Pre-thermalization Production of Dark Matter

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1806.01865, MG, M. Amin

THE 14TH INTERNATIONAL WORKSHOP ON THE DARK SIDE OF THE UNIVERSE

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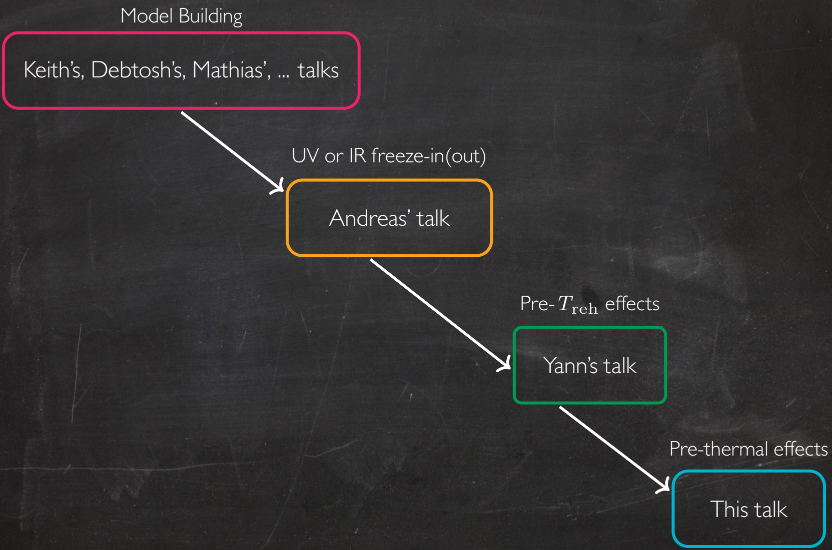
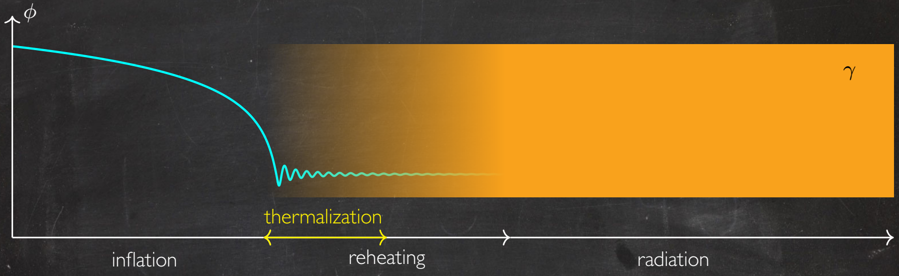
The Path To Thermalization



Freeze-in vs. Freeze-out

The reheating and thermalization processes after inflation have a finite duration



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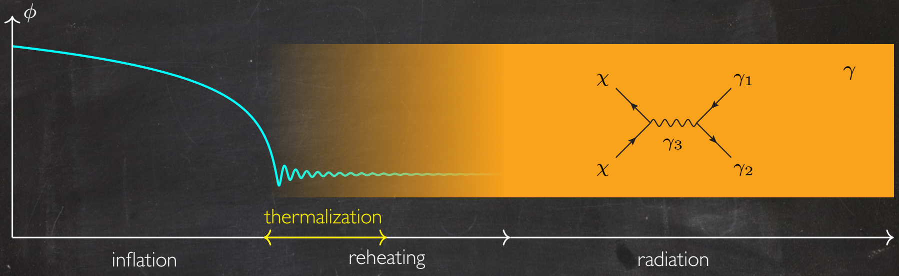
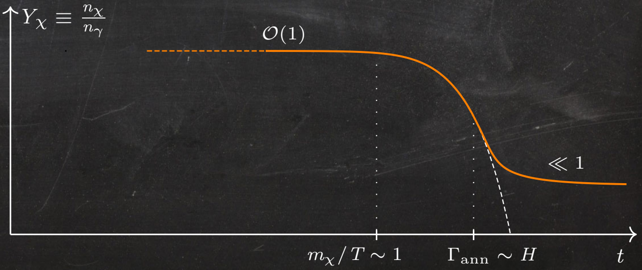


Freeze-out:

- Thermal equilibrium
- Thermal production
- $Y_\chi \longleftrightarrow \langle \sigma v \rangle_{\rm ann}$
- IR dominated



