$f(\boldsymbol{x},\boldsymbol{p},\tau) = f_0(|\boldsymbol{p}|,\tau)[1+\Psi(\boldsymbol{x},\boldsymbol{p},\tau)] \longrightarrow \ddot{\delta} + \mathcal{H}\dot{\delta} - \frac{3}{2}\mathcal{H}^2\left(1 - w\frac{10}{9}\frac{k^2}{\mathcal{H}^2}\right)\delta = 0$$



$$k_{\mathrm{FS}}(a) = \frac{9}{10}\frac{\mathcal{H}^2}{w}$$

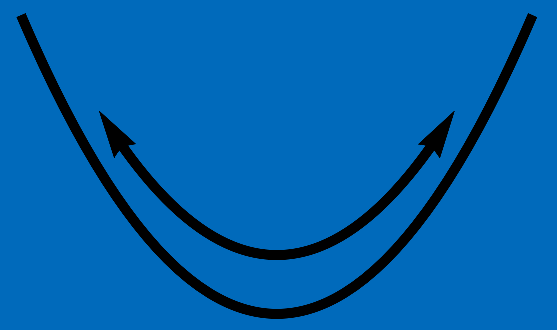
$$k_H(a) \equiv \left[\int_0^a \frac{1}{k_{\mathrm{FS}}(\tilde{a})}\frac{d\tilde{a}}{\tilde{a}}\right]^{-1}$$

$$w \simeq \frac{\delta P}{\delta\rho} = \frac{T_*^2}{3m_{\mathrm{DM}}^2}\frac{\langle q^2\rangle}{a^2}$$

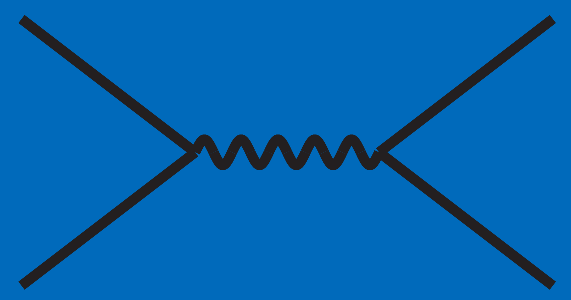
$$k_H(a=1) \simeq 3.5\,h\,\mathrm{Mpc}^{-1}$$

$$w_{\mathrm{WDM}}(a) \simeq 6\times 10^{-15}\,a^{-2}\left(\frac{\mathrm{keV}}{m_{\mathrm{WDM}}}\right)^{8/3}$$

1. Reheating



2. Freeze-in



3. Lyman-α

4. The end?

The perturbed Boltzmann equation

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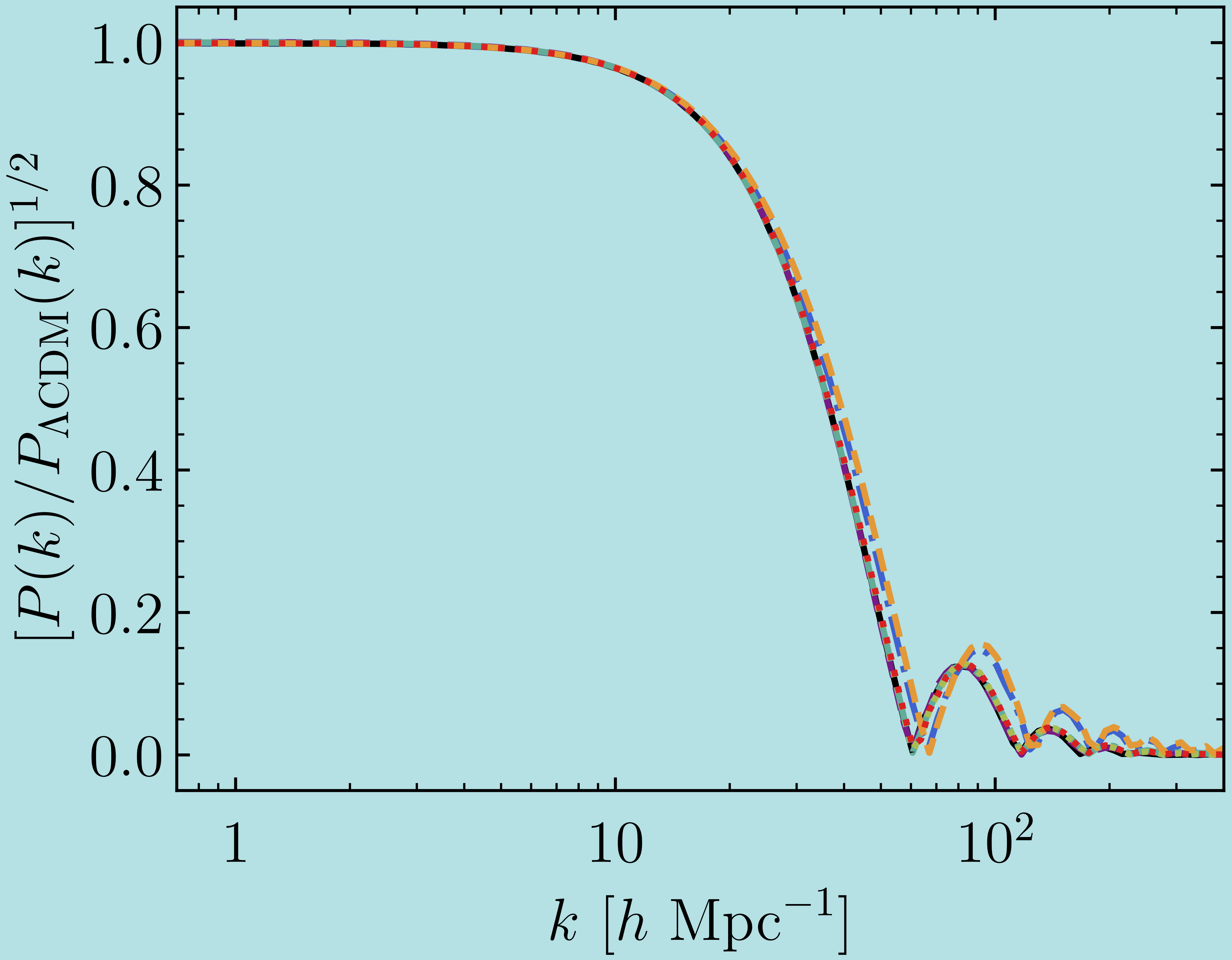

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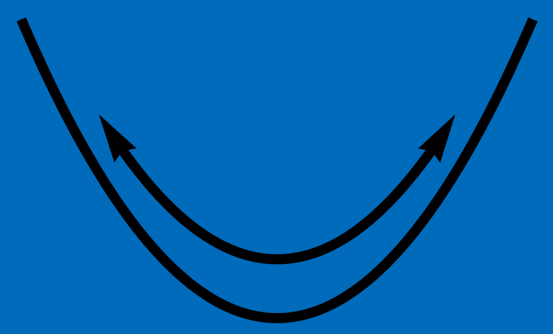
$$k_H(a) \equiv \left[\int_0^a \frac{1}{k_{\mathrm{FS}}(\tilde{a})}\frac{d\tilde{a}}{\tilde{a}}\right]^{-1}$$

$$w \simeq \frac{\delta P}{\delta \rho} = \frac{T_*^2}{3m_{\mathrm{DM}}^2}\frac{\langle q^2 \rangle}{a^2}$$