Freeze-in vs. Freeze-out

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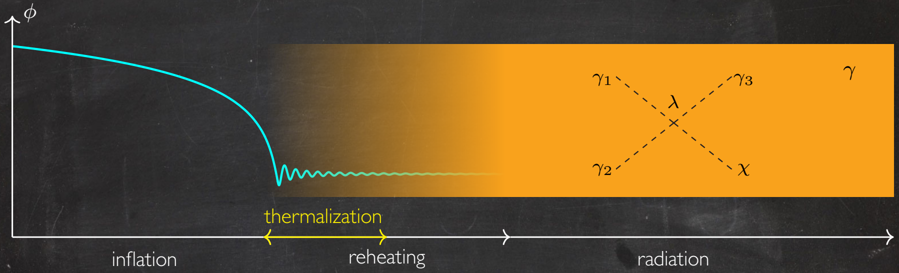
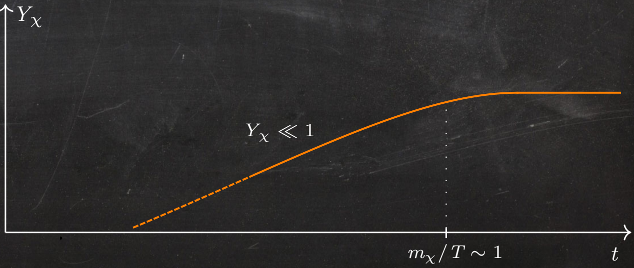


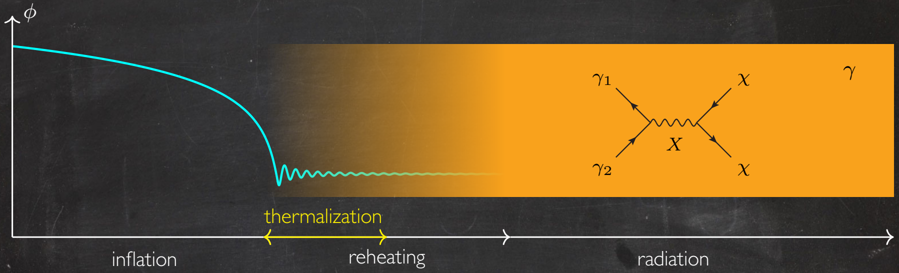
Freeze-in:

- Not in equilibrium
- Thermal production
- $Y_\chi \longleftrightarrow \langle \sigma v \rangle_{\rm prod}$
- IR dominated



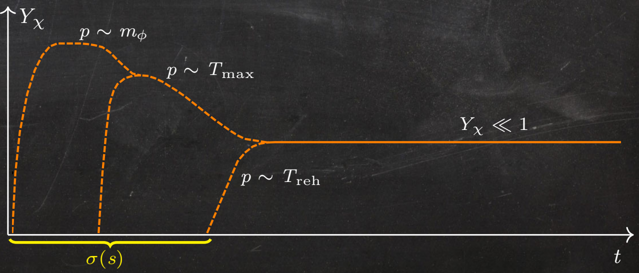
Freeze-in vs. Freeze-out

The reheating and thermalization processes after inflation have a finite duration



Freeze-in:

- Not in equilibrium
- Non-thermal and thermal production
- $Y_\chi \longleftrightarrow \langle \sigma v \rangle_{\rm prod}$
- UV dominated



Pre-thermalization

$$\dot{\rho}_\phi + 3H\rho_\phi + \Gamma_\phi \rho_\phi = 0$$
$$\dot{\rho}_\gamma + 4H\rho_\gamma - \Gamma_\phi \rho_\phi = 0$$
$$\rho_\phi + \rho_\gamma = 3M_P^2 H^2$$

$$\downarrow$$

$$n_\gamma \simeq \frac{\rho_{\rm end}}{m_\phi}\left(\frac{a}{a_{\rm end}}\right)^{-3}\left(1 - e^{-\Gamma_\phi t}\right)$$
$$< g\, n_\gamma^T \sim g\, \rho_\gamma^{3/4}$$

If $\Gamma_\phi / m_\phi \lesssim 10^{-10}$ (Planck suppressed)