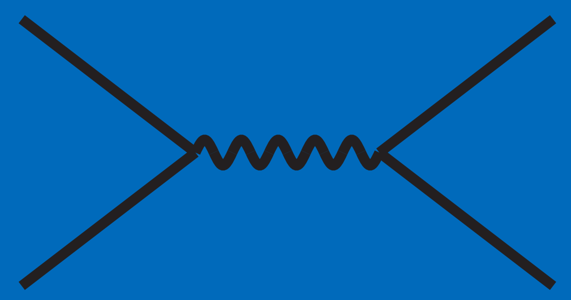
$$w(m_{\mathrm{DM}}) = w_{\mathrm{WDM}}(m_{\mathrm{WDM}})$$

$$m_{\mathrm{DM}} = m_{\mathrm{WDM}}\left(\frac{T_*}{T_{\mathrm{WDM}}}\right)\sqrt{\frac{\langle q^2 \rangle}{\langle q^2 \rangle_{\mathrm{WDM}}}}$$

1. Reheating



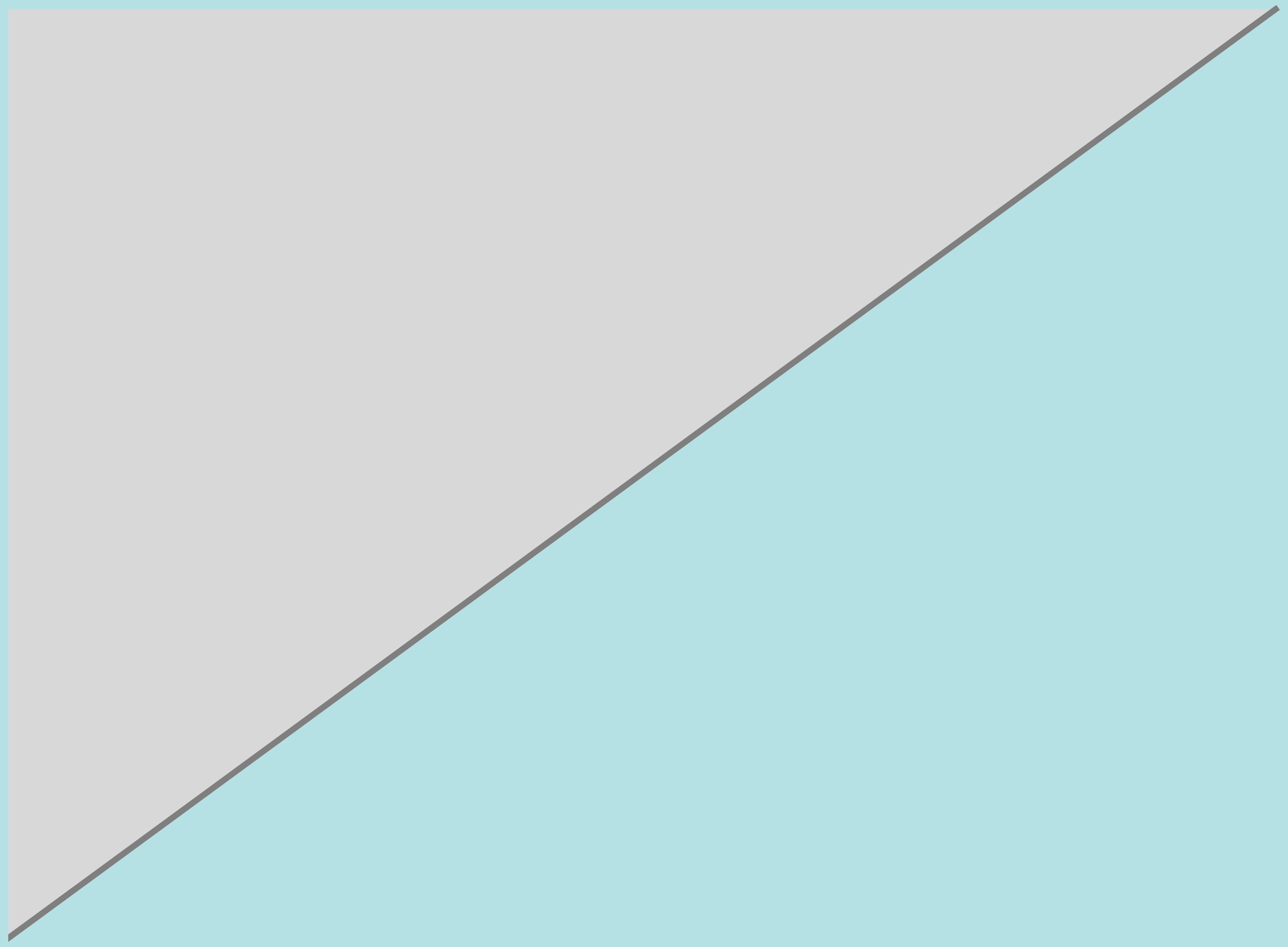
2. Freeze-in



3. Lyman-α

4. The end?

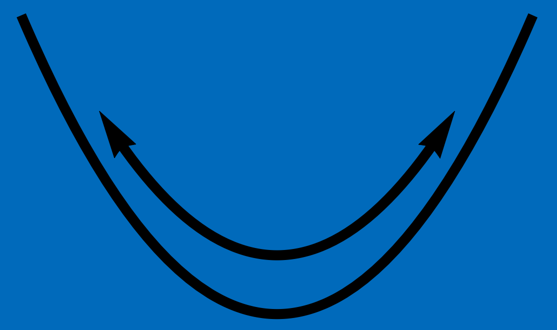
Constraints on dark matter from condensate decay (inflaton, moduli)



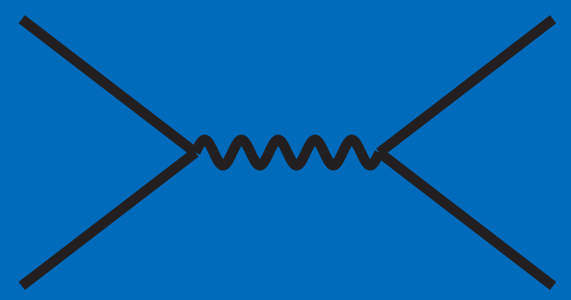
$$\mathrm{Br}_\chi \lesssim 1.5 \times 10^{-4} \left(\frac{g_{*s}^{\mathrm{reh}}}{106.5}\right)^{1/3} \left(\frac{3\,\mathrm{keV}}{m_{\mathrm{WDM}}}\right)^{4/3}$$

(absolute)

1. Reheating



2. Freeze-in



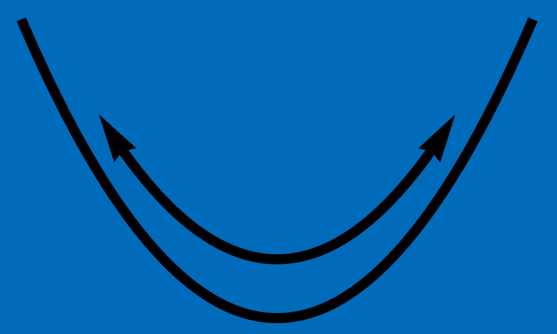
3. Lyman-α

4. The end?

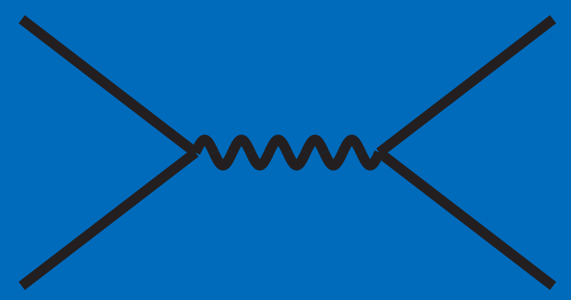
Constraints on dark matter from chain decay $\Phi \to A \to \chi$

$$m_{\rm DM} > \left(\frac{m_{\rm WDM}}{3\ \rm keV}\right)^{4/3} \times \begin{cases} 1.23\ {\rm MeV} \left(\frac{106.75}{g_{*s}^{\rm reh}}\right)^{1/3} \left(\frac{m_\Phi}{3\times 10^{13}\ \rm GeV}\right)\left(\frac{10^{10}\ \rm GeV}{T_{\rm reh}}\right), & \rm R \\ 2.7\,{\rm MeV} \left(\frac{106.75}{g_{*s}^{\rm dec}}\right)^{1/3} \left(\frac{m_A}{3\times 10^{13}\ \rm GeV}\right)\left(\frac{10^{10}\ \rm GeV}{T_{\rm dec}}\right), & \rm NR \end{cases}$$