Pre-thermalization

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$$< g\, n_\gamma^T \sim g\, \rho_\gamma^{1/4}$$

If $\Gamma_\phi / m_\phi \lesssim 10^{-10}$ (Planck suppressed)

$$f_\phi = (2\pi)^3 n_\phi \delta^3(\mathbf{p})$$

$$\downarrow$$

$$\frac{\partial f_\gamma}{\partial t} - Hk\frac{\partial f_\gamma}{\partial k} = \frac{2\pi^2}{k^2} n_\phi \Gamma_\phi \delta(k - m_\phi/2)$$

$$\downarrow$$

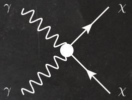
$$f_\gamma(k) \simeq 24\pi^2 \frac{n_\gamma}{m_\phi^3}\left(\frac{m_\phi}{2k}\right)^{3/2} \theta(m_\phi/2 - k)$$

$$\downarrow$$

$$\langle k \rangle \sim m_\phi$$

Pre-thermalization

$$f_\gamma(k) \simeq 24\pi^2 \frac{n_\gamma}{m_\phi^3}\left(\frac{m_\phi}{2k}\right)^{3/2}\theta(m_\phi/2 - k)$$



$$\frac{\partial f_\chi}{\partial t} - Hp_1\frac{\partial f_\chi}{\partial p_1} = -\frac{1}{2p_1}\int \frac{g_\chi\, d^3\mathbf{p}_2}{(2\pi)^3 2p_2}\frac{g_\gamma\, d^3\mathbf{k}_1}{(2\pi)^3 2k_1}\frac{g_\gamma\, d^3\mathbf{k}_2}{(2\pi)^3 2k_2}(2\pi)^4\delta^{(4)}(k_1+k_2-p_1-p_2)$$

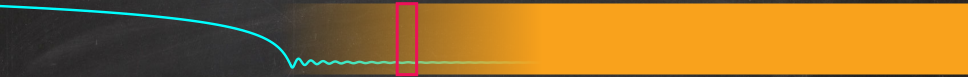
$$\times\left[\,|\mathcal{M}|^2_{\chi\chi\to\gamma\gamma}\ \cancel{f_\chi(p_1)\,f_\chi(p_2)}\ [1+f_\gamma(k_1)]\,[1+f_\gamma(k_2)]\right.$$

$$\left. -\,|\mathcal{M}|^2_{\gamma\gamma\to\chi\chi}\ f_\gamma(k_1)\,f_\gamma(k_2)\,[1-\cancel{f_\chi(p_1)}]\,[1-\cancel{f_\chi(p_2)}]\right]$$

$$\downarrow$$

$$\dot n_\chi + 3Hn_\chi = 18 g_\chi^2 g_\gamma^2\frac{n_\gamma^2}{m_\phi^3}\int_0^{m_\phi^2} ds\,\sqrt{s}\,\sigma(s)\left[\ln\left(\frac{m_\phi+\sqrt{m_\phi^2-s}}{\sqrt{s}}\right) - \frac{\sqrt{m_\phi^2-s}}{m_\phi}\right]$$

Thermalization



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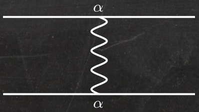
Gauge-interacting γ problematic for small angle scattering

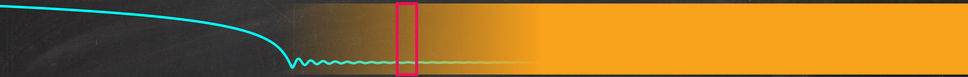
$\neq 0$



$t-$divergent

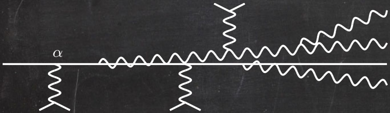
(L. Landau, I. Pomeranchuk, Dokl. Akad. Nauk Ser. Fiz. 92 (1953) 535; A. Migdal, Phys. Rev. 103 (1956) 1811)

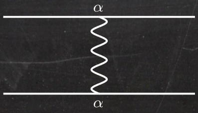
Thermalization



$$\dot{n}_\chi + 3Hn_\chi \;=\; 18 g_\chi^2 g_\gamma^2 \frac{n_\gamma^2}{m_\phi^3}\int_0^{m_\phi^2} ds\,\sqrt{s}\,\sigma(s)\left[\ln\left(\frac{m_\phi+\sqrt{m_\phi^2-s}}{\sqrt{s}}\right)-\frac{\sqrt{m_\phi^2-s}}{m_\phi}\right]$$

Gauge-interacting γ equilibrate through small angle scattering





Including LPM suppression

$$t_\gamma \sim \sqrt{\frac{\tau E}{q_\perp^2}}$$

Elastic screening scale

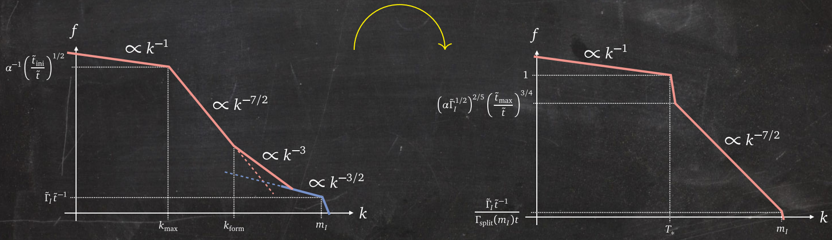
$$m_s^2 \sim \alpha \int d^3k\;\, f_\gamma(k)/k$$

(P. Arnold, G. Moore, L. Yaffe, hep-ph/0111107; hep-ph/0204343; hep-ph/0209353)

Thermalization

$$\dot{n}_\chi + 3Hn_\chi = 18 g_\chi^2 g_\gamma^2 \frac{n_\gamma^2}{m_\phi^3} \int_0^{m_\phi^2} ds\, \sqrt{s}\, \sigma(s) \left[\ln\left(\frac{m_\phi + \sqrt{m_\phi^2 - s}}{\sqrt{s}} \right) - \frac{\sqrt{m_\phi^2 - s}}{m_\phi} \right]$$

Gauge-interacting γ equilibrate through small angle scattering



(K. Harigaya, K. Mukaida, 1312.3097; K. Mukaida, M. Yamada, 1312.3097)

Thermalization

$$\dot{n}_\chi + 3Hn_\chi = 18 g_\chi^2 g_\gamma^2 \frac{n_\gamma^2}{m_\phi^3}\int_0^{m_\phi^2} ds\,\sqrt{s}\,\sigma(s)\left[\ln\left(\frac{m_\phi + \sqrt{m_\phi^2 - s}}{\sqrt{s}}\right) - \frac{\sqrt{m_\phi^2 - s}}{m_\phi}\right]$$

Gauge-interacting γ equilibrate through small angle scattering

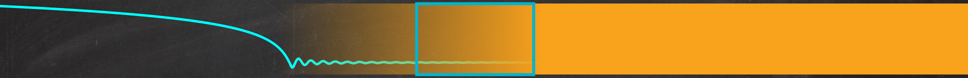
$$\Gamma_\phi t_{\rm th} \simeq \alpha^{-16/5}\left(\frac{\Gamma_\phi m_\phi^2}{M_P^3}\right)^{2/5} \sim 10^{-6,-7}$$

$$\downarrow$$

$$T_{\rm max} \simeq \alpha^{4/5} m_\phi \left(\frac{24}{\pi^2 g_{\rm reh}}\right)^{1/4}\left(\frac{\Gamma_\phi M_P^2}{m_\phi^3}\right)^{2/5}$$

(K. Harigaya, K. Mukaida, 1312.3097; K. Mukaida, M. Yamada, 1312.3097)

Post-thermalization



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$$\Gamma_\phi t_{\rm th} \simeq \alpha^{-16/5} \left(\frac{\Gamma_\phi m_\phi^2}{M_P^3} \right)^{2/5} \ll 1$$

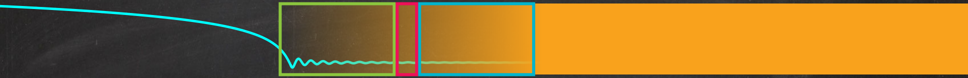
$$\dot{n}_\chi + 3Hn_\chi = \frac{g_\chi^2 g_\gamma^2}{8\pi^4} \int dk_1\, dk_2\, d\cos\theta_{12} \frac{(k_1 k_2)^2 (1 - \cos\theta_{12})}{(e^{k_1/T} \pm 1)(e^{k_2/T} \pm 1)} \sigma(s)$$

$$\downarrow \text{M.B.}$$

$$\simeq \frac{g_\chi^2 g_\gamma^2\, T}{2(2\pi)^4} \int_0^\infty ds\, s^{3/2} \sigma(s) K_1(\sqrt{s}/T)$$