1. Reheating



2. Freeze-in



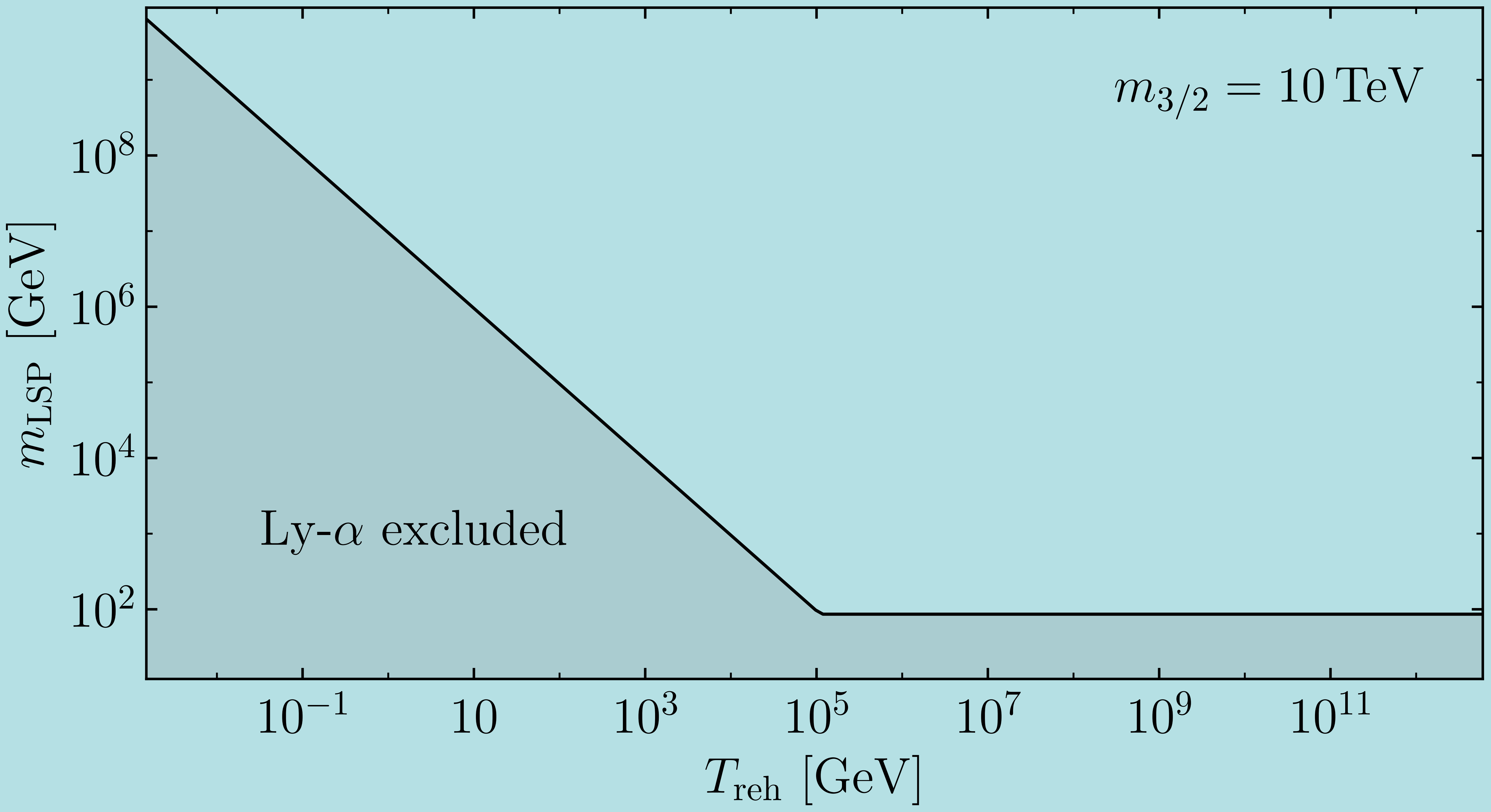
3. Lyman-α

4. The end?

Constraints on dark matter from chain decay

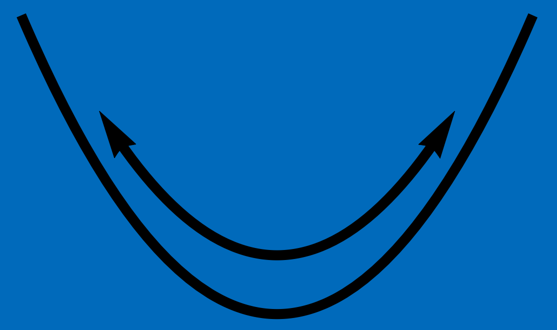
$$\Phi \to \Psi_{3/2} \to \mathrm{LSP}$$

$$m_{\mathrm{LSP}} > \begin{cases} 86\,\mathrm{GeV}\left(\dfrac{m_{\mathrm{WDM}}}{3\,\mathrm{keV}}\right)^{4/3}\left(\dfrac{10\,\mathrm{TeV}}{m_{3/2}}\right)^{1/2}, & T_{\mathrm{reh}} \gg 10^5\,\mathrm{GeV}\left(\dfrac{m_{3/2}}{10\,\mathrm{TeV}}\right)^{1/2} \\ 95\,\mathrm{GeV}\left(\dfrac{m_{\mathrm{WDM}}}{3\,\mathrm{keV}}\right)^{4/3}\left(\dfrac{10^5\,\mathrm{GeV}}{T_{\mathrm{reh}}}\right), & T_{\mathrm{reh}} \ll 10^5\,\mathrm{GeV}\left(\dfrac{m_{3/2}}{10\,\mathrm{TeV}}\right)^{1/2} \end{cases}$$

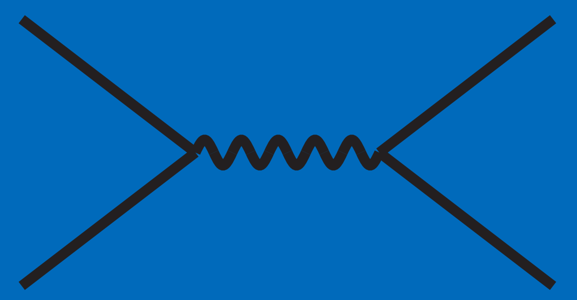


$$\Gamma_{3/2} = \frac{193}{384\pi}\frac{m_{3/2}^3}{M_P^2}$$

1. Reheating



2. Freeze-in



3. Lyman-α

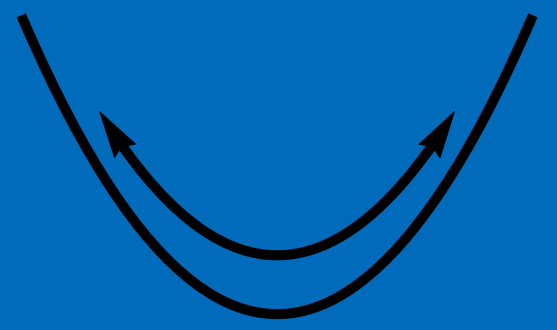
4. The end?