(P. Gondolo, G. Gelmini, Nucl. Phys. B360 (1991) 145)

Non-thermal vs. thermal production



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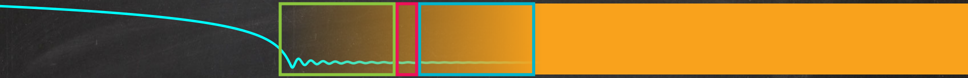
Non-thermal vs. thermal production



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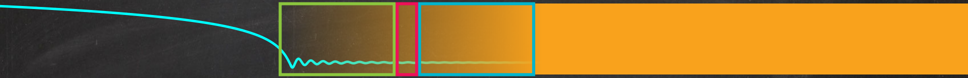
Non-thermal vs. thermal production



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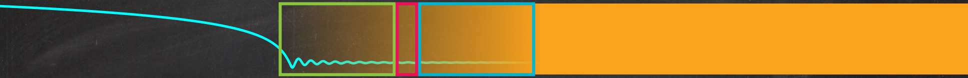
$$\sigma(s) \propto \frac{s^{n/2}}{M^{n+2}}$$

Thermal:

$$Y_\chi^{\rm T}(T_{\rm reh}) \propto \frac{M_P T_{\rm reh}^7}{g_{\rm reh}^{1/2} M^{n+2}} \times \begin{cases} \frac{1}{n-6}\left(T_{\rm max}^{n-6} - T_{\rm reh}^{n-6}\right), & n > -1\,,\; n \neq 6 \\ \ln\left(\frac{T_{\rm max}}{T_{\rm reh}}\right), & n = 6 \end{cases}$$

Non-thermal vs. thermal production



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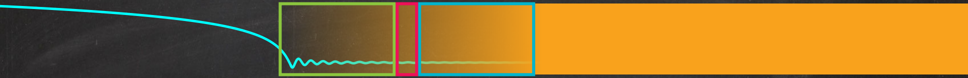
$$\sigma(s) \propto \frac{s^{n/2}}{M^{n+2}}$$

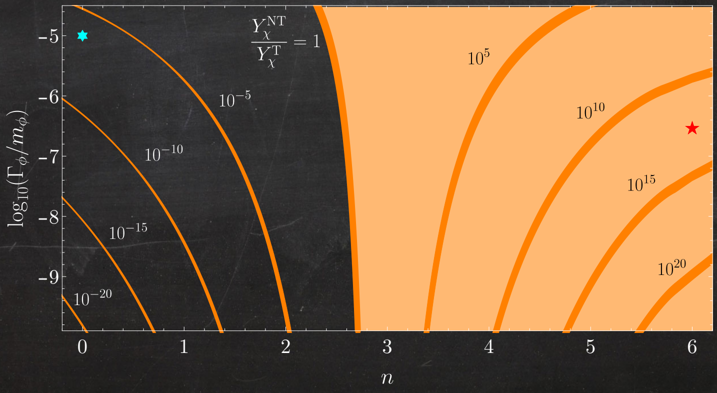
Non-thermal:

$$Y_\chi^{\rm NT}(T_{\rm reh}) \propto g_{\rm reh}^{3/2} \frac{T_{\rm reh}^3 M_P m_\phi^{n-2}}{M^{n+2}} (\Gamma_\phi t_{\rm th})$$

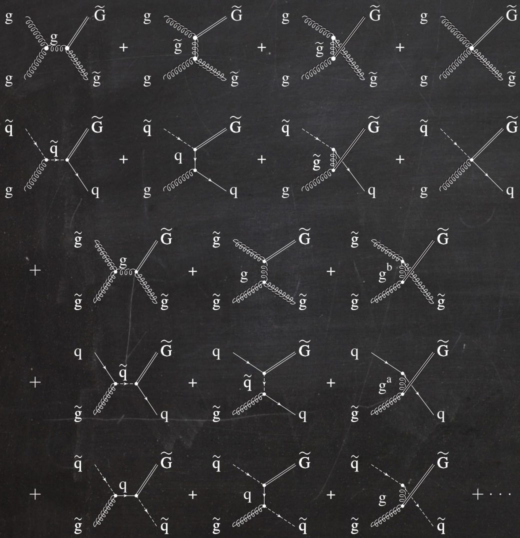
Non-thermal vs. thermal production



$$\sigma(s) \propto \frac{s^{n/2}}{M^{n+2}}$$



$\phi \to g + g$ and weak scale supersymmetry



$$\langle \sigma v \rangle_{\rm NT} = \sum_{i=1}^{3} \frac{16\pi\alpha_i}{M_P^2} |f^{abc}|^2 \left(1 + \frac{m_{\tilde{g}_i}^2}{3m_{3/2}^2}\right)$$

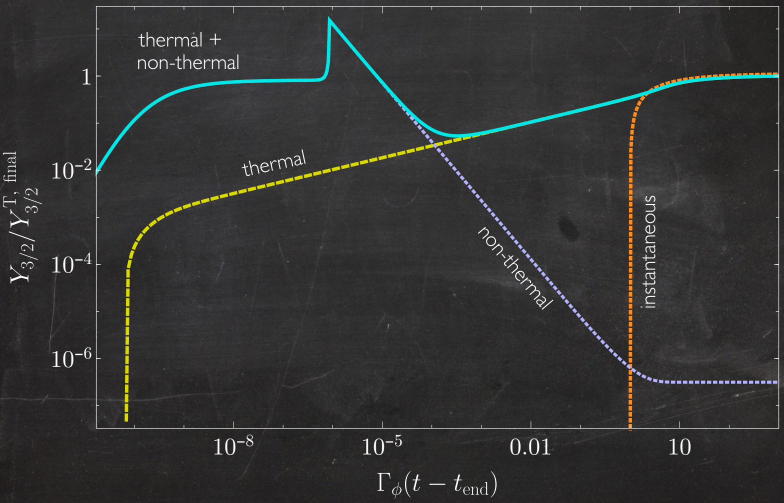
$$\langle \sigma v \rangle_{\rm T} = \sum_{i=1}^{3} \frac{3\pi^2 c_i \alpha_i}{4\zeta(3) M_P^2} \left(1 + \frac{m_{\tilde{g}_i}^2}{3m_{3/2}^2}\right) \ln\left(\frac{k_i}{g_i}\right)$$

(M. Bolz et. al., hep-ph/0012052)

(V. Rychkov, A. Strumia, hep-ph/0701104)

Light Gravitino

$\phi \to g + g$ and weak scale supersymmetry

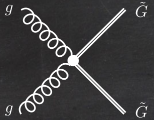


Heavy Gravitino

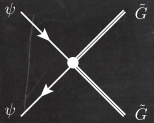
High scale supersymmetry. Only susy state below the inflationary scale is the gravitino

Leading-order universal Goldstino-matter interactions $(F = \sqrt{3} m_{3/2} M_P)$:

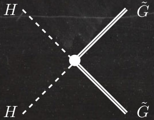
$$\mathcal{L}_{2G} = \frac{i}{2F^2}\left(G\sigma^\mu \partial^\nu \bar{G} - \partial^\nu G \sigma^\mu \bar{G}\right) T_{\mu\nu}$$