Constraints on dark matter from thermal freeze-in

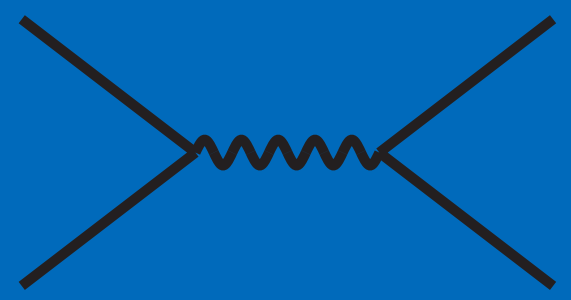
$$m_{\rm DM} > \left(\frac{m_{\rm WDM}}{3\ {\rm keV}}\right)^{4/3} \left(\frac{106.75}{g_{*s}^{\rm reh}}\right)^{1/3} \begin{cases} 7.3\ {\rm keV}\,, & (n=0\,,\ {\rm FF}) \\ 8.5\ {\rm keV}\,, & (n=2\,,\ {\rm FF}) \\ 8.5\ {\rm keV}\,, & (n=4\,,\ {\rm FF}) \end{cases}$$

$$m_{\rm DM} > \left(\frac{m_{\rm WDM}}{3\ {\rm keV}}\right)^{4/3} \left(\frac{106.75}{g_{*s}^{\rm reh}}\right)^{1/3} \begin{cases} 9\,{\rm keV}\ \ln^{-1/2}\left(\frac{T_{\rm max}}{T_{\rm reh}}\right)\,, & n=6\,, \\ 23\,{\rm keV}\left(\frac{T_{\rm reh}}{T_{\rm max}}\right)\,, & n=8\,, \\ 52\,{\rm keV}\left(\frac{T_{\rm reh}}{T_{\rm max}}\right)^{5/3}\,, & n\geq 10\,. \end{cases}$$

1. Reheating



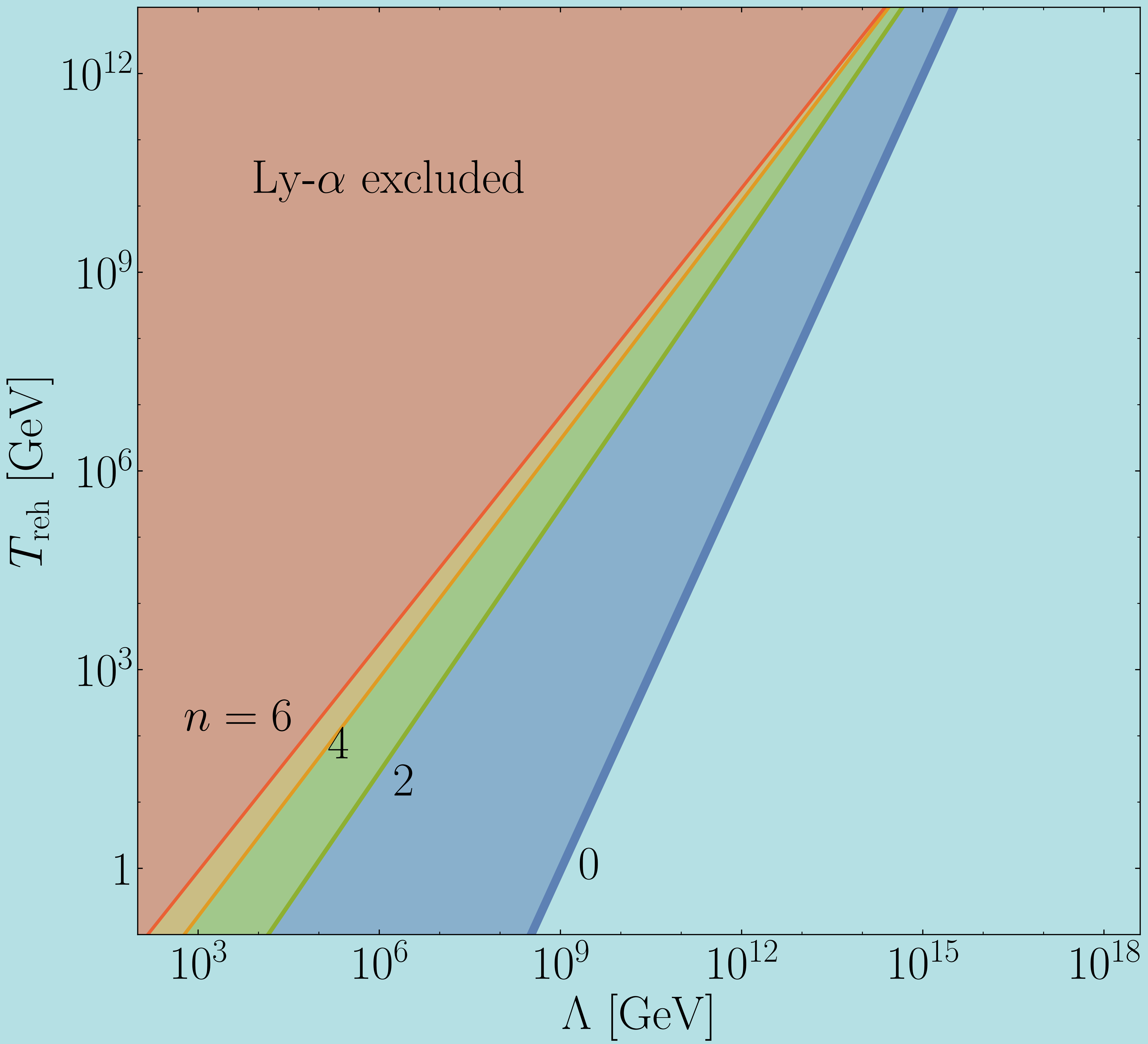
2. Freeze-in



3. Lyman-α

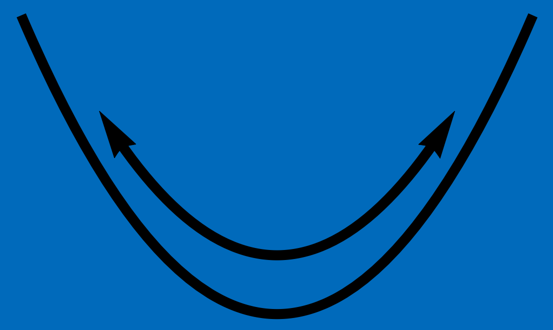
4. The end?

Constraints on dark matter from thermal freeze-in

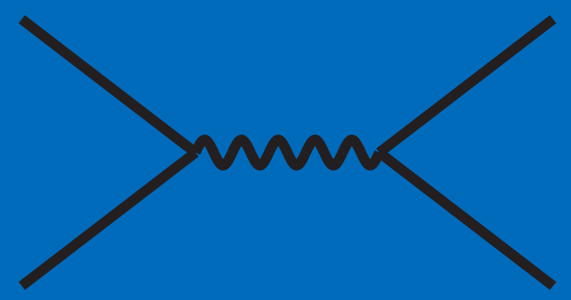


$$\sigma(s) = \frac{s^{n/2}}{\Lambda^{n+2}}$$

1. Reheating



2. Freeze-in



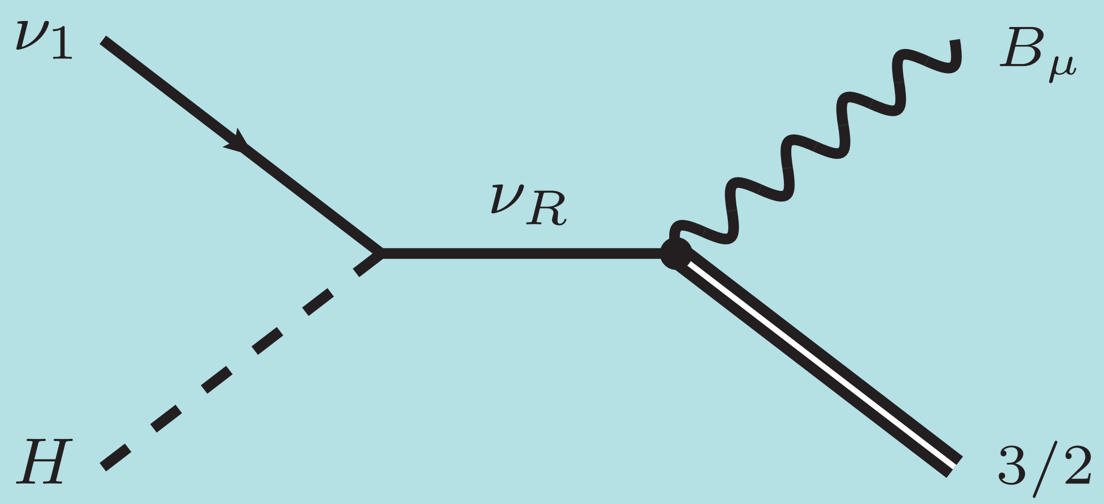
3. Lyman-α

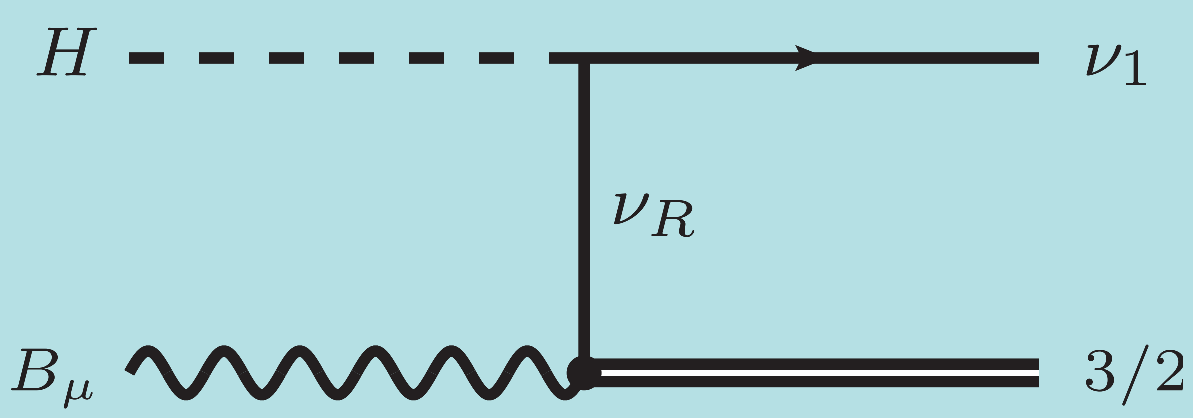
4. The end?

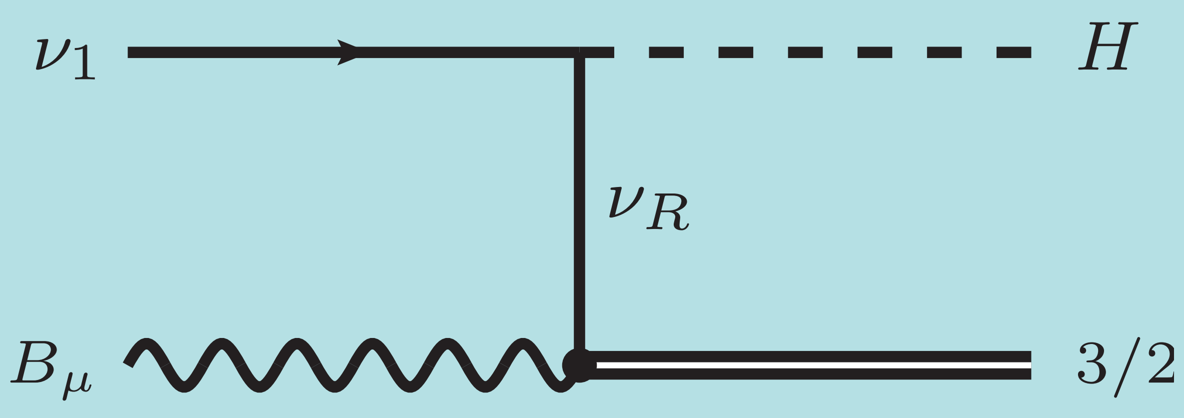
Non-thermal freeze-in: a non-supersymmetric decaying spin-3/2 particle

$$\mathcal{L} = \frac{i\alpha}{2M_P}\bar{\nu}_R\gamma^\mu[\gamma^\rho, \gamma^\sigma]\Psi_\mu F_{\rho\sigma} + \text{h.c.}$$