$(F_\mu^{\lambda\, a} F_{\nu\lambda}^a)$



$(\bar{\psi}\bar{\sigma}_\mu \partial_\nu \psi + \cdots)$



$(\partial_\mu H \partial_\nu H^\dagger + \mathrm{h.c.})$

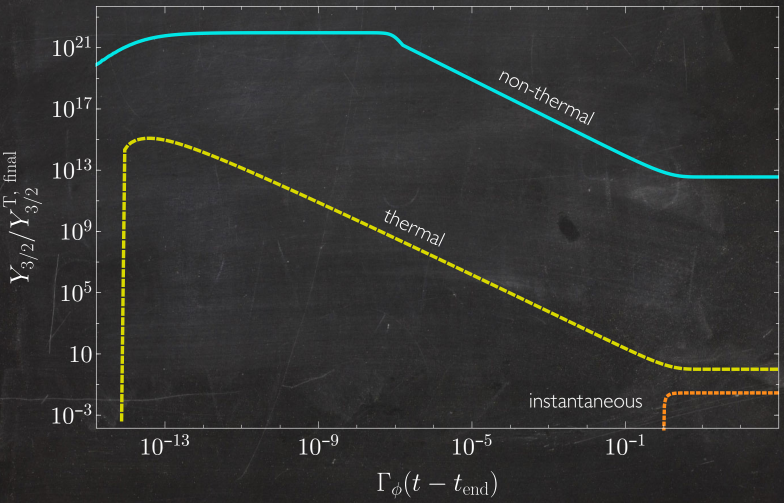
$$\langle \sigma v \rangle_{\mathrm{NT}} = \frac{154 m_\phi^6}{5(64)^2 F^4}$$

$$\langle \sigma v \rangle_{\mathrm{T}} = \frac{6400 \pi^{11} T^6}{(945)^2 \zeta(3)^2 F^4}$$

(E. Dudas, Y. Mambrini, K. Olive, 1704.03008)
(K. Benakli et. al., 1701.06574)

Heavy Gravitino

High scale supersymmetry. Only susy state below the inflationary scale is the gravitino



Heavy Gravitino

High scale supersymmetry. Only susy state below the inflationary scale is the gravitino

Assuming instantaneous reheating and thermalization...

$$\Omega_{3/2}^{\text{inst}} h^2 \simeq 0.11 \left(\frac{0.1\,\text{EeV}}{m_{3/2}} \right)^3 \left(\frac{T_{\text{reh}}}{2.2 \times 10^{10}} \right)^7$$

vs. accounting for their finite duration...

$$\Omega_{3/2} h^2 \simeq 0.11 \left(\frac{0.1\,\text{EeV}}{m_{3/2}} \right)^3 \left(\frac{T_{\text{reh}}}{2.2 \times 10^{8}} \right)^{19/5} \left(\frac{m_\phi}{3 \times 10^{13}\,\text{GeV}} \right)^{24/5} \left(\frac{0.030}{\alpha_3} \right)^{16/5}$$

(similar analysis applies to DM production through heavy spin-2 mediators, N. Bernal et. al. 1803.01866)

Freezing-in dark matter through a heavy invisible Z'

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[a] Saha Institute of Nuclear Physics, HBNI, 1/AF Bidhan Nagar, Kolkata 700064, India
[b] Laboratoire de Physique Théorique (UMR8627), CNRS,
Univ. Paris-Sud, Université Paris-Saclay, 91405 Orsay, France

(1806.00016 [hep-ph])

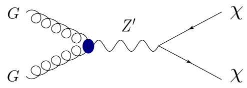

FIG. 1: *Production of dark matter through gluon fusion in the early Universe*

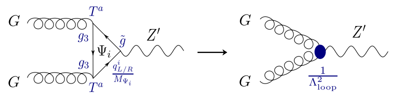

FIG. 4: *Triangle diagram generated containing heavy chiral fermions Ψ_i (left panel), and the resulting effective vertex at low energy (right panel).*

$$\langle \sigma v \rangle n_\gamma^2 = \quad R(T) \approx \begin{cases} 2\times 10^2 \, \frac{\alpha^2}{\Lambda^4}\frac{m_\chi^2}{M_{Z'}^4} T^{10} & \text{(fermionic DM)} \quad \to n = 4 \\ 10^4 \, \frac{\beta^2}{\Lambda^4 M_{Z'}^4} T^{12} & \text{(abelian DM)} \quad \to n = 6 \\ 2\times 10^9 \, \frac{\gamma^2}{\Lambda^4 M_{Z'}^4} T^{16} & \text{(non-abelian DM)} \quad \to n = 10 \end{cases} \tag{14}$$

Conclusion

- UV-dominated freeze-in during reheating is realized for $\sigma(s) \sim s^{n/2}$, $n > 2$
- Thermalization time-scale determines the DM abundance at late times
- Effect important for DM production in very high scale susy models, or for heavy spin-2 mediators. Other models?
- Preheating? $N_{\rm eff}$?

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Thank you