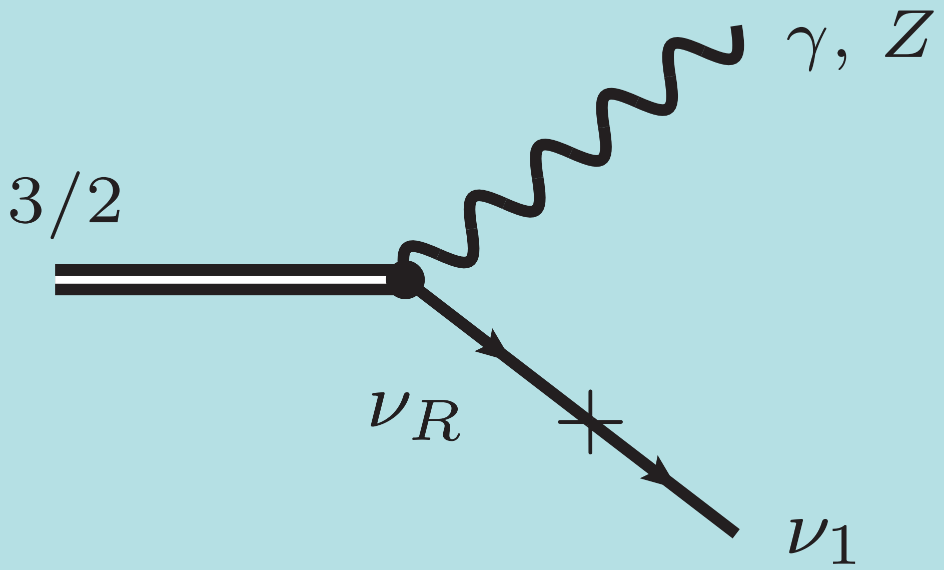
$n = 4$ freeze-in

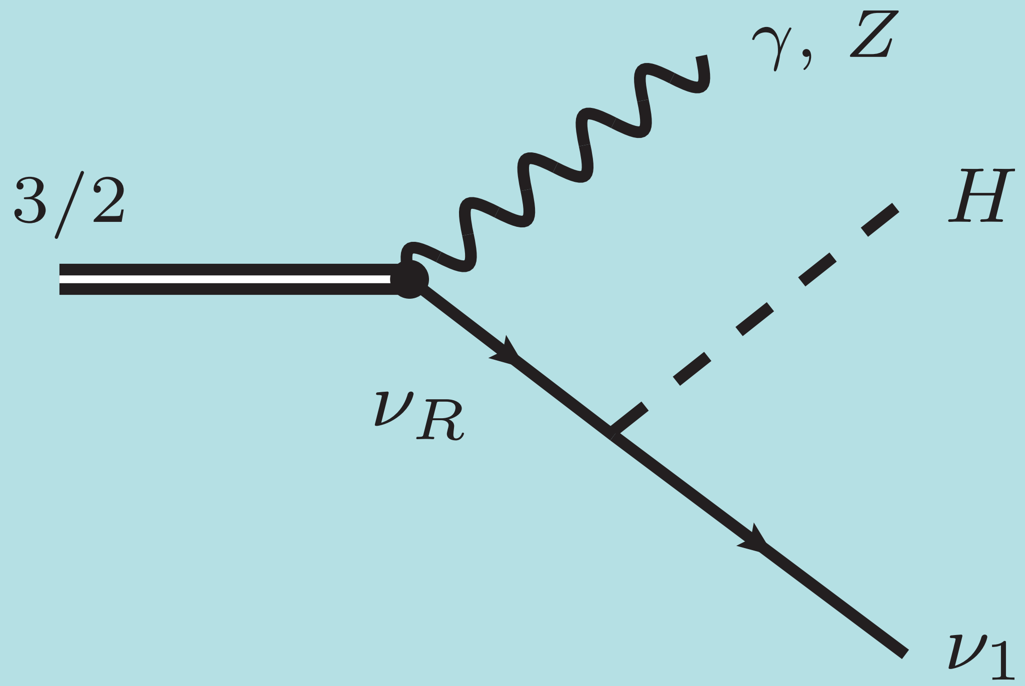




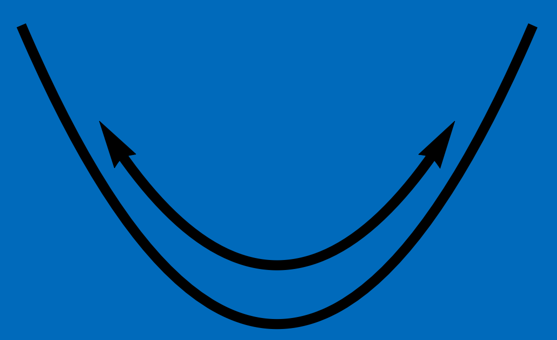


2 & 3-body decays

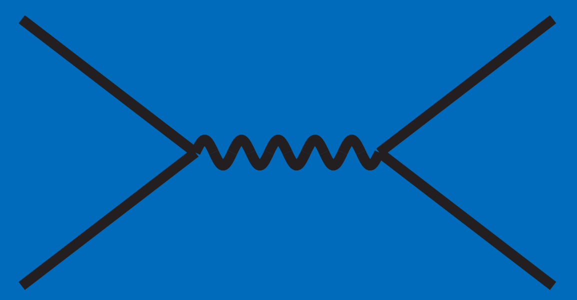




1. Reheating



2. Freeze-in



3. Lyman-α

4. The end?

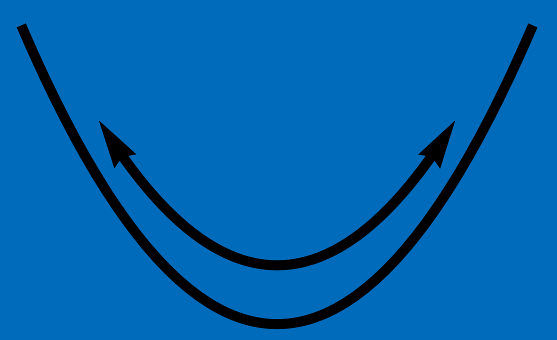
Non-thermal freeze-in: a non-supersymmetric decaying spin-3/2 particle

$$\Omega_{3/2}^{\text{thermal}} h^2 \simeq 0.1 \left(\frac{\alpha_1}{1.1\times 10^{-3}}\right)^2 \left(\frac{106.9}{g_{*s}^{\text{reh}}}\right)^{3/2} \left(\frac{T_{\text{reh}}}{10^{10}\,\text{GeV}}\right)^5 \left(\frac{m_\nu}{0.15\,\text{eV}}\right) \left(\frac{10^{14}\,\text{GeV}}{m_R}\right) \left(\frac{10^4\,\text{GeV}}{m_{3/2}}\right)$$

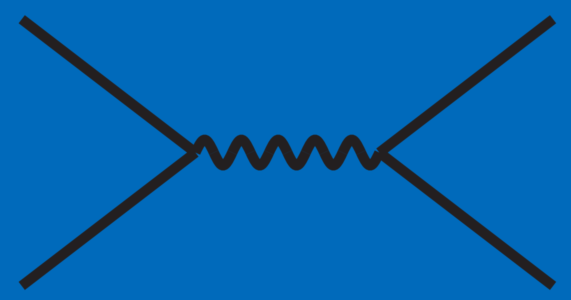
$$\Omega_{3/2}^{\text{non-thermal}} h^2 \simeq 0.1 \left(\frac{\alpha_1}{1.1\times 10^{-3}}\right)^2 \left(\frac{g_{*s}^{\text{reh}}}{106.5}\right)^{7/10} \left(\frac{0.030}{\alpha_{\text{SM}}}\right)^{16/5} \left(\frac{m_\Phi}{3\times 10^{13}\,\text{GeV}}\right)^{14/5} \times \left(\frac{m_\nu}{0.15\,\text{eV}}\right) \left(\frac{10^4\,\text{GeV}}{m_{3/2}}\right) \left(\frac{10^{14}\,\text{GeV}}{m_R}\right) \left(\frac{T_{\text{reh}}}{10^{10}\,\text{GeV}}\right)^{19/5} \left(\frac{\text{Br}_\nu}{7\times 10^{-4}}\right)$$

$$m_{3/2} > 0.44\ \text{keV} \left(\frac{m_{\text{WDM}}}{3\ \text{keV}}\right)^{4/3} \left(\frac{\alpha_{\text{SM}}}{0.03}\right)^{-32/15} \left(\frac{c}{3/5}\right)^{2/15} \times \left(\frac{106.75}{g_{*s}^{\text{reh}}}\right)^{1/5} \left(\frac{10^{10}\ \text{GeV}}{T_{\text{reh}}}\right)^{7/15} \left(\frac{m_\Phi}{3\times 10^{13}\ \text{GeV}}\right)^{23/15}$$

1. Reheating



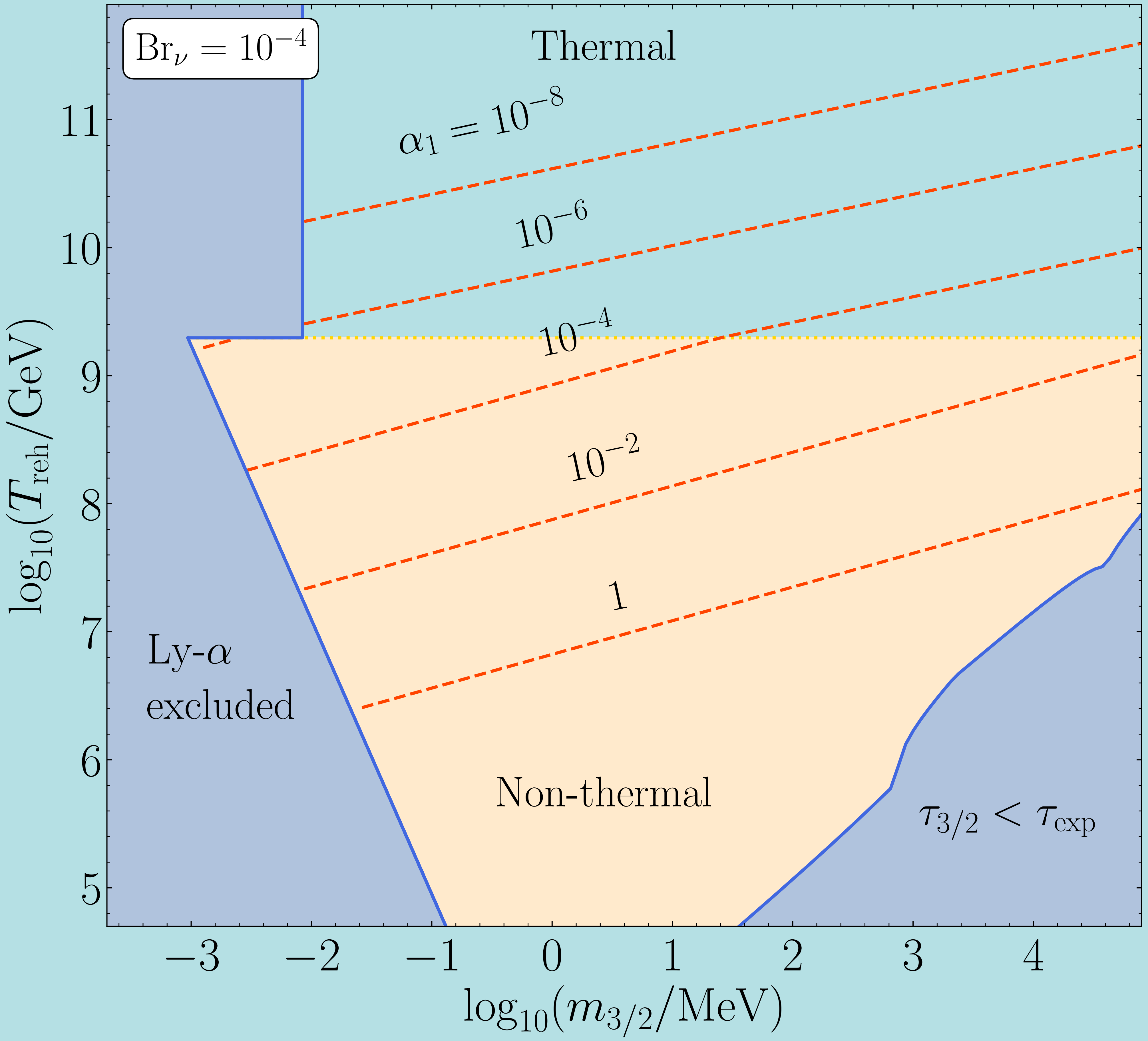
2. Freeze-in

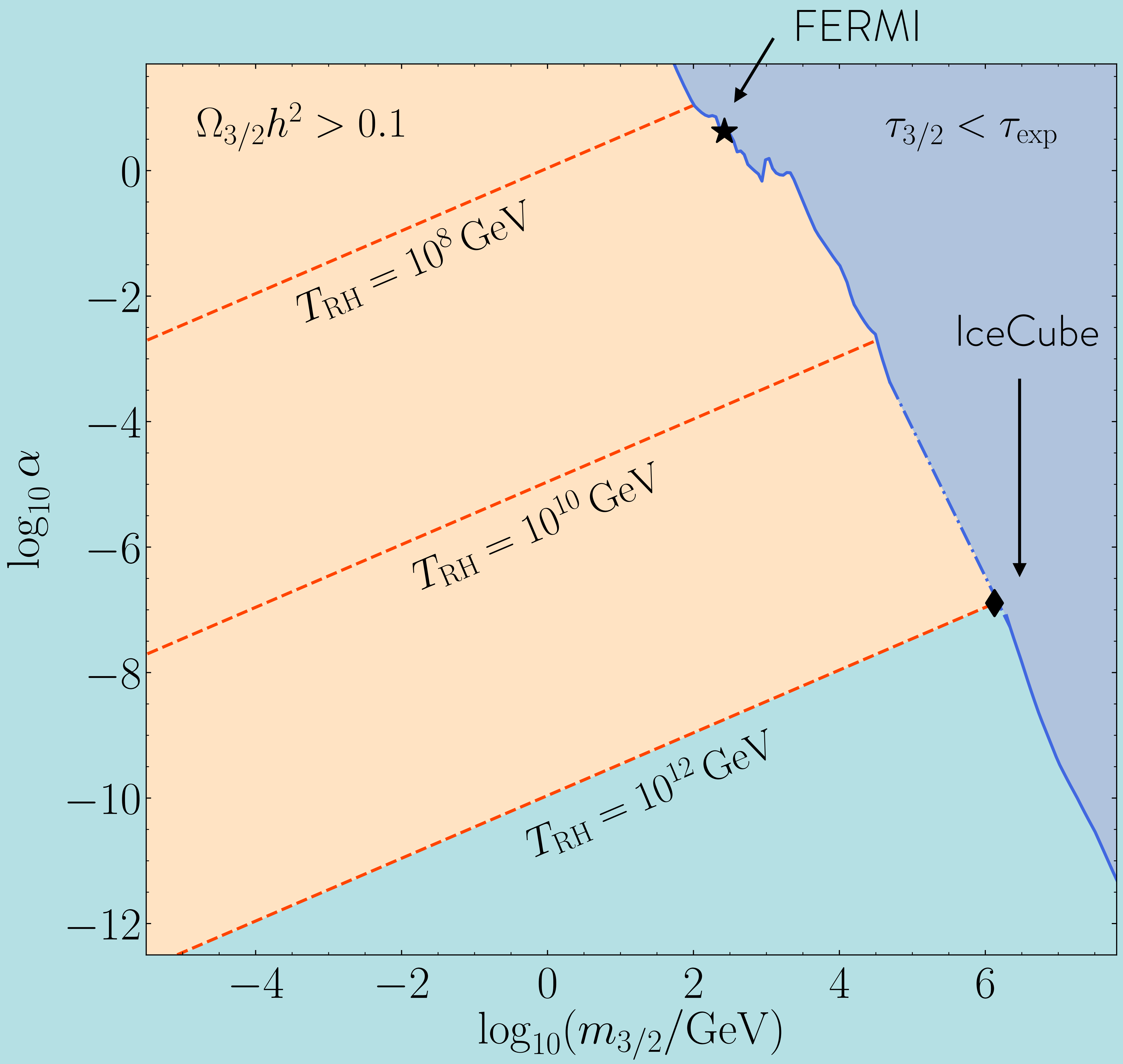


3. Lyman-α

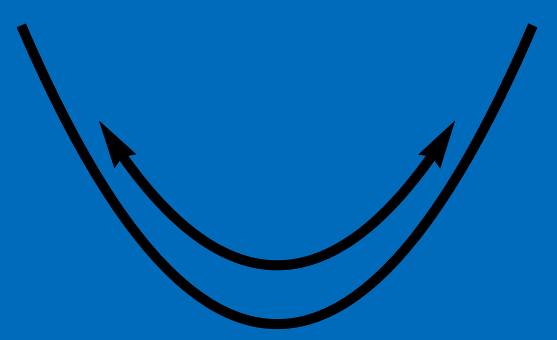
4. The end?

Non-thermal freeze-in: a non-supersymmetric decaying spin-3/2 particle

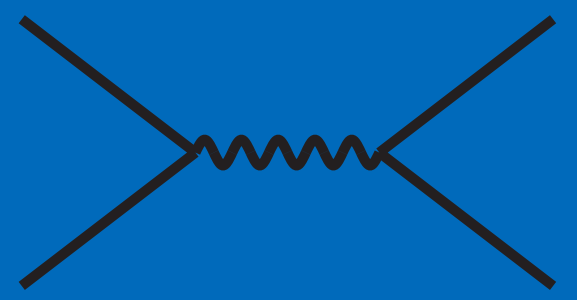




1. Reheating



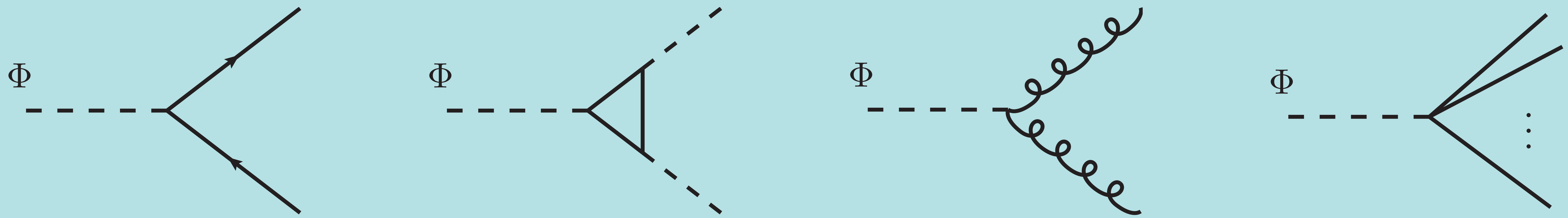
2. Freeze-in



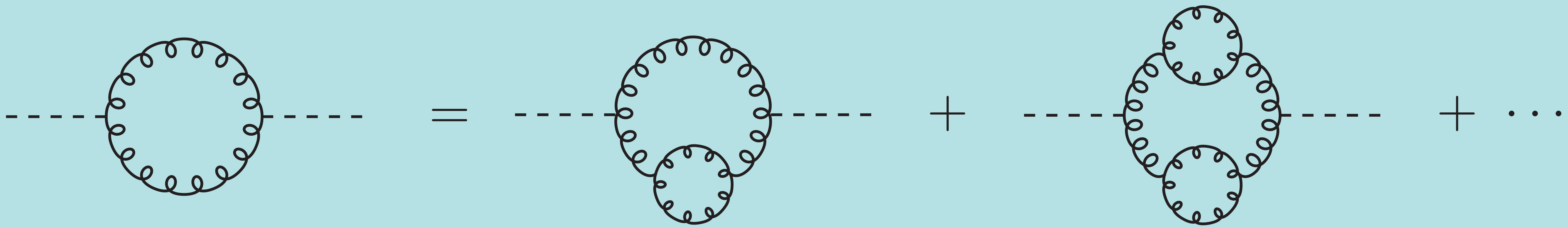
3. Lyman-α

4. The end?

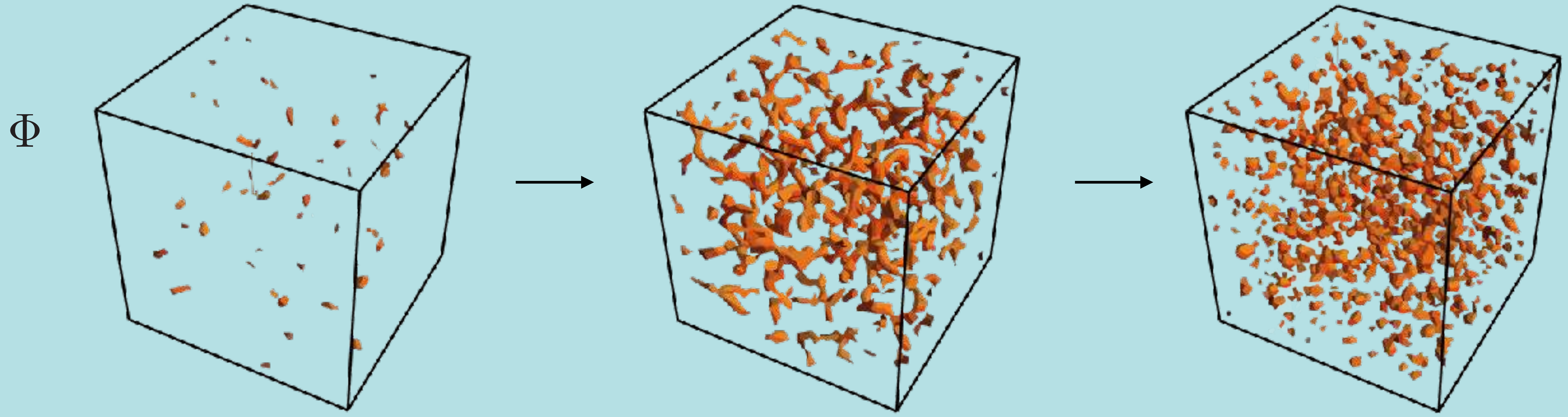
For $k \neq 2$ the nature of the final state matters



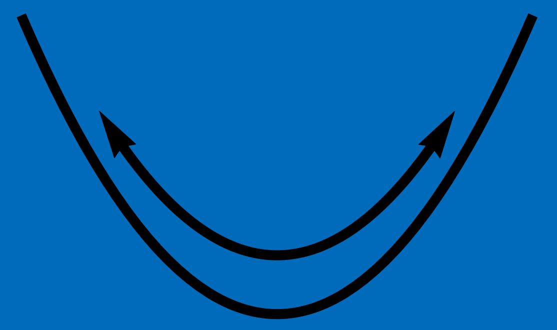
In-medium effects cannot be neglected



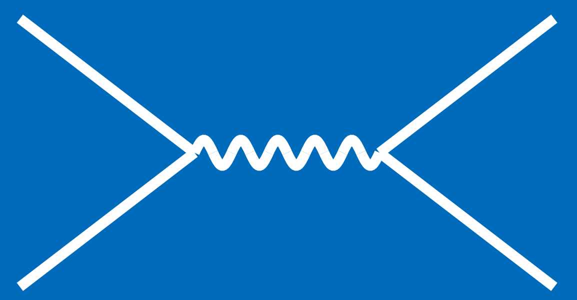
Interactions fragment homogeneous fields



1. Reheating



2. Freeze-in



3. Lyman-α

4. The end?

Out-of-equilibrium decay chain $\Phi \to A \to \chi$

$$\Omega_\chi h^2 \simeq 0.1 \left(\frac{\mathrm{Br}_\chi}{5.5\times 10^{-4}}\right)\left(\frac{m_{\mathrm{DM}}}{1\,\mathrm{MeV}}\right)\left(\frac{T_{\mathrm{reh}}}{10^{10}\,\mathrm{GeV}}\right)\left(\frac{3\times 10^{13}\,\mathrm{GeV}}{m_\Phi}\right)$$

$$\frac{\partial f_A}{\partial t} - Hp\frac{\partial f_A}{\partial p} = -\frac{m_A \Gamma_A}{\sqrt{m_A^2+p^2}} f_A$$

$$t_{\mathrm{dec}} \simeq \begin{cases} \Gamma_A^{-1}\,, & \dfrac{\Gamma_A}{H_A} \ll 1\,, \\ \left(\dfrac{m_\phi \langle q_A\rangle}{2 m_A \Gamma_A \Gamma_\Phi^{1/2}}\right)^{2/3}, & \dfrac{\Gamma_A}{H_A} \gg 1\,. \end{cases}$$

$$\mathcal{C}[f_\chi(p,t)] = \frac{4\pi^4 g_{*s}^{\mathrm{reh}} \mathrm{Br}_\chi \mathrm{Br}_A \Gamma_A m_A}{5 g_A p^2}\left(\frac{T_{\mathrm{reh}}}{m_\phi}\right)^4 \left(\frac{m_\Phi}{2}\right)\left(\frac{a_{\mathrm{reh}}}{a(t)}\right) \times \int_{\left|\frac{2p}{m_\Phi}\frac{a(t)}{a_{\mathrm{reh}}} - \frac{m_A^2}{2pm_\Phi}\frac{a(t)}{a_{\mathrm{reh}}}\right|}^{\infty} \frac{z\,dz}{\sqrt{z^2+\left(\frac{2m_A a(t)}{m_\Phi a_{\mathrm{reh}}}\right)^2}}\,\bar{f}_{\mathrm{R}}(z)$$